COMPARISON OF ACCELERATOR CODES FOR SIMULATION OF LEPTON COLLIDERS

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Abstract

This paper compares simulations between SAD, MADX and the PTC implementation in MADX for the design studies of the FCC-ee. On-momentum and off-momentum optics are explored for the various programs. Particle tracking with and without synchrotron radiation are used to compare amplitude detuning and emittance. Finally, this paper outlines how well-established SAD features such as tapering have recently been integrated into MADX.

INTRODUCTION

Accelerator Codes

The codes tested in the scope of this study are the Strategic Accelerator Design (SAD) code [1], which is written and maintained within the Japanese High Energy Accelerator Research Organization (KEK) and the Methodical Accelerator Design (MADX) code [2], which in turn is maintained by the European Organization for Nuclear Research (CERN). Along with the default MADX environment, this study also examined the Polymorphic Tracking Code (PTC) [3] integration within MADX.

The main uses of SAD is the design, optimisation and simulation of the KEK-B-Factory [4]. As such, whilst it is capable of simulating all kinds of particle accelerators, the code has been developed to include many features relevant to circular lepton colliders. Moreover, these features have been used and compared to real data. MADX developments over the last two decades where much stimulated by the work for the extensive hadron accelerator infrastructure found at CERN. A summary of more recent MADX developments for both hadron and lepton machines can be found in [5]. Nonetheless, all the codes compared in this study are intended as general accelerator design codes and should, in theory, yield similar results when simulating the same machine.

FCC-ee Design

The lattices used for this comparison study are those of the Future Circular Lepton-Lepton Collider (FCC-ee) [6]. The FCC-ee aims to accelerate electrons and positrons to an energy of up to 187.5 GeV and collide these in two or four high luminosity interaction regions. The optics of the FCC-ee lattices are initially designed and optimised using SAD, however, a number of studies are done using MADX and lattice files generated using the SAD-MADX translator [7]. The reliability of these studies depends on the physics in both codes being identical.

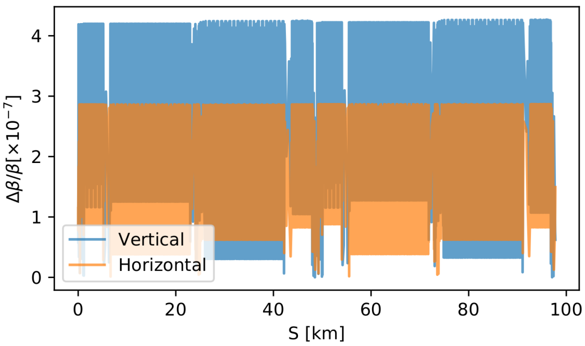

Figure 1: Relative error in β-function obtained from MADX and SAD.

For the purpose of this study the ZZ lattice version *FCCee_z_213_nosol_13* was used as a representative case for tests without radiation, since the ZZ run foresees the most squeezed interaction point optics, whilst the tt lattice version *FCCee_t_213_nosol_13* was used for studies involving radiation, since the radiation is the highest at this energy.

LINEAR OPTICS

The first test performed in this study was to check whether the linear optics obtained from MADX and SAD agree. To do this, the twiss was computed for both programmes and saved to a file. The files were read with a python script that matches the longitudinal location of each entry to map the optical functions computed in SAD to those in MADX. Figure 1 shows the relative error in the β-function between the codes. The error oscillates around order 10^{-7} which indicates some form of numerical error or a different tolerance in the closed orbit search but suggests that the physics agrees. Similar results are observed when plotting the phase advance determined by MADX and SAD.

DETUNING

Once it was established that both codes agree in the linear regime, the next test was to compare the results obtained when deviating from design momentum and closed orbit. This was done by determining the tune in these situations and checking how much it deviates under these conditions. Since the tune is very sensitive to deviations in the optics it is an excellent parameter for bench-marking codes.

Momentum

The momentum detuning was tested by computing the closed twiss of the full ring using MADX, MADX-PTC and SAD whilst introducing momentum offsets. All programs were able to find a stable closed solution for a range of $\delta p/p \in [-0.013, 0.011]$, whilst SAD was also

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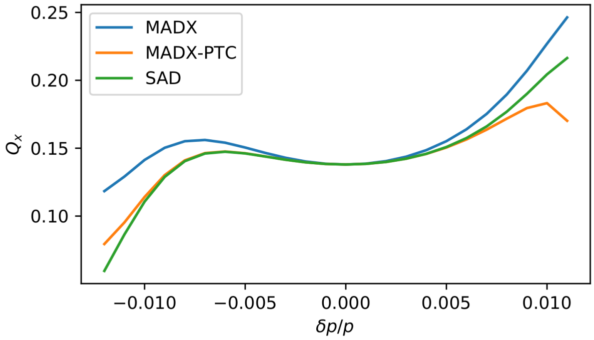

Figure 2: Horizontal momentum detuning obtained using MADX, MADX-PTC and SAD.

