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A FAST NON-LINEAR MODEL FOR THE EBS COMBINED SEXTUPOLE-CORRECTOR MAGNETS

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Abstract

Corrector are often integrated in higher order accelerator magnets. In the new ESRF-EBS storage ring, the sextupoles include additional windings allowing for dipole and skew quadrupole corrections. The accurate modelization of such magnets is not as trivial as it may appear, due to their non-linearities and to the crosstalk between their channels. Changing any corrector current induce non-linear errors in the other corrector channels and in the main sextupole strength, making difficult the trimming of the magnets. A model based on a non-linear excitation curve and quadratic contributions from corrector currents was developed. This model is very fast and was included in the accelerator control system to compute the corrector currents in real-time. It was tested against 3D magnetic simulations and magnetic measurements and compared to a simpler matrix-based model.

INTRODUCTION

The Extremelly Brilliant Source (EBS) is an upgrade of the European Synchrotron Radiation Facility (ESRF) based on a new storage ring [1, 2]. The optics rely on high gradient, compact magnets [3, 4]. This paper focuses on the correctors integrated in the sextupole magnets (Fig. 1) [5]. Four correction channels are available. Combining these channels can generate normal (resp. skew) dipole b_1 (resp. a_1), skew quadrupole a_2 and normal sextupole components b_3 defined as

$$B = \sum_{n>0} (a_n + ib_n) \left(\frac{z}{r_0}\right)^{n-1},$$

where $B = B_Y + iB_X$ is the complex field, $z = x + iy$ the position and r_0 the reference radius.

The operation of the storage ring requires an accurate model of these magnets in order to drive their main and corrector power supplies. Such a model should be sufficiently fast and should work in forward (currents to strengths) and reverse (strengths to currents) directions.

The sextupole magnets are saturated at high current, as shown in Fig. 2. The non-linearities induced by the saturation complicate the computation of strengths: a simple, matrix based model with linear dependence in currents would only work at low sextupole current.

A non-linear model was developed. The mathematical details of the model, and its implementation, are described in the next section. The model predictions are compared to magnetic measurements in a later section.

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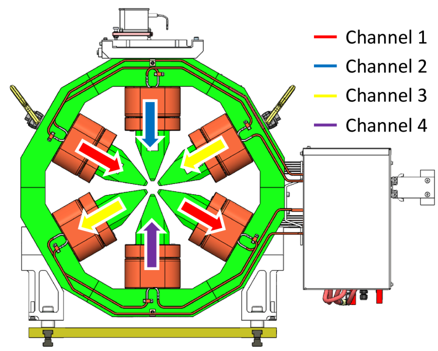

Figure 1: Design view of a combined sextupole-corrector magnet. Each pole is equipped with one sextupole coil and one correction coil. All sextupole coils are connected in series, while correction coils are connected according to the arrows.

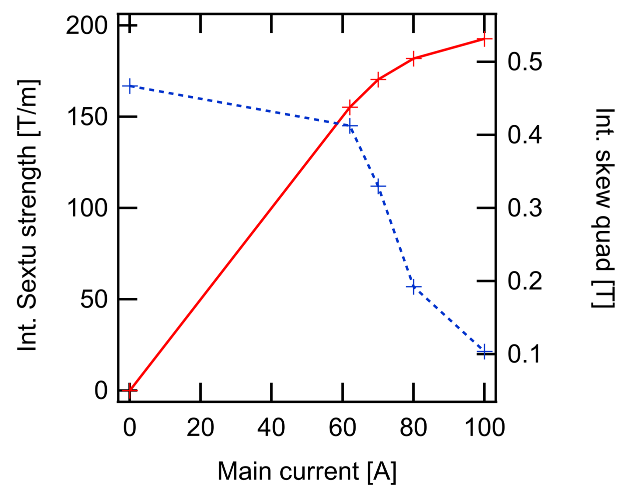

Figure 2: Measured sextupole strength (blue) and skew quadrupole (red, dashed) versus main current.

