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MECHANISM OF LONGITUDINAL SINGLE-BUNCH INSTABILITY IN THE CERN SPS

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Abstract

Understanding the origin of beam instabilities is essential for reaching the highest performance of proton synchrotrons. In the present work, the Oide-Yokoya eigenvalue method of solving the linearised Vlasov equation was used to shed light on the mechanism of longitudinal single-bunch instability in the CERN SPS. In particular, semi-analytical calculations were done for the full longitudinal impedance model, taking into account the RF nonlinearity. The obtained results agree well with macro-particle simulations and are consistent with available beam measurements. For the first time, the instability has been interpreted as a coupling of radial modes within a single azimuthal mode, due to a strong potential-well distortion of the synchrotron-frequency distribution. To avoid this instability, a higher RF voltage is required at a given emittance. Thus, the instability mechanism is very different from the loss of Landau damping, which, in contrast, is mitigated by a lower RF voltage. This understanding also allowed us to optimise the RF voltage programmes during the acceleration cycle for high-intensity bunches used in the AWAKE experiment at CERN.

INTRODUCTION

Longitudinal single-bunch instability as a possible performance-limiting factor was a subject of intense studies for many years [1–11]. The standard approach to evaluate beam stability for a simplified impedance model (for example, a single broad-band resonator) is to solve a linearised Vlasov equation for an initial perturbation of the distribution function. Neglecting potential-well distortion, one can find an instability due to a coupling of different azimuthal modes. This can happen for electron bunches that are typically much shorter than the RF bucket and in presence of a high-frequency impedance. Another type of instability is caused by potential-well distortion resulting in a coupling of two radial modes within one azimuthal mode [6, 7]. An explicit condition required for this instability was found for the double-waterbag model [8]. These two different regimes were studied for different resonant frequencies of the broad-band resonator as an impedance model [11].

For proton bunches in the CERN Super Proton Synchrotron (SPS) the longitudinal single-bunch instability is observed during the acceleration ramp. The attempts to cure this instability in a single RF system by reducing the RF voltage and thus increasing the synchrotron frequency spread were not successful. Instead, a higher RF voltage was more beneficial [12]. In operation the instability is cured by application of a higher-harmonic RF system in the bunch-shortening mode (both RF voltages in phase at the bunch centre). Due to the high complexity of the SPS impedance model [13] (see Fig. 1) the observed instability was studied in macro-particle simulations using the CERN Beam Longitudinal Dynamics (BLonD) code [14]. The simulations through the ramp are consistent with measurements and the agreement has improved with the updated impedance model [15].

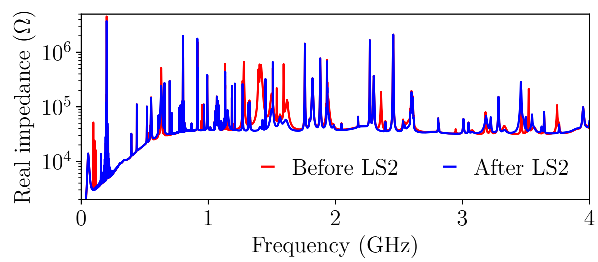

Figure 1: SPS impedance model before and after the impedance reduction campaign during the 2nd Long Shutdown (LS2) finished in March 2021.

A recent study [16] has shown that the azimuthal mode-coupling is possible for the SPS proton bunches. However, a single broad-band resonator impedance was used and the RF system had a frequency 10 times smaller than in reality (and therefore bunches had a significantly smaller emittance to keep the same bunch length). In the present work, the mechanism of the SPS instability is clarified thanks to the numerical solutions of the Vlasov equation using the Matrix Equations for LOngitudinal beam DYnamics calculations (MELODY) code [17] for the full SPS impedance model. The results are compared with the measurements [12] and macro-particle simulations using BLonD.

STABILITY MAPS

The beam stability in simulations is probed by observing the growing oscillations of the bunch position and length. Even though the actual evolution of the bunch parameters depends on the initial seed at the moment of bunch generation, a fast instability still can still be clearly visible. The maximum growth time of instability which can be detected is determined by the number of simulated turns, which corresponds to 2.3 s at the SPS flattop energy of 450 GeV).

In Fig. 2, we present a stability map obtained from a scan in bunch intensity N_p and bunch length $\tau_{4\sigma} = \tau_{\mathrm{FWHM}}\sqrt{2/\ln 2}$ (scaled from the full-width half-

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maximum bunch length τ_{FWHM}) in simulations for 10^6 macro-particles.

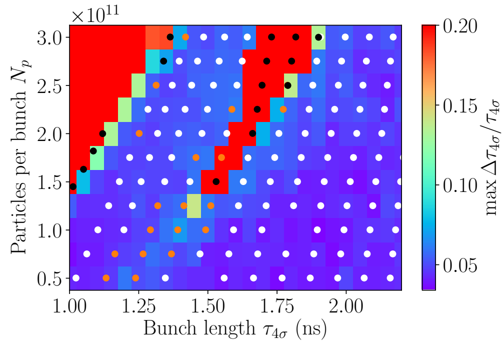

Figure 2: Longitudinal stability map calculated with the MELODY code (circles) and compared with the relative amplitude of bunch-length oscillations in BLonD simulations (colour map). White circles are stable cases, orange circles are cases with growth rates $\mathrm{Im}\,\Omega/\omega_{s0} \in (0, 0.002)$ (weak instability) and black circles correspond to growth rates greater than $0.002\omega_{s0}$.