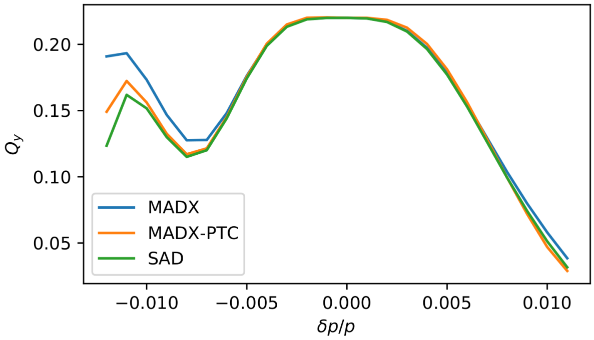

Figure 3: Vertical momentum detuning obtained using MADX, MADX-PTC and SAD.

able to find a stable solution for a slightly larger range of $\delta p/p \in [-0.015, 0.014]$. Figures 2 and 3 show the horizontal and vertical tunes determined by all three codes over the interval in which all codes were able to find a stable solution in steps of $\delta = 0.001$.

From Figs. 2 and 3 one can see that there is a generally good agreement on the momentum detuning between all three codes, with a slightly better agreement between MADX-PTC and SAD than between MADX and the other two codes. All three codes show that there is a region of $\delta p/p \approx \pm 0.003$ where the tune is almost constant which is a result of the chromaticity correction by the sextupoles in the lattice. Beyond this region, all codes show that the tune starts to change and rapidly approaches the integer resonances.

Amplitude

In SAD, the detuning due to particle amplitude needs to be computed from data obtained by tracking particles at various amplitudes. To do this, one hundred particles with a horizontal offset were tracked in SAD, MADX and MADX-PTC. The particles were generated at the interaction point at locations corresponding to one hundred horizontal beam sizes, σ_x, in steps of one σ_x. In order to track in MADX, the finite element lattice had to be converted to a sequence made of thin slices. The results from both codes were read in using a python script and the tune and action were determined using the harpy algorithm [8]. The results from this analysis are shown in Fig. 4.

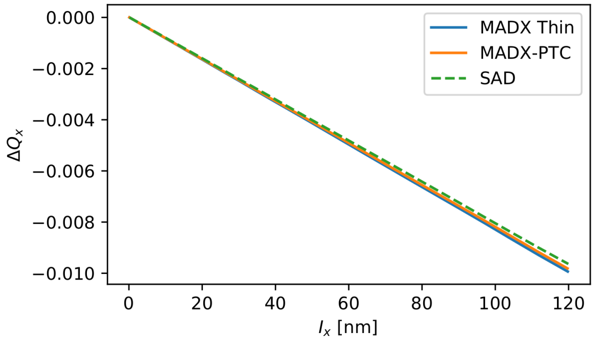

Figure 4: Horizontal amplitude detuning obtained using MADX, MADX-PTC and SAD.

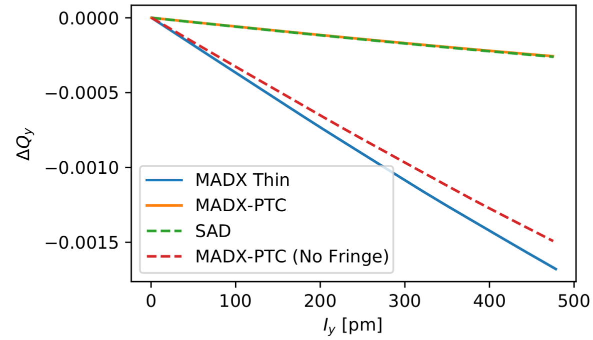

Figure 5: Vertical amplitude detuning obtained using MADX, MADX-PTC and SAD.

Figure 4 shows that there is almost perfect agreement between the detuning behaviour in all three codes. In the course of this study it was found that it is very important to have a sufficient number of slices in MADX and integration steps in SAD and MADX-PTC for the final focus doublet and the interaction region sextupoles. By design, these magnets are very strong and the β-functions are large in these locations. It was found that convergence was reached when the EPS parameter was set to 0.01 in SAD, the number of integration steps was set to 15 in MADX-PTC and the number of thin slices was set to 20 in MADX for these elements.

The same was repeated with 30 particles, spaced by one vertical beam size each and the results are shown in Fig. 5. In this case, there is very good agreement between MADX-PTC and SAD, however, the results in MADX differed greatly from the ones of the other codes. A key difference between MADX thin tracking and MADX-PTC is that the former does not include fringe fields, whilst in the latter fringe fields from dipoles are included. Indeed, they could have a larger impact than in the horizontal plane since the detuning is significantly lower. The effect of this was checked by repeating the MADX-PTC simulation but with all fringe fields turned off. This is also shown in Fig. 5. In that case, a much better agreement is observed between MADX and MADX-PTC.