SEXTUPOLE-CORRECTOR MAGNET MODELS

Linear Model

The first model developped was linear in corrector currents and non-linear in the main sextupole current i_0. It writes

$$\begin{pmatrix} a_1 \\ b_1 \\ a_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ k(i_0) \end{pmatrix} i_0 + \mathbf{M}(i_0)\,\mathbf{I},$$

where $\mathbf{I} = {}^T(i_1, ..., i_4)$ are the corrector currents and the coefficients of the matrix $\mathbf{M}$ depend non-linearly in the main current.

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This model is simple to implement. The matrix coefficients were determined for a set of main currents, with corrector channels off and on. They were interpolated versus main current with cubic splines.

The accuracy of the model was estimated from 3D simulations of a sextupole-corrector magnet, using the Radia magnetostatic code [6–8]. Relative errors up to 2% were obtained on the sextupolar component b_3. Given the 0.1% sextupole strength accuracy needed for the operation of the machine, this model was considered as not suitable.

Quadratic Model

A higher order model was developed to better reproduce the non-linearities of the combined sextupole-correctors. At a given main sextupole current, the strengths are

$$\begin{pmatrix} a_1 \\ b_1 \\ a_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ k\,(i_0) \end{pmatrix} i_0 + \mathbf{M}\,(I_0)\,\mathbf{I} + \begin{pmatrix} {}^{T}\mathbf{I}\,\mathbf{F}^1\,(i_0)\,\mathbf{I} \\ \vdots \\ {}^{T}\mathbf{I}\,\mathbf{F}^4\,(i_0)\,\mathbf{I} \end{pmatrix},$$

where the last term describes the quadratic contribution of corrector currents. The $\mathbf{M}$ and $\mathbf{F}^p$ matrices depend on the main current. Using short hand tensor notations, the model writes

$$c^j = k^j i_0 + m^j_k i^k + f^j_{kl} i^k i^l,$$

where the k^j, m^j_k and f^j_{kl} are functions of i_0.

It is often more convenient to express the multipole components as linear forms. This gives for the skew dipole:

$$a_1 = \left(i_1, ..., i_4, i_1^2, i_1 i_2, ..., i_4^2\right) \cdot \begin{pmatrix} g_1 \\ \vdots \\ g_4 \\ g_{11} \\ g_{12} \\ \vdots \\ g_{44} \end{pmatrix},$$

where we have introduced the notation $g_k = m^1_k$ and $g_{kl} = f^1_{kl}$. Similar linear forms can be written for the other components.

Symmetries

This model has a large number of coefficients: without additional assumption, 81 parameters should be determined at each value of i_0. This would imply a significant number of simulations or magnetic measurements. However, this number can be reduced by using the symmetries of the magnet.

Let us focus on the skew dipole. Its linear dependencies in i_1 and i_3 should have opposite signs, whereas it does not vary with i_2 and i_4, etc. The skew dipole symmetries can be described by a sparse matrix $\mathbf{S}^1$:

$$a_1 = \left(i_1, ..., i_4, i_1^2, i_1 i_2, ..., i_4^2\right) \mathbf{S}^1 \begin{pmatrix} g_1 \\ \vdots \\ g_{44} \end{pmatrix},$$

where

$$\mathbf{S}^1 = \begin{pmatrix}
1 & 0 & \dots & & & & & \dots & 0 \\
0 & \dots & & & & & & \dots & 0 \\
-1 & 0 & \dots & & & & & \dots & 0 \\
0 & \dots & & & & & & \dots & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & & \dots & 0 \\
0 & \dots & & & 0 & 1 & 0 & \dots & 0 \\
0 & \dots & & & & & & \dots & 0 \\
0 & \dots & & & 0 & -1 & 0 & \dots & 0 \\
0 & \dots & & & & & & \dots & 0 \\
0 & \dots & & & 0 & -1 & 0 & \dots & 0 \\
0 & \dots & & 0 & -1 & 0 & & \dots & 0 \\
0 & \dots & & & 0 & -1 & 0 & \dots & 0 \\
0 & \dots & & & & & & \dots & 0
\end{pmatrix}.$$

Similar matrices can be found for b_1, a_2 and b_3.

