

TRACKING COMPLEX RE-CIRCULATING MACHINES WITH PLACET2

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Abstract

We present the latest version of the multi-particle tracking package PLACET2. This software was designed to track multiple electron bunches through re-circulating machines with complex topologies, such as the recombination complex of the Compact Linear Collider (CLIC), energy-recovery linacs such as the Large Hadron-Electron Collider (LHeC), racetracks and others. This update also expands the capabilities of PLACET2 to track heavier particles such as muons. In addition to simulation, PLACET2 was also developed to allow beamline optimization scans, evaluating beam properties and tuning the beamline parameters at runtime either standalone or accessing the optimization tools present in the Octave and Python packages, with which it interfaces. This paper presents and benchmarks PLACET2's latest features, such as coherent and incoherent synchrotron radiation, long and short wakefields and power extraction.

INTRODUCTION

As the use of charged-particle accelerators becomes more common within the scientific, industrial and medical communities, the operational requirements of such machines becomes more demanding. This in turn increases the need for effective simulation tools during both design and operation phases of these facilities. Tracking software packages such as PLACET [1] and Elegant [2] model a charged-particle bunch as a collection of macro-particles and transports it through the accelerator lattice. In this paper, we present the most recent update to PLACET2 [3], and what new features this code offers. We will focus on the implementation of coherent synchrotron radiation (CSR), how its evaluation compares to known results and what impact it can have on beam dynamics.

FEATURES OF PLACET2

PLACET2 is the first tracking code that offers the possibility to track several multi-particle bunches through re-circulating topologies such as those needed for energy recovery linacs like the LHeC or complex recombination schemes such as the one present in CLIC's Drive-Beam, see Fig. 1. The main advantage PLACET2 has over its predecessors is the capacity to describe the lattice as a series of interconnected beamlines and transfer bunch from one beamline to the other in a realistic manner. This enables the tracking of multiple bunches through different paths, simultaneously performing optimization scans that target all possible pathways. It also allows the study of time-dependent interaction between bunches such as long range wakefields.

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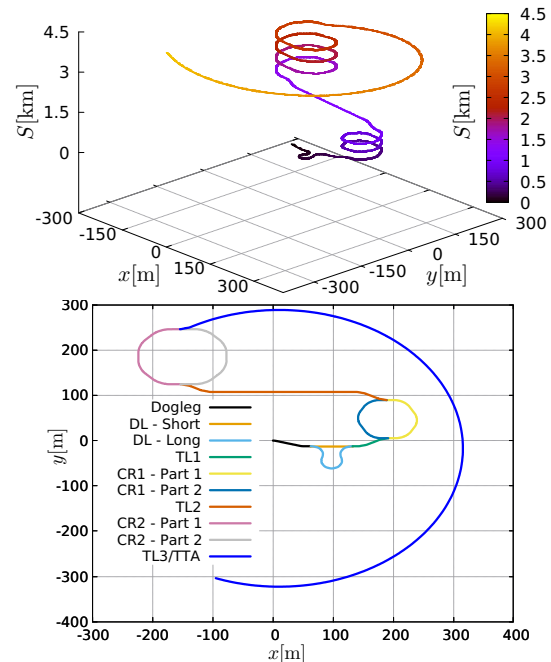


Figure 1: CLIC Drive-Beam recombination complex: On the top panel, uncoiled single-pathway lattice developed for MAD-X and PLACET. On the bottom panel, 24-pathways lattice developed for PLACET2.

Even though its development has targeted complex topologies, PLACET2 is, like its predecessor, an extremely versatile tool capable of tracking non-linear and time-dependent effects such as coherent and incoherent radiation, transverse and longitudinal wakefields, and chromatic effects. With this update, we also added to PLACET2 the ability to track different bunch species and to create particle distributions with non-uniform weights. The former was done with the possibility of future muon colliders in mind, while the latter to allow for more computationally permissive halo studies.

COHERENT SYNCHROTRON RADIATION

The emission of CSR often has a significant impact on both the beam energy profile and the transverse emittance. In order to accurately model the impact of CSR on the bunch distribution along the magnet, PLACET2 divides the bending magnet in a user-defined number of sectors and applies the CSR wakefield at the end of each sector. Other non-linear effects such as incoherent synchrotron radiation and wakefields are modelled in a similar manner.

