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BASIC DESIGN STUDY FOR DISK-LOADED STRUCTURE IN MUON LINAC

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Abstract

The world's first disk-loaded structure (DLS) at the high-velocity part of a muon LINAC is under development for the J-PARC muon $g-2$/EDM experiment. We have simulated the first designed constant impedance DLS to accelerate muons from $\beta = 0.7$ to 0.94 at an operating frequency of 1296 MHz and a phase of −10 degrees to ensure longitudinal acceptance and have shown the quality of the beam meets our requirements. Because the structure needs a high RF power of 80 MW to generate a gradient of 20 MV/m, a constant gradient DLS with the higher acceleration efficiency is being studied for lower operating RF power. In this paper, we will show the cell structure design yielding a gradient of 20 MV/m with lower RF power.

INTRODUCTION

The Muon $g-2$/EDM Experiment at J-PARC

The muon anomalous magnetic moment ($g-2$) has been measured by two experiments at Brookhaven National Laboratory [1] and Fermi National Accelerator Laboratory [2]. The world average has 4.2σ discrepancy from the Standard Model (SM) prediction [3]. The contribution of New Physics beyond the SM is expected in this discrepancy.

A novel method of measurement for the $g-2$ and the muon electric dipole moment (EDM) is under development at J-PARC. This experiment plans to use some unprecedented techniques: thermal muonium production, laser ionization, muon acceleration, and 3D spiral injection, as shown in Fig. 1 [4]. The muon LINAC accelerates the 5.6 keV muon beam ionized from muoniums to 212 MeV keeping its low transverse emittance to make the muon beam circulate in the compact and weak focusing storage magnet for its lifetime. The muon LINAC has four different types of structure matching the change in muon velocity for high-efficiency acceleration and suppressing the emittance growth during acceleration, as shown in Fig. 2. The disk-loaded structures (DLS) take charge of acceleration from 40 MeV to 212 MeV in the last high-velocity section of about 10 m in length. For the above reasons, the DLS section requires an acceleration gradient of 20 MV/m, keeping a low normalized transverse emittance of 1.5π mm mrad or less, and a small momentum spread of 0.1% or less at the end of DLS.

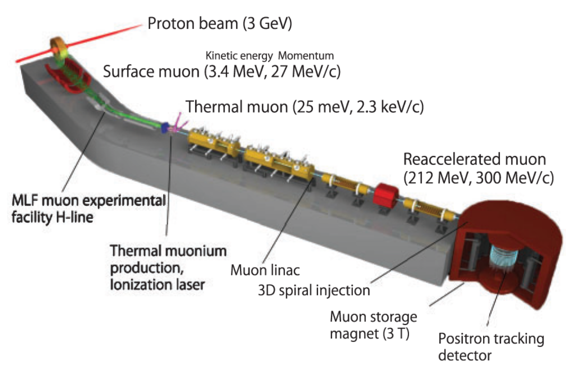

Figure 1: Schematic view of the muon $g-2$/EDM experiment at J-PARC [4].

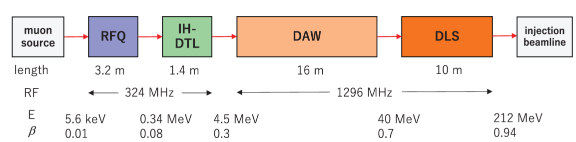

Figure 2: Schematic configuration of the muon LINAC [4].

The First Design

The high-velocity section of the first design consists of four constant impedance (CI) type DLSs, each about 2 m in length, and three transport lines connecting the DLSs, and installed with two quadrupole magnets for transverse focusing. The accelerating mode is TM01-$2\pi/3$ and the selected operating frequency of 1296 MHz and the synchronous phase of −10 degrees yield a sufficient longitudinal acceptance. The length of each cell in the DLS is determined as

$$D = \beta\lambda/3 \tag{1}$$

where β is the beam velocity normalized by the speed of light at the start point of each cell, and λ is the wavelength of a 1296 MHz wave. β is calculated in terms of the energy gain assuming a constant accelerating gradient of 20 MV/m as

$$\text{Energy Gain} = 20\,\text{MV/m} \times \cos(-10\,\text{deg}) \times D' \tag{2}$$

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where D' is the length of the adjacent cell on the upstream. An input RF power of about 80 MW is required in this design for an accelerating gradient of 20 MV/m.