Estimation of Model Parameters

Let $I_n = (i_{1n}, ...i_{4n})$ be a vector of corrector currents and a_{1n} the corresponding skew dipole. Combining N vectors of currents and dipole measurements gives

$$\begin{pmatrix} a_{11} \\ \vdots \\ a_{1N} \end{pmatrix} = \begin{pmatrix} i_{11} & \dots & i_{41}^2 \\ \vdots & & \vdots \\ a_{1N} & \dots & i_{4N}^2 \end{pmatrix} \mathbf{S}^1 \begin{pmatrix} g_1 \\ \vdots \\ g_{44} \end{pmatrix} = \mathbf{H} \begin{pmatrix} g_1 \\ \vdots \\ g_{44} \end{pmatrix}.$$

The model parameters at main current i_0 can then be estimated from a pseudo-inverse of $\mathbf{H}$:

$$\begin{pmatrix} g_1 \\ \vdots \\ g_{44} \end{pmatrix} = \mathbf{H}^+ \begin{pmatrix} a_{11} \\ \vdots \\ a_{1N} \end{pmatrix},$$

and the same approach can be applied to the other components. One should notice that one single multipole measurement gives the four components a_1 to b_3, further reducing the number of measurements needed for the parameter estimation. At a given main current i_0, the complete set of parameters can be deduced from measurements with eight current vectors:

$$I_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad I_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, I_3 = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad I_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},$$

$$I_5 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \quad I_6 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, I_7 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \quad I_8 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}.$$

The main sextupole strength is determined by I_1, the linear and quadratic terms of channel 1 and 3 are estimated from I_2 and I_3, the terms of channels 2 and 4 are given by I_4 and $_5$ and the remaining current vectors are for the crossed terms.

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Implementation

Five input currents (main power supply plus four corrector channels) are available to drive four multipole channels. We decided to set the b_3 contribution of the correctors to zero in all cases. This simplifies the computation of the currents from strengths, which is performed as follows:

- The main sextupole current is obtained from the excitation curve, using Newton's method
- The parameters of the quadratic model are interpolated at this current
- The corrector currents are obtained by inverting the matrix of the model

The model above was implemented as a C++ library. It relies on the Eigen library for matrix computations [9].

It is initialized with a table of parameters measured at various currents and with an additional calibration parameter which links the strength of a given magnet to a reference. Cubic spline interpolation functions of the main current are initialized for all parameters of the model. This step takes a few milliseconds on a single CPU.

After initialization, the computation of strengths from currents or currents from strengths takes about 5 µs CPU time.

MAGNETIC MEASUREMENTS

After a first validation based on simulations, the quadratic model was tested on a real magnet. The magnetic measurements were performed with a moving stretched wire bench developed at the ESRF [10]. The field multipoles were computed from measurements of the magnetic field integral on a circle.

The number of corrector current vectors was increased from 8 (see above) to 15 in order to improve the quality of the data.

The measurements have shown that the magnetic hysteresis [11] is the main limitation of the model: the measured values depend on the magnet history. To make the measurements repeatable, the hysteresis implied cycling the correctors with a sequence that is not compatible with operation in the machine (at maximum current, the correctors can change the sextupole by more than 20%).

Table 1 shows sample test results measured at 90 A. It corresponds to a saturated working point: the sextupole efficiency is reduced by 17% and the skew quadrupole efficiency is reduced by 69% (Fig. 2). The relative errors on the sextupole components are in the range of 0.1%, which remains acceptable for operation, and the relative errors in the corrector strengths are in the range of 1%.

CONCLUSION

The outputs of the non-linear sextupole-corrector model are in very good agreement with 3D simulation results: the relative errors are in the range of 10^{-4}. The model computation time is 5 microseconds per magnet on a single CPU, which is compatible with its integration in the ESRF control system. It correctly predicts the effect of the magnetic saturation on the sextupole and the corrector channels. However, real magnets are affected by magnetic hysteresis. This hysteresis is not included in the model, leading to discrepencies between its predictions and magnetic measurements. The next step will be the implementation of an hysteresis model, in which this non-linear model should be integrated.

Table 1: Sample Test Results at 90 A Main Current (i.e. Saturated)

		Test 1		**Test 2**	
		Spec.	**Meas.**	**Spec.**	**Meas.**
a_1	(T mm)	0	0.034	0	0.003
b_1	(T mm)	6	5.96	0	0.097
a_2/r_0	(T)	0	-0.003	0.13	0.1287
b_3/r_0^2	(T / mm)	0.46984	0.46916	0.46984	0.46956

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