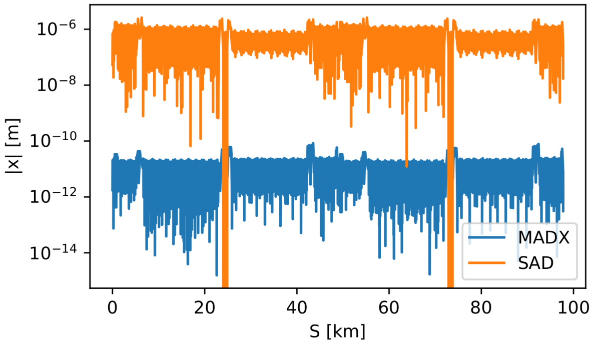

Figure 6: Residual horizontal orbit after tapering in MADX and SAD.

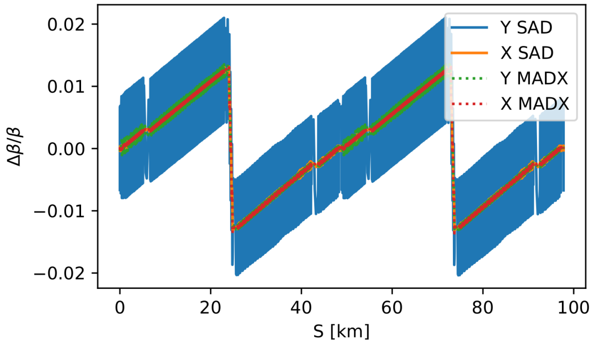

Figure 7: Deviation of β-functions between non-radiating and tapered radiating lattices in MADX and SAD.

RADIATION

Tapering

One of the most important aspects for high energy lepton colliders is how radiation affects the beam dynamics. In the case of the FCC-ee, the large amount of synchrotron radiation results in the beam energy varying drastically throughout the machine. Without correcting for this energy change, the beam would experience bending, focusing and higher order magnetic effects that differ from what it would experience in the absence of radiation, leading to large horizontal orbits and changes in the optical functions.

To compensate for this, the magnet strengths have to be locally adjusted so that the beam experiences an effect equivalent to what it would experience without radiation. This compensation can be activated in SAD by switching on the tapering flag. More recently (since version 5.6.00), a similar functionality has been introduced in MADX. In order to check, whether the new MADX tapering is as effective as in SAD, the horizontal orbit obtained from both codes is plotted in Fig. 6.

Figure 6 shows that the MADX implementation of tapering results in an even smaller residual orbit compared to SAD. Again, one possible cause for the difference in residual orbit might be different tolerances in the closed orbit search. Moreover, the optics obtained from both implementations reduces the change in the β-function compared to the case without radiation to the order of 1 %, as presented in Fig. 7.

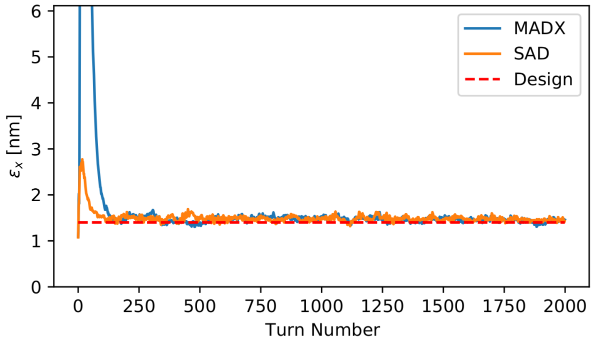

Figure 8: Emittance computed from tracking 5000 particles in MADX and SAD.

Emittance from Tracking

The radiation behaviour of the lattice can also be tested by tracking particles through the tapered lattices whilst including radiation damping and excitation effects. 5000 Gaussian distributed particles were initiated and tracked for 2000 turns in tapered MADX and SAD lattices. The slices and integration steps in both cases were set up in the same way as for the amplitude detuning study and all radiation effects were included. At each turn, the average position and canonical momentum of the particles was computed in order to compute the horizontal emittance. The results are presented in Fig. 8, including the design emittance.

Figure 8 shows that after an initial increase, the particles in both codes very quickly converge to a distribution characterised by an emittance almost identical to the design emittance. The initial increase is significantly larger in MADX, however, the convergence shows that the radiation effects from tracking in both codes are in good agreement.

DISCUSSION

As shown above, MADX is able to compute linear optics that are almost identical to those found by SAD. The study also showed that there is a good agreement between SAD and MADX-PTC when modelling non-linear effects as long as the simulations are set up in the right way and it also highlighted that since the vertical detuning is very small, the lack of fringe fields in MADX can have a relatively large effect in this case. This information is very useful for potential dynamic aperture studies using both codes.

Moreover, it was demonstrated that the recent MADX tapering implementation matches that of SAD and that the codes bare similar results for dynamics with radiation. This sets the path for future studies that can build on these findings, including emittance calculations using matrix methods and tapered lattices, effects from solenoids and studies including beam-beam effects.

ACKNOWLEDGEMENTS

We would like to thank Katsunobu Oide for his help with the SAD code.

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