ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB027

REVIEW OF ACCELERATOR LIMITATIONS AND ROUTES TO ULTIMATE BEAMS*

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Abstract

Various physical and technology-dependent limits are encountered for key performance parameters of accelerators such as high-gradient acceleration, high-field bending, beam size, beam brightness, beam intensity and luminosity. This paper will review these limits and the associated challenges. Possible figures-of-merit and pathways to ultimate colliders will also be explored.

INTRODUCTION

As accelerators and colliders are being pushed to ever higher performance, the question of ultimate limitations naturally arises. A diverse set of physical limitations constrain the maximum acceleration gradient, the achievable bending field, the beam size, the beam brightness, and the luminosity. In addition, technology-dependent limits are also being encountered, e.g. ones related to material properties (critical current, tensile properties,...), while other boundaries are set by the accelerators' societal imprints (e.g. size, cost, electrical energy).

BENDING AND ACCELERATION

Superconducting accelerator magnets based on Nb-Ti, deployed till now, cannot reach field levels much above 8-9 T, as achieved at the LHC. The next generation of magnets, using Nb_3Sn superconductor, may reach fields up to 16 T [1], which is the target of the high-field magnet development for the FCC-hh. Advanced high-temperature superconductors may eventually allow for accelerator magnets with 20–30 T [2]. According to present knowledge, this path forward is unlikely to yield practical field levels above 100 T. Hence, the route of advancing macroscopic accelerator magnets may terminate at "only" about an order of magnitude higher fields than the present state of the art. To make a much larger step, crystals or nano-structures could offer a promising avenue. The effective field in a bent crystal can reach the equivalent of 1000 Tesla or more [3], at least a factor 100 above the LHC's dipole magnets. To minimize particle losses (e.g. caused by lattice vibrations), such crystals should be operated at cryogenic temperature. As the bending angle is determined by the crystal curvature, the acceleration could not be induced by changing the "dipole field" as in today's synchrotrons, but, e.g., by using induction acceleration [4]. Synchrotron radiation in the crystal channel may yield the ultimate emittance $\varepsilon^{\mathrm{QM}}$ (see below) [5] or be shielded [6].

A similar situation is found for the acceleration structures. Advanced conventional warm copper cavities or superconducting cavities may be pushed to 100 MV/m or perhaps a few times this value. Much higher acceleration fields are possible only if a new technology is deployed. It has been well demonstrated that plasma can sustain gradients of many GV/m. Currently, the main challenges for deploying this technique are beam quality, stability, staging, energy efficiency, and positron acceleration. Ultimately, thanks to much higher electron densities of $n_e \approx 10^{24}$ m^{-3}, crystals or nanotubes could reach gradients of order 100 TV/m [7]. The new technique of thin film compression provides a path to single-cycle coherent X-ray pulses and TeV/cm acceleration at solid state densities [8].

It is intriguing that the ultimate electromagnetic fields for either bending or acceleration can be obtained in crystalline structures. This, in fact, was the main motivation for organizing the 2020 ARIES workshop on "Applications of Crystals and Nanotubes for Acceleration and Manipulation" (ACN2020) [9].

ULTIMATE FIELD LIMITS

One "ultimate limit" on the acceleration $|dv/dt| = |qE/m|$ of a particle with mass m, charge q and velocity v, in an electric field E, follows from the Heisenberg uncertainty principle for energy E_b and time t [10] as

$$\Delta E_b\ \Delta t \geq \frac{\hbar}{2} \rightarrow \left|\gamma^3 \frac{dv}{dt}\right|_{\max} = 2\frac{mc^3}{\hbar}\ ,$$

which depends on the mass m of the charged particle. The equivalent maximum electrical field is

$$E_{\max} = 2m^2c^3/(q\hbar)\ , \qquad (1)$$

which for electrons amounts to $E_{\max,e} \sim 2.6\times10^{18}$ V/m, and for protons to $E_{\max,p} \sim 10^{25}$ V/m.

A more mundane limit arises if the average energy of photons emitted by synchrotron radiation becomes appreciable compared to the energy of the particle [11]. Classically computed, the average photon energy is $E_\gamma = 4/(5\sqrt{3})\hbar c\gamma^3/\rho$. Requiring this to be much smaller than the particle energy $E = \gamma mc^2$ yields the inequality

$$B\,\gamma \ll \frac{m^2c^2}{\hbar e}\frac{5\sqrt{3}}{4}. \qquad (2)$$

For electrons, the right-hand side evaluates to 9.6×10^9 T, for protons to 3×10^{16} T. At larger values of γ, this may

* This work was supported, in part, by the European Union's Horizon 2020 Research and Innovation Programme under grant agreement No. 101004730 (I.FAST).
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ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB027

become a significant constraint. For example, for a 3 TeV electron beam, requiring the average photon energy to stay below 1% of the beam energy, the magnetic field needs be less than 16 T.

