# NONLINEAR CORRECTORS TUNING FOR THE COLLECTOR RING ISOCHRONOUS MODE 

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## Abstract

One of the operating modes for the Collector Ring (CR) under construction in Darmstadt is the isochronous mode, in which the captured ions circulate with an equal period regardless of their momentum. The measurement of the orbital period T by the time-of-flight sensors makes it possible to precisely determine the mass to the charge ratio of the ion under study. For this, the change of the circulation period dT should not exceed $10^{-6}$ for $\mathrm{dT} / \mathrm{T}$ in the entire momentum acceptance of the $0.62 \%$. Modeling in the Strategic Accelerator Design code showed that without nonlinear effects compensation, the orbital period variation is $10^{-5}$. In this work, the parameters of nonlinear correctors, which are sextupoles and octupoles in CR, are determined, necessary for the isochronous mode implementation.

## INTRODUCTION

The Collector Ring is being designed as a cooler for antiprotons and rare isotopes and will be part of the FAIR facility at the GSI Helmholtz Centre for Heavy Ion Research [1]. Three modes are planned - isochronous mode, antiprotons and rare isotopes. In this article, the implementation of the isochronous mode is considered. In it, the ions of different species, but with the same mass-tocharge ratio $\mathrm{m} / \mathrm{q}$, should have equal periods of revolution. Thus, the isochronous Collector Ring mode implements the mass spectrometer mode. Achieving the required accuracy of the $\mathrm{m} / \mathrm{q}$ measurement imposes a strict condition on the value of the spread over the time of the particle revolution. The discrepancy dT between the rotation time of an arbitrary particle and the period T of a particle in an equilibrium orbit without momentum shift, normalized by the period $T$, will be called nonisochronicity. Its level should not exceed $10^{-6}$ along the entire acceptance in coordinates and momentum. The main parameters of the modes are given in Table 1.

The condition of the rotation period independence from the momentum requires first of all the zeroing of the slip factor $\eta=\alpha_{p}-1 / \gamma^{2}$. In the linear approximation, this means creating optics with the desired dependence of the dispersion function, giving $\alpha_{\mathrm{p}}=1 / \psi^{2}$ for each energy value. It is planned to work on 3 fixed values of $\gamma$, but in the range of values of magnetic rigidity, which will allow to study many different isotopes and rare ions.

The structure of the ring has two axes of symmetry that divide it into four superperiods.

Table 1: Regimes Parameters

| Circumference |  | 221.45 m |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Regime | Anti- <br> protons | Rare <br> Isotopes | Isochronous Mode |  |  |

## TUNING

To work out the tuning mechanism, the energy with $\gamma=1.43$ was selected, required momentum acceptance of which optics is $0.22 \%$.

The tuning consisted of iterating on changing the strengths of nonlinear correctors, followed by checking the result by tracking the distribution.

## Nonlinear Correctors

Achieving a high degree of isochronicity requires zeroing $\eta$ not only in the linear approximation, but also in the higher orders. The presence of non-linear effects can seriously disrupt $\eta=0$, leading to a large spread over $d T / T$. Non-linear elements should be used to suppress such effects. The Collector Ring project includes the use of sextupoles and octupoles. In the context of the article, optimization using sextupoles is considered. There are 24 units at our disposal, divided into 6 groups of 4 units, powered by groups and located in each of the CR four quadrants. Power supply scheme for CR sextupoles is shown in Fig. 1.


Figure 1: Power supply scheme for CR sextupoles.

Maximum gradient of the sextupole with magnetic field $B$ is $d^{2} B_{y} / \mathrm{dx}^{2}=10 \mathrm{~T} / \mathrm{m}^{2}$ and its effective length is 0.5 m .

## Distribution

To measure the quality of the isochronous mode implementation a test distribution was used. The number of particles in it was selected to check as many extreme cases of the particle position in the 5-dimensional space of transverse coordinates and momentum $\left\{x, x^{\prime}, \mathrm{y}, \mathrm{y}^{\prime}, \mathrm{dp} / \mathrm{p}\right\}$ as possible. On the other hand, the number of particles should not be too large to reduce the time of each tracking. As a result, the problem of creating a distribution was factorized into two 2-dimensional coordinates and one 1dimensional momentum distributions, between which all possible combinations of five coordinates were sorted out.