This design was estimated by beam tracking simulation to meet the requirements described above [5], however, we need consideration about the accelerating gradient and the phase slip. This paper describes the calculation method of the RF properties of the traveling wave in the DLS for β-varying acceleration and the status of the cell design of the quasi-constant gradient (quasi-CG) type for a part of the DLS that β is from 0.7 to about 0.8.

CALCULATION OF TRAVELING WAVE

The traveling wave in the DLS is obtained by the superposition of the standing waves in two different boundary conditions: Neuman boundaries ($E_T = 0$) and Dirichlet boundaries ($H_T = 0$) calculated by using Autofish solver in SUPERFISH [6]. Since the accelerating mode is set to $2\pi/3$, the standing waves in the DLS are calculated for every 180 degrees, i. e., with the same boundary conditions at both sides of the 1.5 cells, as shown in Fig. 3. The length of 1.5 cells is set as 1.5 times D determined by Eq. (1), and the cylinder diameter is adjusted so that the resonant frequency is 1296 MHz in the Neuman boundaries. The quality factor (Q), and the shunt impedance per unit length (Z) are calculated in the solver as

$$Q = \frac{2\pi f U}{P} \quad \text{and} \quad Z = \frac{\left|E_0'\right|^2}{P/1.5D} \tag{3}$$

respectively where f is the operating frequency, U is the stored energy in 1.5 cells, P is the power dissipation in 1.5 cells, and $\left|E_0'\right|$ is the accelerating gradient averaged over 1.5 cells. The group velocity of the traveling wave is expressed as

$$v_g = \frac{P}{W_{\mathrm{TW}}} = \frac{\frac{1}{2}\int E_{r,\mathrm{TW}}(r)H_{\phi,\mathrm{TW}}(r)\, dS}{W_{\mathrm{TW}}} \tag{4}$$

where W_{TW} is the stored energy per unit length, $E_{r,\mathrm{TW}}$ and $H_{\phi,\mathrm{TW}}$ are the transverse electromagnetic fields, and S is any cross-sectional plane [7]. Since the traveling wave with $U = 2$ J is the superposition of the two standing waves with $U = 1$ J, the group velocity at the left side of 1.5 cells is calculated by using $E_{r,\mathrm{SW(Dirichlet)}}$ and $H_{\phi,\mathrm{SW(Neuman)}}$ which are normalized electromagnetic fields to be $U = 1$ J as

$$v_g = \frac{\frac{1}{2}\int_0^b E_{r,\mathrm{SW(Dirichlet)}}(r)H_{\phi,\mathrm{SW(Neuman)}}(r)\, 2\pi r dr}{2\,\mathrm{Joules}\,/\,1.5D}. \tag{5}$$

These calculation processes are based on a tool package [8].

CELL DESIGN FOR CG-TYPE

Four kinds of structures were designed under the following conditions for the comparison of their accelerating gradient.

- The iris aperture $2a$ of the first cell is fixed to 40 mm.