Yet another "ultimate limit" on electromagnetic acceleration is given by the breakdown of the QED vacuum. The Sauter-Schwinger critical field for e^+e^- pair creation, E_{cr} is defined such that its product with the electron Compton wavelength equals the rest energy of the electron,

$$\hbar/(m_e c)\, eE_{cr} \sim m_e c^2 \,, \tag{3}$$

which amounts to $E_{cr} \approx 1.3 \times 10^{18}$ V/m — a value which, for electrons, is only slightly below the value from the uncertainty relation. In a similar manner, the Sauter-Schwinger critical magnetic field B_{cr} can be defined as

$$\hbar/(m_e c)\, ecB_{cr} \sim m_e c^2 \,, \tag{4}$$

which evaluates to $B_{cr} \approx 4 \times 10^9$ T.

At about 13 times the Schwinger-Sauter critical field, i.e. beyond the breakdown of the QED vacuum, the proton is predicted to become unstable, with a lifetime comparable to the usual lifetime of the neutron, and to decay into a neutron, positron and electron-neutrino [12].

LUMINOSITY

The luminosity of a collider can be written as

$$L = \frac{f_{rep} n_b N_b^2}{4\pi\sigma_x^*\sigma_y^*} F_{geom} = \frac{1}{4\pi}\frac{P_{wall}\eta}{\gamma m_p c^2}\frac{N_b}{\sigma_x^*\sigma_y^*} F_{geom} \,, \tag{5}$$

where f_{rep} denotes the repetition rate (or revolution frequency), n_b the number of bunches per pulse (or ring), N_b the bunch population, $\sigma^*_{x,y} = (\beta^*_{x,y}\varepsilon_{x,y})^{1/2}$ the rms horizontal or vertical beam size at the collision point, P_{wall} the wall-plug power, $P_{beam} = f_{rep} n_b N_b \gamma m_p c^2$ the beam power, $\eta = P_{beam}/P_{wall}$ the efficiency of converting wall-plug into beam power, m_p the particle mass, and $F_{geom} \approx 1$ a geometric factor, which includes the so-called hour-glass and pinch effects (and which we will drop in the following). The quantum mechanical limit for the normalized emittance is [13] $\gamma\varepsilon^{QM}_{x,y,z} \approx \lambda\!\!\!^{-}_p/2 \approx 0.2$ pm, with $\lambda\!\!\!^{-}_p$ the particle's de Broglie wavelength; the numerical value applies to electrons.

Optimistically, we may ignore both beamstrahlung [14, 15] (a limit from the collision process) and Oide effect [16] (a limit from the final focusing magnets), and assume that all three emittances are quantum limited, namely that $\gamma\varepsilon_{x,y,z} \approx \lambda\!\!\!^{-}_p/2$ (which in the case of polarized electrons would require bunches consisting of single particles, with a bunch charge of e). Taking into account the hourglass effect, we require $\beta^*_{x,y} \approx \sigma_z \approx \varepsilon_z/(\Delta p/p)_{rms}$. Inserting these relations into (5), the ultimate luminosity becomes [13]

$$L_{ult} \approx \frac{\gamma}{\pi}\frac{P_{wall}\eta}{\lambda\!\!\!^{-}_p \hbar c}\left(\frac{\Delta p}{p}\right)_{rms} \,, \tag{6}$$

which, for constant wall-plug power P_{wall} and efficiency η, increases linearly with beam energy γ, since the relative rms momentum spread of the beam must not exceed a few percent. However, as elementary cross sections decrease as $1/\gamma^2$, the luminosity should increase as γ^2. Equation (6) reveals that this can only be achieved if the product of wall-plug power and efficiency is increased linearly with the beam energy. Using energy recovery concepts, the effective efficiency η (beam power for a given wall-plug power) can be raised to values much beyond 100%. For convenience, we note that, in the case of electrons, $\lambda\!\!\!^{-}_e \hbar c \approx 1.2 \times 10^{-38}$ Jm^2. As an example, we consider an e^+e^- collider with 1 TeV c.m. energy, for which $\gamma \approx 10^6$. In this case, the ultimate luminosity (6) becomes $L_{ult} \approx (\gamma/10^6)\ (P_{wall}\eta/(1\text{MW}))\ (\Delta p/p)_{rms}/(1\%)\ 2.7 \times 10^{51}$ cm^{-2}s^{-1}, which is more than 17 orders of magnitude higher than today's state of the art.

