CONSIDERATION OF TRIPLE-HARMONIC OPERATION FOR THE J-PARC RCS

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Abstract

The wideband magnetic alloy (MA) cavities are employed in the J-PARC RCS. The dual-harmonic operation, in which each MA cavity is driven by superposition of the fundamental accelerating voltage and the second harmonic voltage, significantly improves the bunching factor and is indispensable for acceleration of the high intensity beams. The original LLRF control system was replaced with the new system in 2019, which can control the amplitudes of the higher harmonics as well as the fundamental and second harmonics. Therefore, we are considering triple-harmonic operation which uses the third harmonic voltage additionally for further improvement of the bunching factor during acceleration. In this study, the triple-harmonic pattern for the RCS is designed. The beam simulation results show the improvement of the bunching factor compared to the current dual-harmonic operation. In this article, we describe the detail of the triple-harmonic pattern is described. The longitudinal beam simulation and the results of the high power test are presented.

INTRODUCTION

The wideband magnetic alloy (MA) cavities are employed in the J-PARC RCS. The dual-harmonic operation, in which each MA cavity is driven by superposition of the fundamental and the second harmonic voltage, significantly improves the bunching factor and is indispensable for acceleration of the high intensity beams. The bunching factor is defined as the ratio of the peak current to the average current of the bunched beam and is desirable as large as possible in high intensity accelerator to alleviate the space charge effects.

The original LLRF control system was replaced with the new system in 2019, which can control the amplitudes of the higher harmonics as well as the fundamental and the second harmonics [1]. This made it possible to use the third harmonic voltage for the beam acceleration. Therefore we consider the triple-harmonic operation which uses the third harmonic voltage additionally.

The triple-harmonic operation can form a flat and wide potential well and further improvement of the bunching factor is expected. On the other hand, the parameter optimization is not trivial due to the increased number of the parameters. In this study, an effective scheme to determine the parameters of the triple-harmonic operation is developed and the triple-harmonic pattern using the scheme is designed. In this article, we describe the detail of the triple-harmonic pattern is described. The longitudinal beam simulation and the results of the high power test are presented.

TRIPLE-HARMONIC OPERATION

The longitudinal beam motion can be considered as the oscillation in the potential generated by the RF voltage and other sources. In general, for a matched beam to the RF bucket, the bunch shape is almost identical to the potential well shape. Thus, to improve the bunching factor, it is necessary to form a wide and flat potential well.

The RF voltage waveform and the potential well of the dual-harmonic operation for the different ratio of the second harmonic \( V_2 \) to the fundamental voltage \( V_1 \) when the energy gain of synchronous particle \( \delta E_x = 0 \) are shown in Fig. 1. For the dual-harmonic operation, as \( V_2/V_1 \) increases, the width of the potential well increases steadily. On the other hand, the flatness of the potential well becomes worse where \( V_2/V_1 \) is more than 0.5 since the RF waveform oscillates around 0 deg. If another harmonic can flatten this oscillation, a flat and wider potential well can be formed compared to the dual-harmonic operation and further improvement of the bunching factor can be realized.

Figure 1: The RF voltage waveform and the potential well of dual-harmonic operation for different \( V_2/V_1 \).

The third harmonic \( V_3 \) is one of the solutions. In the Fig. 2 the RF waveform and the potential well of the dual-harmonic and the triple-harmonic operation at \( V_2/V_1 = 0.8 \) are shown. As shown in this figure, the oscillation of the RF waveform in the vicinity of 0 deg. of the dual-harmonic operation has approximately the same length as \( V_3 \). By adding the \( V_3 \), the RF waveform can be smoothed out and a flat and wide potential well can be formed. Thus, an improved of
the bunching factor can be realized by the triple-harmonic operation.

![Image](image.png)

Figure 2: The comparison of the RF waveform and the potential well for the dual-harmonic and the triple-harmonic operation where $V_2/V_1 = 0.8$ and $V_3/V_1 = 0.2$.

