# MOGA OPTIMIZATION OF SUPERCONDUCTING LONGITUDINAL GRADIENT BEND BASED ON NbTi WIRE* 

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## Abstract

Multi-bend achromat lattices with unit cells have been used in diffraction-limited storage ring designs. The longitudinal gradient bend can reduce the horizontal emittance below the theoretical minimum of a given magnet structure, and generally the horizontal emittance reduces with the peak field grows. Therefore superconducting longitudinal gradient bend (SLGB) can produce higher peak field value and quasi-hyperbolic field profile to minimize emittance at location of radiation and generate better hard X-rays. NbTi conductor, rather than $\mathrm{Nb}_{3} \mathrm{Sn}$ conductor, is selected to keep the design and manufacture of SLGB magnet as simple as possible. In this paper, how the field profiles of race-track type coil and solenoid coil change with their geometric parameters is studied, and multi-objective genetic algorithm is used to optimize SLGB magnet structure considering Hefei Advanced Light Facility lattice design demand and NbTi critical current.

## INTRODUCTION

It is known a superconducting bend magnet was used in Advanced Light Source (ALS) and it was formed to significantly enhance the capability and capacity of ALS in the hard X-ray region without compromising the performance of ALS in the VUV and soft X-ray region [1]. With the development of research on longitudinal gradient bend in diffraction-limited storage ring (DLSRs) designs [2,3], Paul Scherrer Institute is planning a major upgrade of the Swiss Light Source including the use of high field longitudinal gradient bends [4]. The ALS-U project investigated a superconducting Superbend magnet using $\mathrm{Nb}_{3} \mathrm{Sn}$ conductor, though it was unselected [5]. Hefei Advanced Light Facility (HALF) was brought forward several years ago, and its dominant radiation will be located in the VUV and soft X-ray region [6] as the beam energy of HALF was chosen to be about 2.2 GeV . The use of superconducting longitudinal gradient bend (SLGB) can help HALF to build a hard X-ray source. At the electron beam energy of 2.2 GeV and a peak SLGB field of 5 T, the SLGB beamline will have a critical photon energy of 16.1 keV and be good sources of photons to 40 keV (see Fig. 1).

Considering the demand of B-field profile along the beam path (Fig. 1) amd the basic geometry construction of SLGB (Fig. 2), the design and fabrication of SLGB are challenging. $\mathrm{Nb}_{3} \mathrm{Sn}$ conductor has higher critical current than NbTi

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Figure 1: (left) Brightness of a SLGB (red line) versus the normal conducting bend (blue line). (right) Optimized Byfield profile along the beam path by lattice design.


Figure 2: (left) Geometric parameters of race-track coil and solenoid coil, $(r, w, h, t, l)$ for race-track coil and ( $R, W, H, T$ ) for solenoid coil. (right) NbTi critical current density from WSTC (measured and fitted).
conductor, but it has poor mechanical properties after heat treatment which makes its mechanical design and fabrication hard. The critical current of NbTi decreases approximately linearly as B-field increases (Fig. 2). The maximum field in the coils can be reduced by optimizing the parameters of coils and then NbTi may meet the B -field requirement of SLGB.

In this paper, we study how the coil's geometric parameters influence the distribution of B-field, including the Bfield profile along the beam path, the ratio of the B-field
integral along the available longitudinal length to the central peak B-field $\left(B_{p}\right)$, the ratio of maximum B-field in the coil $\left(B_{m}\right)$ to $B_{p}$, and the homogeneity of the integration of B-field. Based on the above studies, we can reduce the number of coils' geometric parameters to be optimized, and thus improve the optimization speed.

To meet requirements of the SLGB, first of all, the B-field integral along the available longitudinal length should be equal to the given value. When the geometric parameters of coils are set randomly within a certain range, the current density of the coil can be calculated and the B-field profile is obtained at the same time. To get a peak B-field and reduce the load line of NbTi conductor, MOGA is used to search for the best geometric parameters. The SLGB only using racetrack coil and using both racetrack coil and solenoid coil are all optimized, and we analyzed the impact of solenoid coil on B -field profile and the load line of NbTi conductor.

## IMPACT OF COIL GEOMETRIC PARAMETERS ON B-FIELD

The B-field profile along the beam path can be calculated by Biot-Savart Law. The coils consist of many similarshaped small cross section coils, which can be treated as line current approximately. Then we can simplify triple integrals to single integrals, and the impact of coils' geometric parameters on the B-field will be find out. In order to make the result easy to present, we choose certain values of $h, l$ and $R$, and then how the By-field profile changes with the value of $r / h$ and $R / H$ can be observed in Fig. 3. It is find that small r leads to small FWHM of By-field profile. The By-field profile of step-function is shown in Fig. 1, which is well optimized for reducing emittance, and its longitudinal magnet length is about 460 mm while its B-field integral along the beam path is $0.4 \mathrm{~T} \cdot \mathrm{~m}$. As field enhancement increases, hyperbola function field can reduce emittance more than step function field [2], and SLGB using race-track type coil can generate quasi-hyperbolic field profile.


Figure 3: (left) By-field generated by race-track coil along the beam path varies with different $r / h(h=2.5 \mathrm{~cm}, l=12 \mathrm{~cm})$. (right) By-field generated by solenoid coil along the beam path with different $R / H(R=18 \mathrm{~cm})$.