In the space $\left\{x, x^{\prime}\right\}$, as well as in $\left\{y, y^{\prime}\right\}$, a phase ellipse consistent with the ring structure was created, the emittance of which was equal to $100 \mathrm{~mm} * \mathrm{mrad}$, and its position in the main axes corresponded to the azimuth of the symmetry axis, on which the Twiss parameter $\alpha=0$. For $d p / p$, the distribution is linear: (-dp/p, -dp/2p, 0, dp/2p, dp/p). The total number of points used was $8 * 8 * 5=320$.


Figure 2: Test distribution in the azimuth of the straight section symmetry axis.

Figure 2 shows phase ellipses corresponding to the azimuth of the straight section symmetry axis. Sets of coordinates from ellipses have been used in all possible combinations to create the distribution. All points belong to the boundary of the maximum phase ellipse for a given acceptance. It is not necessary to take into account the internal points, since the non-isochronicity for them is less than the external ones.

A control distribution was created separately. It required a larger number of points to obtain more detailed information about the resulting form of non-isochronicity. It used 6400 points.

## Tracking

The tracking of the particle distributions was done in the Strategic Accelerator Design (SAD) code [2], which allows running 6D fully symplectic calculations. Tracking was performed through one revolution along the ring, the start and end points coincided and were located on the line of straight section symmetry near the injection site. It was possible to perform tracking on a set of revolutions, but this was not necessary, because due to the fractional betatron frequency at each subsequent revolution, the particle will have a new value of the betatron phase, gradually filling the phase space. In this work, an orthogonal approach was used, in which a set of the phases values of betatron oscillations was generated in advance.

## Optimization

The optimization algorithm was based on performing iterations, which results refined the strengths of sextupoles 6 families in order to reduce the non-isochronicity. The numerical value of the non-isochronicity for a given set of sextuple strength $\{\mathrm{S} 1, \mathrm{~S} 2, \mathrm{~S} 3, \mathrm{~S} 4, \mathrm{~S} 5, \mathrm{~S} 6\}$ was calculated by the constructed target function. It returned the maximum of the absolute deviation of the rotation time for all the particles of the test distribution relative to the time of the equilibrium particle.

The optimization steps were performed sequentially across the families. At the beginning, a single family was used to achieve the minimum value of non-isochronicity. Then, as soon as the next step led to its increase, the previous value was saved and the process was started again for the next family. The process did not end on one pass through the families of sextupoles, since the search for the optimum in the next family shifts the previous ones from the optimum. Practice has shown that after about 10 passes, the situation stabilizes and subsequent passes through the families cease to improve the non-isochronicity.

## Results

During the optimization process, a non-isochronicity value of $1.46^{*} 10^{-5}$ was achieved, which is 1.63 times better than the corresponding non-optimized value of $2.39^{*} 10^{-5}$. The result of including sextupoles with new strength values in the model was verified using a control distribution. The non-isochronous values for each point are shown in the histogram of Fig. 3.


Figure 3: Control Distribution. Red shows the distribution of particles depending on the degree of their nonisochronicity after optimization. Gray for comparison distribution without optimization.

For comparison, the gray color shows the tracking result for the case with the sextupoles turned off. It shows that after the corrections are enabled, the distribution view becomes more symmetrical and concentrated near zero.

## CONCLUSION

A lower level of non-isochronicity of the relative value in the unoptimized case is obtained, but the required order of magnitude of $10^{-6}$ has not yet been reached. In the course of the work, results were obtained that simplify further proceedings with the problem of constructing nonisochronous optics, for which the optimization algorithm will be improved and the use of octupoles will be included.

## REFERENCES

[1] A. Dolinskii et al., "Collector ring project at FAIR," Physica Scripta, vol. 2015, no. T166, p. 014040 , Nov. 2015. doi:10.1088/0031-8949/2015/t166/014040
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