The main parameters are the ring circumference, $C = 6911.554$ m, and the transition gamma, $\gamma_{\mathrm{tr}} = 17.951$. The SPS is equipped with a double RF system with frequencies of 200 MHz and 800 MHz and the corresponding RF voltages used in these simulations were $V_{200} = 7.2$ MV and $V_{800} = 0$ MV. The maximum amplitude of the bunch-length oscillations in simulations is divided by the initial bunch length (colour code) and indicates the bunch stability. This amplitude is always non-zero even for stable cases due to the finite resolution of the bunch profile used for the bunch length calculation.

Solving the linearised Vlasov equation with MELODY using the Oide-Yokoya method [6] provides a straightforward way to evaluate single-bunch stability. For given parameters one obtains the eigenvalues Ω that may have an imaginary part due to either a real instability or some numerical noise. In the former case there are two eigenvalues that have the same real part and absolute value and opposite signs of the imaginary part. Additionally the shape of the corresponding eigenvectors must be described by a regular function. In the case of noise these two criteria are not satisfied and the corresponding solutions are considered stable. In Fig. 2 on top of the stability map obtained in BLonD simulations, there are points calculated using MELODY with 5-10 azimuthal modes included in the analysis. In general, one can see that stable solutions (white dots) correspond to a small bunch-length oscillation amplitude observed in BLonD simulations, while it is large for most of unstable solutions (black dots). We also indicate the cases with a weak instability (orange points) for which the growth rate is below $0.002\omega_{s0}$ (here ω_{s0} is the bare angular synchrotron frequency) and it should not be clearly observed in simulations. There is an overall good agreement between both methods of beam stability evaluation.

Two Different Instability Mechanisms

The stability map in Fig. 2 shows a non-monotonic dependence of the instability threshold on the bunch length. There is even an unstable 'island', also observed in past studies [18]. As one can see in Fig. 3, the synchrotron frequency distribution $\omega_s(\epsilon)$ is significantly modified by the potential-well distortion. On the right-hand side of the instability island, $\omega_s(\epsilon)$ is a monotonic function and we neither expect nor observe any instability. Inside the island, there is an overlap of synchrotron frequencies at a wide range of emittances resulting in a coupling of radial modes within the single azimuthal mode. The highest growth rate occurs at the fourth, octupole, azimuthal mode ($m = 4$). For smaller bunch lengths, the synchrotron frequency has the shape typical for operation in the bunch-lengthening mode (RF voltages are in counter-phase at the bunch centre) in a double RF system, but it remains monotonic and the bunch is stable. For even shorter bunches we reach a completely different regime: an azimuthal mode coupling instability (in this example, modes with $m = 3$ and $m = 4$ are coupled).

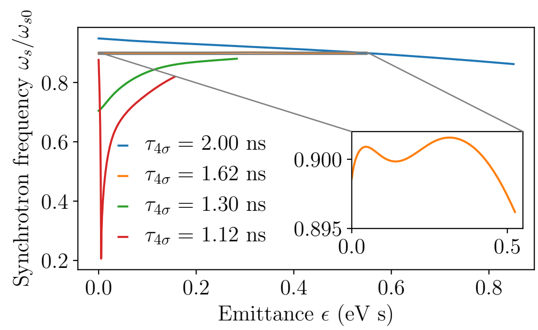

Figure 3: Synchrotron frequency as a function of emittance ϵ for different bunch lengths and intensity 2×10^{11} particles on the SPS flat top with $V_{200} = 7.2$ MV and $V_{800} = 0$.

Comparison with Measurements

During the previous studies, the macro-particle simulations were compared with dedicated measurements. However, the impact of the transfer function of the measurement system was not corrected for bunch profiles. In Fig. 4 the corrected data of bunch intensity versus extraction bunch length is summarised (white triangles). It agrees well with BLonD simulations during the ramp with initial bunch parameters similar to those in measurements (squares). The bunches with intensity below 2×10^{11} were found to be stable during acceleration and their parameters lie in the stable region at the SPS flat top (background colour). For higher intensities, an instability during the ramp is clearly seen both in simulations and measurements.

We also used the MELODY code to calculate a map as a snapshot of beam stability at any time during the cycle. We have found that at the cycle time of 3000 ms (about 100 GeV) bunches with an emittance of 0.3 eV s (corresponds to 1.3 ns at flat top) and intensity above 1.7×10^{11} particles are in

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the unstable region where the radial mode coupling occurs (see Fig. 5). Thus, the SPS single-bunch instability observed in previous measurements has been due to a radial mode coupling.