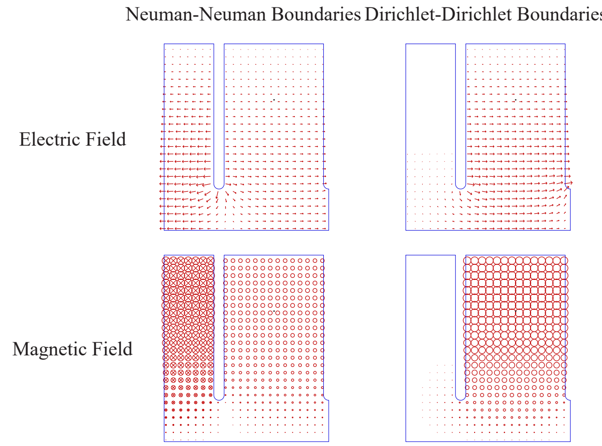

Figure 3: Standing waves in half of 1.5 cells.

- $2a$ of the last cell ($2a_{\mathrm{last}}$) is set to 37 mm, 38 mm, 39 mm, or 40 mm. The case of $2a_{\mathrm{last}} = 40$ mm corresponds to the CI type described above.
- $2a$ of the other cells is determined as the linear function of the cell number (n):

$$2a(n) = 40 + \frac{2a_{\mathrm{last}} - 40}{32} \times n. \tag{6}$$

To evaluate the average accelerating gradient E_0 of each cell, E_0 is calculated using the parameters above. The attenuation factor is calculated as

$$\alpha(n) = -\frac{1}{2}\frac{dP(n)}{P_{\mathrm{in}}dz} = \frac{\pi f}{v_g(n)Q(n)}. \tag{7}$$

Using this attenuation factor, the power dissipation per cell is calculated as

$$\Delta P(n) = P_{\mathrm{in}} \exp\left(-2\sum_{i=1}^{n-1}\alpha(i)D(i)\right)\times\left(1-\exp\left(2\alpha(n)D(n)\right)\right)$$
$$\simeq P_{\mathrm{in}}\prod_{i=1}^{n-1}\left(1-2\alpha(i)D(i)\right)\times 2\alpha(n)D(n) \;\; (\because \alpha D \ll 1). \tag{8}$$

With the above calculations, E_0 is given as

$$E_0(n) = \sqrt{\Delta P(n) \times Z(n)D(n)}\,/\,D(n) \tag{9}$$

where the quality factor in Eq. (7), and the shunt impedance per unit length are average values of the two types of boundaries. The calculated E_0 of the four structures are shown in Fig. 4. Since the structure with $2a_{\mathrm{last}} = 38$ mm (the green open circles in Fig. 4) had the most uniform gradient among the four, this is designated as the quasi-CG type and compared with the CI type in the next section.

DISCUSSIONS

The calculations of the relation between the phase slip and the accelerating gradient are performed in General Particle Tracer (GPT) [9]. The simulation setups are as follows.

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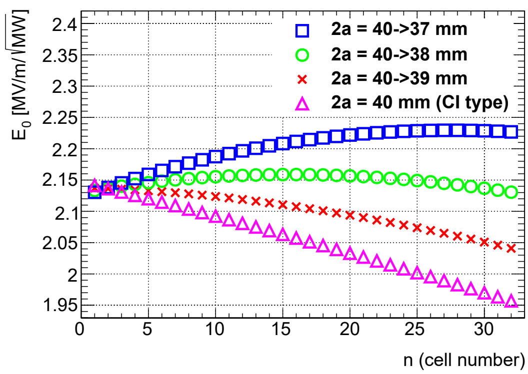

Figure 4: Comparisons of the accelerating gradients for the four designs.

- The field maps are calculated by SUPERFISH as described above.
- The traced particle is one muon with an initial kinetic energy of 42.7 MeV.