Implementation of the CSR Kick

The treatment of CSR in PLACET2 follows [4]. At the end of each sector of length ct , we start by computing a 1D mesh $\lambda(z)$ containing the particle density along the bunch. Where z is defined from $z_{\min} = 0$ at the tail of the bunch to z_{\max} at the head of the bunch. Given an ultra-relativistic bunch, the change of energy for a particle at position z is computed as

$$\frac{dE(z, \phi)}{d(ct)} = -\frac{2Nr_c mc^2}{3^{1/3} \rho^{2/3}} \left\{ \frac{\lambda(z - z_s) - \lambda(z - 4z_s)}{z_s^{1/3}} + \int_{z - z_s}^z \frac{\lambda'(z')}{(z - z')^{1/3}} dz' \right\}, \quad (1)$$

where N is the number of particles in the bunch, ρ is the magnet's bending radius, ϕ is the current angular position of the bunch, e is the electron charge, and $z_s = \rho \phi^3 / 24$ is the CSR slippage length.

Notice that the integral in Eq. (1) (in the following identified as I_{CSR}) is not defined when $z' = z$. This poses an issue when computing this integral numerically. To avoid truncating the integral when the computation approaches the singularity, we opted to rewrite it using integration by parts as,

$$I_{\text{CSR}}(z, \phi) = \int_{z - z_s}^z \frac{\lambda'(z')}{(z - z')^{1/3}} dz' = \frac{3}{2} \left\{ z_s^{2/3} \lambda'(z - z_s) + \int_{z - z_s}^z (z - z_s)^{2/3} \lambda''(z') dz' \right\}. \quad (2)$$

Despite requiring the computation of the second derivative of the particle density (λ''), this formulation avoids the singularity and it's therefore more robust.

Downstream of the bending magnet, the evolution of the CSR wakefield has been implemented following [5]. We found that the behavior of the CSR in this regime can be reasonably approximated by an exponential decay of the CSR wakefield computed at the exit of the bending magnet with a decay constant of $L_{\text{overtaking}} = (24\sigma_z \rho^2)^{1/3}$, where σ_z is the rms bunch length. This approximation is similar to the one implemented in Elegant [6].

Benchmark

In order to benchmark the implementation, we chose to reproduce the results of a seminal paper on CSR by Dohlus, [7]. PLACET2 was used to generate and track a bunch distribution with 10^6 particles of initial energy $E = 0.1$ GeV and a longitudinal Gaussian distribution with $\sigma_z = 50 \mu\text{m}$. All transverse coordinates were set to 0. The bunch was tracked through a 30 cm magnet with a total angle of $\phi_m = 0.2$ rad. The magnet was divided in 1 cm sectors for the purpose of the CSR evaluation and the bunch longitudinal mesh was set such that $N_{\text{bins}} = 60$. As shown in Fig. 2, the evolution of the CSR wakefield computed by PLACET2 shows perfect agreement with Fig. 2 of [7] as the bunch travels through a bending magnet. Additionally, the bunch was tracked through a 50 cm drift downstream of the magnet to compare

with Fig. 3 of [7]. As shown in Fig. 3, the exponential decay approximation accurately models the wakefield amplitude decrease after the magnet.

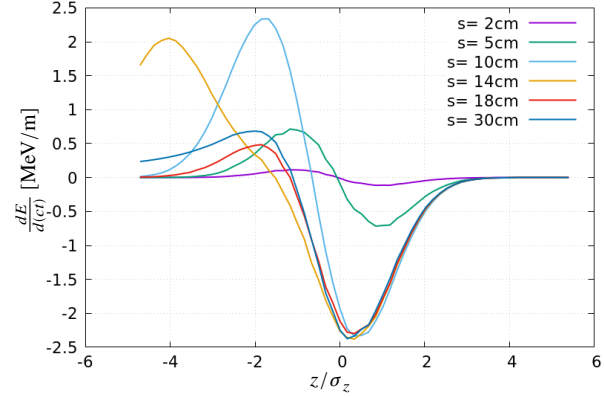


Figure 2: CSR benchmark: Evolution of the CSR wakefield generated by 1 nC, $\sigma_z = 50 \mu\text{m}$ Gaussian line bunch as it travels through a 1.5 m radius dipole.

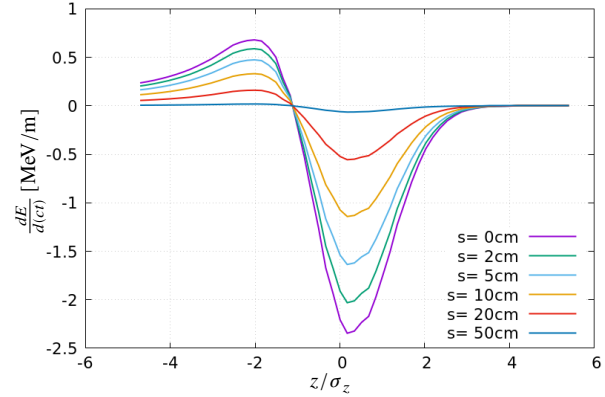


Figure 3: CSR downstream benchmark: Evolution of the CSR wakefield downstream of the magnet. Bunch and magnet parameters are similar to those of Fig. 2.