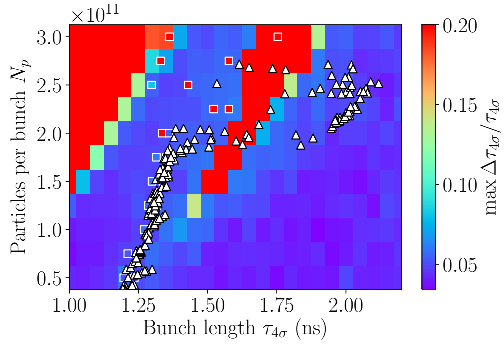

Figure 4: Comparison of stability map obtained from BLonD simulations at the flat top (background colour) and through the ramp (squares) with measurements (triangles).

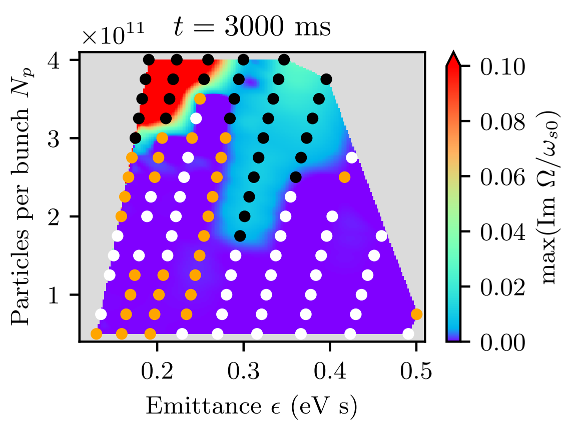

Figure 5: Stability map for the ramp obtained with MELODY. Background colour is the growth rate. Colours of the circle have the same meaning as in Fig. 2.

Since the instability mechanism is understood its cure would be an increase of the 200 MHz RF voltage as well as using the 800 MHz RF system to reduce the harmful impact of potential-well distortion. This was demonstrated experimentally since the 800 MHz RF system is used in the SPS operation for many years.

OPTIMISATION OF THE AWAKE CYCLE

The Advanced WAKefield Experiment (AWAKE) at CERN SPS [19] requires the shortest bunch length for a bunch intensity of 4×10^{11} (maximum available from the SPS injector, Proton Synchrotron). To obtain stable and reproducible bunches, the optimisation of the RF voltage programme during the cycle is necessary [20].

By analysing the equation that defines beam stability [21] one can show that beam and machine parameters can be combined in a dimensionless parameter

$$\zeta = \frac{qh\omega_{\mathrm{RF}}N_p \,\mathrm{Im}\, Z/n}{V_{200}\cos\phi_{s0}},$$

where q is the charge, ω_{RF} is the RF frequency, and $\mathrm{Im}\, Z/n = 1\,\Omega$ is the inductive impedance used for impedance normalisation. In the code MELODY the calculations are performed for the parameter ζ and the bunch length in radians $\omega_{\mathrm{RF}}\tau_{4\sigma}$, which allows us to obtain the stability map for any point of the cycle by re-scaling and interpolation of about 20 stability maps.

From the map at the 400 GeV flat top of the AWAKE cycle with maximum voltage in both RF systems operating in a bunch shortening mode and the impedance model expected after the impedance reduction campaign (2018-2020), we find that a minimum emittance of 0.3 eV s ($\tau_{4\sigma} = 1$ ns) is required for bunch stability at 4×10^{11} particles. To maintain bunch stability through the cycle the ranges of RF voltages were obtained as shown in Fig. 6. One can see that the range of possible parameters is wide (the oscillating pattern is due to the interpolation of a limited number of maps). Note also the $V_{800}/V_{200} = 0.1$ is sufficient up to 3000 ms during the ramp, while the ratio should be increased to 0.18 in the second part of the ramp.

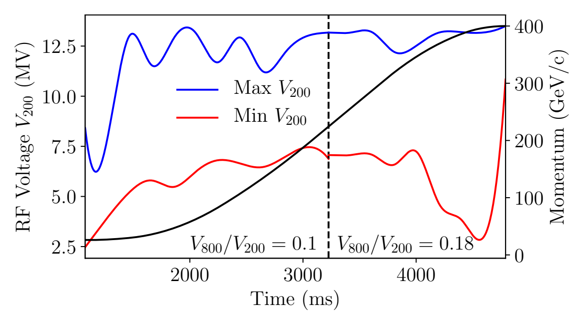

Figure 6: Range of RF voltages that keep bunches with $N_p = 4.0 \times 10^{11}$ particles and emittance $\epsilon = 0.31$ eV s stable throughout the acceleration cycle (black line).

CONCLUSION

We have studied the longitudinal stability of single bunches in the SPS under different conditions. We have found that the instability observed in the measurements is due to the coupling of the radial modes. The results obtained using the MELODY code agree well with the BLonD simulations and available beam measurements. As this instability can be mitigated by a higher 200 MHz RF voltage and usage of a higher-harmonic RF system, optimal RF voltage programmes for future operation have been found.

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