A limit on the useful luminosity in e^+e^- colliders may arise from synchrotron radiation emitted in the field of the opposite bunch, the so-called *beamstrahlung* [15, 17], which degrades the luminosity spectrum of high-energy linear colliders. This effect becomes more severe the higher the collision energy [18]. The strength of the beamstrahlung is characterized by the parameter $\Upsilon \equiv \gamma(|B|+|E|/c)/B_{cr}$ [14], with B (E) the local magnetic (electric) field. The standard approach is to stay in a regime with $\Upsilon < 1$ and to limit the ratio N_b/σ^*_x, by colliding flat beams ($\sigma^*_x \gg \sigma^*_y$), An alternative approach at high energy is to operate at $\Upsilon \gg 1$ and to profit from the quantum suppression of beamstrahlung [19]. Here, both the number of beamstrahlung photons emitted per electron (positron) in the collision and the resulting energy spread scale as $(N_b\sigma_z)^{1/3}$ [19], which goes to zero in the limit of vanishing rms bunch length σ_z.

COLLIDER SIZE

A circular tunnel of about 87 km circumference had almost been completed for the SSC in Texas. The proposed CEPC and FCC projects require tunnels of similar size, 80–100 km. Somewhat larger tunnel circumferences of 233 and 300 km were considered for the VLHC in Illinois and for the Eloisatron in Italy, respectively.

A floating collider in the Gulf of Mexico ("Collider in the Sea") is proposed with a circumference of 1900 km [20]. Six times larger still would be a collider on the moon, with a circumference of up to 10, 900 km, that could be realized in collaboration with NASA [21].

It is intriguing that the depth of a tunnel on earth is limited to about 33 km (Mohorovicic discontinuity between the Earth's crust and the mantle), which constrains the length of a straight linear tunnel to 650 km [22, 23], while there is no similar size constraint for a circular tunnel.

Need the linear tunnel be straight? A study for CLIC [24] concluded that for a 3 TeV collider laser-straight tunnels are much preferred, in view of not only an additional 20% emittance growth from synchrotron radiation in the perfectly aligned case, but, in particular, the challenging vertical emittance preservation, that would be rendered much more difficult for a curved tunnel. The normalized emittance growth due to the earth curvature ρ can be estimated — in

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB027

an optimistic smooth approximation — as

$$\frac{d(\gamma\varepsilon_y)}{ds} \approx \frac{55}{48\sqrt{3}} \frac{r_e^2\gamma^6}{\alpha} \frac{\beta_y^3}{\rho^5} , \tag{7}$$

where α is the fine-structure constant, $\rho \approx 6,400$ km the radius of the earth, and we have approximated Sand's dispersion invariant as $\mathcal{H}_y \approx \beta_y^3/\rho^2$. The linac beta function typically scales as $\beta_y \propto \gamma^{1/2}$ [25], so that (7) yields a scaling $d(\gamma\varepsilon_y)/ds \propto \gamma^{15/2}/\rho^5$. We write $\beta_y = \beta_0\gamma^{1/2}/\gamma_0^{1/2}$ and $\gamma = \gamma_0 + as$, where a is the accelerating gradient in units of γ per meter. With $\gamma \gg \gamma_0$, integrating (7) yields

$$\Delta(\gamma\varepsilon_y) \approx \frac{55}{24\ 17\sqrt{3}} \frac{r_e^2}{\alpha a} \frac{\beta_0^3}{\gamma_0^{3/2}\rho^5}\ \gamma^{17/2} . \tag{8}$$

As an example, consider $\beta_0 \approx 1$ m, $\gamma_0 \approx 1$, and $a \approx 200$/m, which corresponds to a gradient of 100 MV/m for electrons. Requiring $\Delta(\gamma\varepsilon_y) < \gamma\varepsilon_y^{\mathrm{QM}}$, we find a limit of $\gamma \leq 10^6$. In other words, the maximum electron energy is limited to 500 GeV for a curved tunnel following the earth surface, if the vertical emittance growth shall not exceed $\varepsilon_y^{\mathrm{QM}}$.

The size limit for a terrestrial circular machine is set by the earth circumference to 40,000 km, as already considered by E. Fermi in 1954 [26, 27].

Assuming operation at the Schwinger-Sauter critical field, (3) or (4), the Planck energy of 10^{28} eV can be reached by either a circular or linear collider with a size of about 10^{10} m, or about a tenth of the sun-earth distance [6]. A Planck-scale linear collider of this size was first considered in Ref. [7], where it was judged to be "not an inconceivable task for an advanced technological society".

