BUNCH LENGTH CHARACTERIZATIONS FOR THE
SOLARIS INJECTOR LINAC

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Abstract
During 2020 the first characterization of bunch length and bunch profile in the Solaris injector LINAC has been performed since the start of its operation. In absence of more sophisticated bunch length diagnostics, we have adopted an inversion algorithm applied to beam energy spectra. In practice, the method applies a transformation matrix which maps the particle energy into the particle longitudinal coordinate along the bunch. The construction of this matrix is made analytically, based on the solution of the Liouville equation for the study of the longitudinal beam dynamics. The analytic approach has been benchmarked with experimental measurements of the beam properties along the machine and cross-checked with other tools, as particle tracking and/or beam optics codes. The final results are presented. Moreover, a new diagnostic station at the end of the LINAC has been installed which will host experiments of coherent radiation emission that will be used to confirm the validity of our observations. Preliminary simulations of the coherent spectra are finally reported.

INTRODUCTION
Bunch length diagnostics is of primary importance in any accelerator facility. It provides a huge variety of applications, ranging from the optimization of accelerator’s performances itself to the delivery of light pulses of controlled duration for different kind of experiments. We show an application of the fully analytic solution of the Liouville equation for the longitudinal beam dynamics. This turns to be a powerful tool for calculations of beam dynamics as well as for beam diagnostics. In fact, we show the possibility of retrieving the temporal profiles of the accelerated bunches in accelerator facilities not provided by any of the complex diagnostics, but at least possessing a dispersive line to measure the beam energy spectrum. We have developed theory and experiments [1] for the SOLARIS injector LINAC [2]. In this paper we also provide simulations of coherent radiation spectra that will be measured in order to confirm the results obtained by the combined use of Liouville theory and the experimental measurements of beam energy spectra.

ANALYTIC SOLUTION OF THE LIOUVILLE EQUATION
The 1D Liouville equation describing the longitudinal phase space of a particle beam is following:

$$\frac{\partial}{\partial t} \rho(z,p_z;t) = \hat{L} \rho(z,p_z;t)$$

(1)

where $\rho$ is the phase space density, $p_z$ is the longitudinal momentum and the Liouville operator is:

$$\hat{L} = \left[ \frac{\partial H}{\partial z} \frac{\partial}{\partial p_z} - \frac{\partial H}{\partial p_z} \frac{\partial}{\partial z} \right].$$

(2)

The hamiltonian is $H$, while the Hamilton equations can be expressed as:

$$\frac{dp_z}{dt} = -\frac{\partial H}{\partial z} = F_z$$

$$\frac{dz}{dt} = \frac{\partial H}{\partial p_z} = v_z$$

(3)

where $F_z$ is the longitudinal force and $v_z$ is the longitudinal velocity. Equation (1) admits the formal solution:

$$\rho(z,p_z;t) = e^{\int_0^t dt \hat{L} \rho(z,p_z;0)}.$$  

(4)

Moreover, using the Eqs. (3), Eq. (4) can be further reduced to a more explicit form:

$$\rho(z,p_z;t) = e^{-\int_0^t dt F_z \frac{\partial}{\partial z} - \int_0^t dt v_z \frac{\partial}{\partial p_z}} \rho(z,p_z;0) =$$

$$\rho \left( z - \int_0^t dt v_z, p_z - \int_0^t dt F_z \right).$$

(5)

The power of this solution lays in the fact that the knowing the single-particle dynamics allows for a complete description of the whole beam in the phase-space at any time [1,3].

RELATION BETWEEN BEAM ENERGY SPECTRUM AND LONGITUDINAL BUNCH PROFILE
The transfer line from the injector LINAC to the SOLARIS storage ring acts as a dispersive line where the beam...
energy spectrum is measured (see Fig. 1). A general condition verified in dispersion lines is [4]:

\[ Y_D(y) = \int_{-\infty}^{\infty} Y(y - D_y \Delta E(z)) T(z) \, dz \]  

(6)

where \( Y_D(y) \) is the vertical profile of the beam in the vertical dispersive line, \( Y(y) \) is the vertical profile that the beam would have on a non-dispersive (straight) line at an equivalent distance from the origin of the lattice, and \( T(z) \) is the longitudinal profile of the electron bunch (\( z = ct \) for ultrarelativistic beams, therefore the longitudinal profile can also be thought as a temporal profile). The correlated energy spread is \( \Delta E(z) \), \( E_r \) is the average beam energy and the vertical dispersion function is \( D_y \). Equation (6) states that whenever \( D_y = 0 \), i.e. on a straight line, \( Y_D(y) \rightarrow Y(y) \), since the vertical profile of the beam measured on the straight path is time-integrated. Inverting [5] Eq. (6) in terms of \( T(z) \) allows, in principle, for the reconstruction of the temporal profile of the bunches starting from the measurement of the dispersed profile \( Y_D(y) \), which apart from a conversion factor is the beam energy spectrum profile. The latter inversion can be eventually performed by discretizing the integral equation and reducing the problem to a matrix inversion [1]. Equation 6 neglects the uncorrelated energy spread with respect to the correlated energy spread, which is not always possible, but it is for the parameters of the SOLARIS injector LINAC.

For the transfer line of the SOLARIS injector \( D_y = 1 \) m and \( E_r = 536 \) MeV. The energy spectrum profile \( \propto Y_D(y) \) is measured in the transfer line on a screen placed at the maximum of the dispersion function. The non-dispersed profile is instead measured on a screen placed before the beam dump. The correlated energy spread of the beam is analytically calculated by propagating the beam from the electron gun through the accelerating section via Eq. (5), while considering the proper phase space at the exit of the gun [6–8] (measured and benchmarked by ASTRA simulations [9]) and the proper energy gain in the RF sections (experimentally measured as reference). Finally the reconstructed profile has been the one shown in Fig. 2.

COHERENT RADIATION SPECTRA

In order to confirm the result shown in Fig. 2 we are planning to perform measurements of coherent radiation spectra. Indeed, coherent radiation is certainly related to the bunch longitudinal profile through the so-called form factor [10]. A sketch of the experimental setup to be adopted for experiments of production and detection of coherent radiation is shown in Fig. 3. Considering the longitudinal bunch profile reported in Fig. 2, we have calculated the...
expected Coherent Diffraction Radiation spectral angular distribution that is going to be measured. This calculation, reported in Fig. 4 will serve as reference for identifying the working spectral range of the detectors to be chosen. The visible modulation at 3 GHz is due to the fact that in a single bucket injected into the storage ring there are three electron bunches. This time structure is created through a selection performed by the 100 MHz chopper on the 3 GHz modulated electron current exiting the thermoionic gun cavity [1].

CONCLUSIONS

The bunch length in the SOLARIS injector LINAC has been retrieved by a combined use of measurements and theoretical expectations. In particular we have demonstrated that a fully analytic approach based on the solution of the Liouville equation for the longitudinal dynamics yields the same results than other numerical but slower approaches and it might speed up the process of bunch length retrieval starting from measurements of beam energy spectrum. On the other hand, the validity of this method shall be confirmed by more direct measurements of the bunch length, for example exploiting Coherent Diffraction Radiation. These measurements will be performed on a newly-installed optical table at the end of the accelerator placed before the beam dump as in Fig. 5, which shows the basic setup we have now installed to measure the spatial properties of the beam and its transverse position. After the later characterization, detection of coherent spectra will be realized in the sub-THZ range with the aim of reconstructing the longitudinal bunch profile.

![Figure 5: New diagnostic station placed before the beam dump for coherent radiation studies.](image)

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REFERENCES