Figure 4: (left) The B-field integral (along the beam path) changes with $r \& h$ while $l=12 \mathrm{~cm}$. (right) The B-field integral (along the beam path) changes with $R \& H$. The maximum longitudinal magnet length is 46 cm .

The B-field integral is most important for synchrotron storage ring bend magnets. When the coil geometric parameters are known, we can calculate the current density by the B -field integral requirement, then the B -field profile and $B_{p}$ can be figured out. The relationship between B-field integral and $B_{p}$ with different parameters is shown in Fig. 4. It is obviously that the ratio of B-field integral to $B_{p}$ of race-track type coil is bigger than that of solenoid coil.

For superconducting magnets, the maximum B-field $\left(B_{m}\right)$ in coil is most important. With different coil parameters and current density calculated by the demand of B-field integral along the beam path, $B_{m}$ and $B_{p}$ are different. In Fig. 5, we study how the ratio of $B_{m}$ to $B_{p}$ changes with coil parameters. It can be analyzed that $B_{m}$ is located at $(0, h+t / 2, l / 2+r)$. When the SLGB is composed of race-track type coil and solenoid coil, the maximum B-field is thought to be at the same point approximately. The integrated harmonics normalized to the fundamental are expected to be as small as possible, and how the deviation of B-field integral changes with $h$ and $l$ of race-track type coil is shown in Fig. 6. The minimum integral is along the path from $(-46,0,0.4)$ to $(46,0,0.4)$ and the maximum integral is from


Figure 5: (left) The maximum B-field in the racetracktype coil changes with $w \& t(r=1.5 \mathrm{~cm}, l=12 \mathrm{~cm})$. (right) The B-field generated by solenoid coil at the maximum B-field point in racetrack-type coil changes with $W \& T$. $(r, w, h, t, l)=(1.5,3,2.5,5,12) \mathrm{cm}$ is chosen for the solenoid coil and $R=18 \mathrm{~cm}, H=h+t-T$.
$(-46,0.4,0)$ to $(46,0.4,0)$. It is obvious that small $h$ and big $l$ lead to good integral uniformity, but $h$ can not be less than a certain value considering the vacuum chamber diameter and cryostat structure.


Figure 6: (left) The relative deviation between the |By|-field integral along the path from $(-46,0,0.4)$ to $(46,0,0.4)$ and along the standard beam path. (right) The relative deviation between the |By|-field integral along the path from ( $-46,0.4,0$ ) to $(46,0.4,0)$. The coordinate unit here is cm .

## MOGA FOR SLGB OPTIMIZATION

When the B-field integral along the beam path is known, the operating current desnsity $(J)$ can be calculated with random coil parameters within certain range, then we can get $B_{m}$ and $B_{p}$. Considering the critical current of NbTi wire, like Fig. 2, the critical current density $\left(J_{c}\right)$ can be thought approximately reducing linearly with critical B-field $\left(B_{c}\right)$ :

$$
\begin{equation*}
J_{c}=-a B_{c}+b . \tag{1}
\end{equation*}
$$

Then with calculated $B_{m}$ and $J$, we get the load line of SLGB:

$$
\begin{equation*}
\text { load line }=\left(a B_{m}+J\right) / b . \tag{2}
\end{equation*}
$$

We want to find out the minimum load line of different $B_{p}$. Coil parameters are given randomly in reasonable range, which is determined by the limit of NbTi wire bending radius and requirement of field integral uniformly etc. Considering the goal of $B_{p}$ and load line, MOGA is used to optimize coil parameters. Two kind of SLGBs compose of only race-track type coil and both race-track type coil and solenoid coil are optimized. The load lines of different $B_{p}$ are shown in Fig. 7 and Fig. 8. Bgoal here is set up for convenient comparison. The load line of race-track type coil will increase with the use of solenoid coil, but the B-field profile of SLGB will be adjustable.

All the analysis above is based on linearity condition, in fact the load line will be about $20 \%$ lower with the use of iron yoke and core. The ratio of B-field integral to $B_{p}$ will change with iron yoke thickness, there need adjustment of current to fit the difference between linearity and nonlinearity condition.The magnetic field and By-field along the beam path of a optimized solution are shown in Fig. 9 and Fig. 10 separately with $B_{p}=4.5 \mathrm{~T}(h=2.5 \mathrm{~cm}, l=12 \mathrm{~cm})$ and its load line is about $78 \%$.

## CONCLUSION

The optimization of SLGB based on NbTi wire was completed with MOGA. A model with peak B-field along the


Figure 7: (left) Original MOGA optimization result of racetrack type coil. (right) Load line of $\left(B_{p}\right)$ in the range of $(3.8,5.0) \mathrm{T}$. The color refers to the best results of different generation ( $h=2.5 \mathrm{~cm}, l=12 \mathrm{~cm}$ ).


Figure 8: Original MOGA optimization result of SLGB composed of race-track type coil and solenoid coil ( $h=2.5 \mathrm{~cm}$, $l=12 \mathrm{~cm})$.


Figure 9: (left) Magnetic field distribution in the coils and in the iron yoke. (right) Magnetic field distribution in the superconducting coils.


Figure 10: Designed B-field profile (blue) and simulation result (red).
beam path of 4.5 T is given, and its load line is below $80 \%$. The optimization result could be reference for lattice design, especially for SLGB B-field profile design. There is still some work to do with simplifying the non-linear calculation to linear calculation approximately.

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