SCATTERING OFF THERMAL PHOTONS

The lifetime of electron and positron beams in a storage ring is ultimately limited by Compton scattering off thermal photons, with a maximum lifetime τ_{therm} given by [28, 29] $\tau_{\mathrm{therm}}^{-1} \approx n_{\mathrm{ph}} c \sigma_{\mathrm{C}}$, where $n_{\mathrm{ph}} \approx 20.2\ \mathrm{T}^3\ \mathrm{cm}^{-3}$ [28] denotes the density of thermal photons, and σ_{C} the Compton cross section that, for LEP, was ~ 0.665 barn, limiting the LEP beam lifetime to ~25 h [29]. At higher beam energies ($\gtrsim$100 TeV), also pair production [30] in the thermal photon field becomes important.

High-energy protons in space interact with photons of the 3 K cosmic microwave background, through the delta resonance, as $p+\gamma \to \Delta^+ \to n+\pi^+$ or $p+\phi^0$, restricting the proton energy to $\leq 5\times10^{19}$ eV (Greisen–Zatsepin–Kuzmin limit) after the passage of long distances (~160 million light-years). At 300 K, the maximum energy is reduced, by a factor 100, to 5×10^{17} eV, and the relevant distance by a factor $\sim 10^6$ to "only" 160 light-years.

MUON ACCELERATION

Recently a renewed interest in muon colliders can be noted. A disadvantage of muons is that they are unstable and decay with a rather short lifetime. The number of turns a muon survives in a storage ring or collider, n_{surv}, depends only on the dipole magnetic field B, as was already noticed by Budker [31]. Assuming a dipole filling factor of $F_{\mathrm{dip}} \approx 0.7$ the relation is $n_{\mathrm{surv}} = F_{\mathrm{dip}} eB/(2\pi)\tau_{\mu,0}/m_\mu \approx 300\ F_{\mathrm{dip}}\ B$[T], where $\tau_{\mu,0} \approx 2.2\ \mu$s denotes the muon lifetime at rest, and $m_\mu c^2 \approx 105.66$ MeV the muon rest mass. With 20-T magnets the muons survive for about 4000 turns. Neutrino radiation resulting from the muon decay may limit the maximum beam energy [32], at least for circular muon colliders, to a few 10's of TeV. Another key question to address is: How do we accelerate muons in less than a few thousand turns or a in a single passage to extremely high energies? It appears that, as of today, plasma or crystal/nanostructure acceleration would be the most suitable approaches. Implications for total voltage and power should be considered.

FIGURES-OF-MERIT

Common figures-of-merit (FoM) are either luminosity L or beam power per electric wall-plug power P_{wall}, or integrated luminosity L_{int} per consumed energy $E_{\mathrm{wall}} = \int P_{\mathrm{wall}} dt$. The correct figure of merit should be the physics reach per wall-plug power. This, however, depends on the specific physics programme. As an example, for hadron colliders the mass discovery reach scales with centre-of-mass energy E_{cm} and integrated luminosity L_{int} as $M \propto E_{\mathrm{cm}}^{2/3} L_{\mathrm{int}}^{1/6}$ [33, 34], which would suggest the following figure-of-merit:

$$\mathrm{FoM}_{\text{hadron collider}} = E_{\mathrm{cm}}^{2/3} L_{\mathrm{int}}^{1/6} / E_{\mathrm{wall}} . \tag{9}$$

CONCLUSIONS AND OUTLOOK

Present colliders operate far from the ultimate limits on beam density, luminosity, accelerating gradients, and energy. Energy recovery may prove essential to increase the luminosity of future highest-energy colliders with the square of the collision energy, at affordable wall-plug power.

The 2020 Update of the European Strategy Update (ESU) for Particle Physics [35] has set priorities for future high-energy accelerator development over the next 5–7 years. Beside emphasizing the need for an electron-positron collider Higgs factory, a key element is the further advance of SC magnet technology, flanked by R&D on energy recovery, muon colliders, and high-gradient acceleration.

Far-future options for accelerators have been assessed through a recent survey of the ARIES APEC network [36]. The survey outcome anticipates, over the next 10–15 years, the realization of high-energy high-current energy recovery facilities, the wide application of crystal bending, and a Gamma Factory [37]. On a longer 20–30 year time scale, two types of muon colliders, plasma acceleration, crystal or nanostructure acceleration are envisioned, as is the possibility of detecting gravitational waves using storage rings.

This paper has focused on ultimate limitations defined by physics, and not on the technology advancements. However, the ESU 2020 and the APEC survey strongly suggest that also the present state of the art in accelerator technology still leaves large room for further significant improvements.

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