CONSIDERATION OF TRIPLE-HARMONIC PATTERN FOR THE RCS

The RF voltage waveform of the triple-harmonic operation $V(\phi)$ are expressed in Eq. (1). $\phi_1, \phi_2$ and $\phi_3$ are the phase offsets of each harmonic and $\phi_1 = 0$ when the fundamental harmonic is used as a reference. To realize the effective triple-harmonic operation, it is necessary to determine $V_1, V_2/V_1, V_3/V_1, \phi_2$ and $\phi_3$ so that the bunching factor is optimal,

$$V(\phi) = V_1 \sum_{i=1}^{3} \left[ \frac{V_i}{V_1} \sin (i\phi + \phi_i) \right].$$  

(1)

$V_1$ is mainly related to the RF bucket area. $V_1$ has to be determined considering the longitudinal beam emittance. The other parameters are related to the shape of the potential well. In this study, these parameters are determined using the Hofmann-Pedersen distribution [2]. Assuming that the beam follows the Hofmann-Pedersen distribution, the bunch shape can be calculated from the potential well. Quasi bunching factors can be calculated from the potential well without any beam simulations. The parameters can be adjusted in such a way that the quasi bunching factor is optimal. Using this method, if $V_1$ and $\Delta E_s$ and the hamiltonian of the extreme (= boundary) particle are given, all the rest parameters of the triple-harmonic operation can be obtained. Note that the beam is accelerated quasi-adiabatically in the RCS, so this method is valid, but in non-adiabatic processes, this method cannot be adapted.

Figure 3 shows the designed triple-harmonic pattern for the RCS. The triple-harmonic pattern is designed using the same $V_1$ as the original pattern, i.e. $V_1$ is not optimized. The other parameters are obtained using the method mentioned above. As the result of designing the triple-harmonic pattern, it is found that $V_2/V_1$ became larger and should be changed depending on $\Delta E_s$.

![Image](image.png)

Figure 3: The comparison of the designed triple-harmonic pattern and the current dual-harmonic pattern.

Figure 4 shows the results of the longitudinal beam simulation using the designed triple-harmonic and current dual-harmonic pattern. The beam simulation is carried out using BLoND developed by CERN [3]. The simulations using the triple-harmonic pattern are carried out for the two different injection methods.

![Image](image.png)

Figure 4: The longitudinal beam simulation results for the dual-harmonic and triple-harmonic operation. The top, middle and bottom figures show the bunching factor, momentum filling factor and $dp/p$, respectively. The horizontal axis corresponds to the number of turns.

In the RCS, the multi-turn injection method is used and the beam is continuously injected until 307 turns. In the current dual-harmonic operation, longitudinal painting injection, the momentum offset injection and the second harmonic phase sweep, are employed to achieve a large bunching factors [4,5]. With this manipulation, a large bunching factor of about...
0.4 is achieved after the injection, while a decrease of the bunching factor is unavoidable with these painting scheme.

The simulation results denote that triple-harmonic operation without the momentum offset injection can improve the bunching factor during the injection significantly. This is because that the triple-harmonic operation realizes a large bunching factor without having to distribute the beam widely in the RF bucket and also the RF bucket shape matches well to the injection beam shape. Furthermore, when the momentum offset injection is introduced, the bunching factor after the injection achieves 0.5. In Fig. 5, the comparison of the bunch shape is shown. As shown in Fig. 5, the triple-harmonic operation can form the wide and flat bunched beam.

In Fig. 4, the momentum filling factor and $dp/p$ of the beam during the acceleration are also depicted. The momentum filling factor is defined as the ratio of $dp/p$ of the beam to the RF bucket height and is a convenient parameter to evaluate how much of the beam is stuck in the RF bucket. It is found that the momentum filling factor and $dp/p$ of the designed triple-harmonic pattern become small. This means that the RF bucket area is too large compared to the longitudinal beam emittance. The small momentum filling factor leads to the reduction of the bunching factor. It is expected that the bunching factor can be improved more by optimizing $V_3$ so that the momentum filling factor is comparable to the current dual-harmonic operation.

**HIGH-POWER TEST**

A high power test was carried out in which the triple-harmonic pattern was applied to all the 12 RF cavities of the RCS. In Fig. 6, the measured gap voltage and phase of each harmonic are shown. We confirmed that the RF cavities can be driven by the triple-harmonic pattern without problems. One can see that the amplitudes and phases of the harmonics up to the third harmonic are well controlled as programmed.

**SUMMARY**

The triple-harmonic operation for the J-PARC RCS is under consideration toward further improvement of the bunching factor. By adding the third harmonic voltage, a flat and wide potential well can be formed. The triple-harmonic pattern for the RCS is designed using Hofmann-Pedersen distribution so that the bunching factor is optimal. The longitudinal beam simulation results shows the significant improvement of the bunching factor compared to the current dual-harmonic operation. We confirmed that the RF cavities can be driven by the triple-harmonic pattern without problems. The harmonic components of the gap voltage are well controlled as programmed. As an outlook, we are planning to demonstrate the triple-harmonic operation in the beam experiment.

**REFERENCES**