The phase slip at the middle of a cavity of each cell is calculated as

$$\phi(n) - (-10) = 360 f t_{\mathrm{beam}}(n) - 120n \tag{10}$$

where $t_{\mathrm{beam}}(n)$ shows the arrival time of the muon to the end of the n-th cell. Since the longitudinal electric field at each position ($E_z(z)$) should include $\cos(\phi)$, the average accelerating gradient is calculated as

$$E_0(n)\cos(\phi(n)) = \frac{1}{D(n)} \int_{D(n)} E_z(z)\, dz \tag{11}$$

This gives the energy gain per unit length. The results of the comparison between the quasi-CG type and the CI type are shown in Fig. 5, where the input RF power of 72 MW was set so that the phase slip of the last cell to be roughly zero in the quasi-CG type. The quasi-CG type has smaller phase slips and more uniform accelerating gradients than those of the CI type at the same input RF power. The energy gain of the quasi-CG type is 35.8 MeV closing to the design value of about 38 MeV and more than that of the CI type. However, the effective gradient of about 19 MV/m × cos(−10 deg) is lower than the expected one of 20 MV/m × cos(−10 deg). Increasing the input RF power to solve the deterioration in energy gain causes another problem: larger phase slip at the end of the structure. We consider that this problem arises from imperfect matching cell-length with beam energy in the quasi-CG structure.

The summary of simulated parameters of the quasi-CG type DLS is shown in Table 1.

SUMMARY AND PROSPECTS

We got the better solution with the quasi-CG type DLS with respect to small phase slip and uniform accelerating gradient. To get solutions with less energy deviation from the design, more rigorous simulations will be needed.

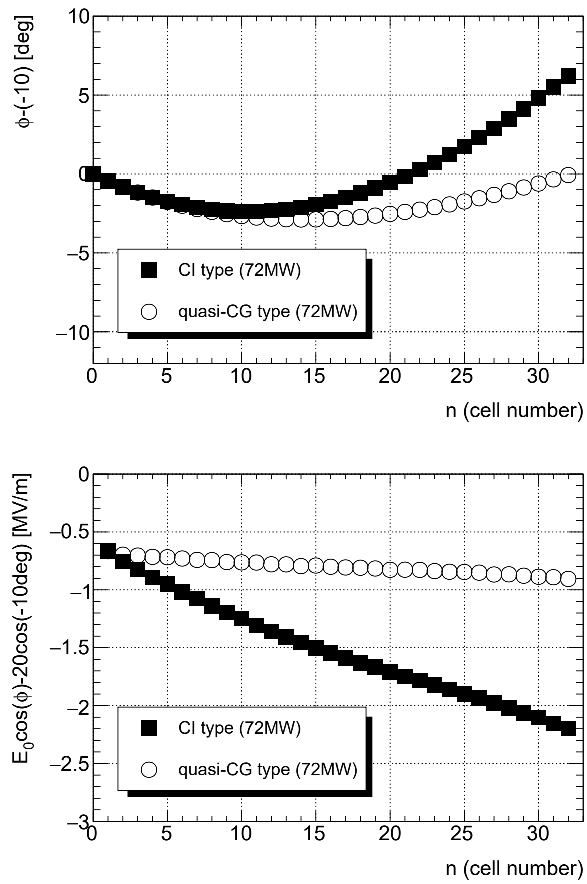

Figure 5: Comparison of the slips of the phase and the energy gain per unit length.

Table 1: Summary of Simulated Parameters of the Quasi-CG Type DLS

Input beam energy	42.7 MeV ($\beta = 0.702$)
Output beam energy	78.5 MeV ($\beta = 0.819$)
Operating frequency (f)	1296 MHz
Accelerating mode	TM01-$2\pi/3$
Synchronous phase	−10 deg
Number of regular cells	32
Input RF power (P_{in})	72 MW
Accelerating gradient (E_0)	~19 MV/m
Cell length (D)	54–63 mm
Disk thickness	5 mm
Iris aperture ($2a(n)$)	38–40 mm
Cylinder diameter ($2b(n)$)	179.5–180.3 mm
Quality factor ($Q(n)$)	17 000–19 000
Shunt impedance ($Z(n)$)	28–36 MΩ/m
Group velocity/speed of light	0.82–0.96 %
Filling time	0.69 μs
Field attenuation factor ($\alpha(n)$)	0.083–0.086

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