As a final benchmark, we chose to compare PLACET2's evaluation of CSR to that of its predecessor's (which, from now on, we will refer to as PLACET1 for clarity) under the same conditions ($N = 10^6$ and $N_{\text{bins}} = 60$). In addition to the implementation of I_{CSR} , the two codes differ in how they compute and interpolate $\lambda(z)$ and $\lambda'(z)$, with PLACET1 applying a Savitzky-Golay filter to smooth the mesh. Figure 4 shows the computed CSR wakefield for PLACET2 and PLACET1 with different polynomial orders (O) and half filter lengths (h_f). While PLACET1's is effective at reducing numerical noise when tracking bunches with low N , the results of Fig. 4 show us that applying such a filter may inadvertently lead us to underestimate the impact of CSR on the bunch. Additionally, this method places the burden of the user to apply reasonable filter parameters with since, as shown in the figure, an incorrect choice of those may cause the wakefield to be underestimated by over 50%. For these two reasons, we opted against applying any form of function smoothing in PLACET2's implementation.

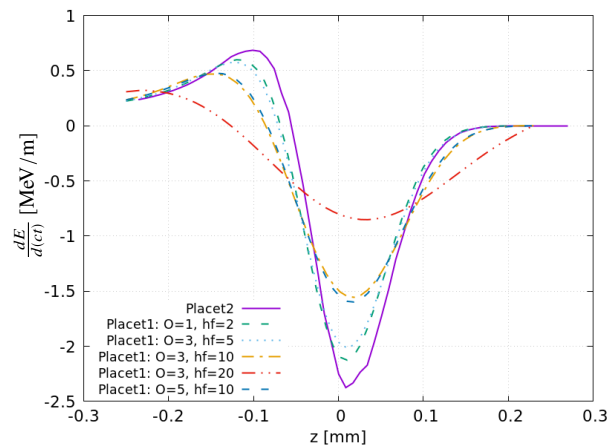


Figure 4: Comparison between PLACET2 and PLACET1 while using different fitting parameters. Bunch and magnet parameters are similar to those of Fig. 2 at $s = 30$ cm.

APPLICATION: DRIVE-BEAM BUNCH COMPRESSOR

As an example of application, we chose to evaluate the effect of CSR on an arc bunch-compressor for the Compact Linear Collider (CLIC) Drive-Beam accelerator. CLIC's Drive-Beam is projected to have a 180° arc between linacs one and two. This arc serves a double purpose: reducing the overall footprint of the linac tunnel, and acting as a bunch-compressor, reducing the bunch length from 3 mm to 1 mm. The energy of the electron beam at this stage is of 300 MeV, allowing us the possibility of exploring compact designs, such as the one denoted in Fig. 5, which is a simple four-bend achromat which tunable R_{56} . The combination of high bunch charge (8.4 nC) with low bending radius (≈ 56 cm) dipoles and relatively short σ_z makes this design highly susceptible to the effects of CSR.

Figure 6 shows the horizontal emittance (ε) as the bunch travels through the bunch-compressor and its relationship to the bunch length. We can also verify that PLACET2's results are in general agreement with a theoretical estimation performed in accordance to [8]

$$\varepsilon = \varepsilon_0 \sqrt{1 + \frac{\eta^2 + (\beta\eta' + \alpha\eta)^2}{\beta} \frac{\sigma_{\delta, \text{CSR}}^2}{\varepsilon_0}}, \quad (3)$$

where ε_0 is the initial horizontal emittance, α and β are the horizontal twiss functions, η and η' are horizontal dispersion

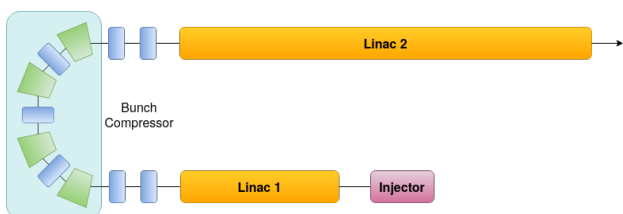


Figure 5: Four-bend bunch-compressor schematics for the CLIC Drive-Beam accelerator.

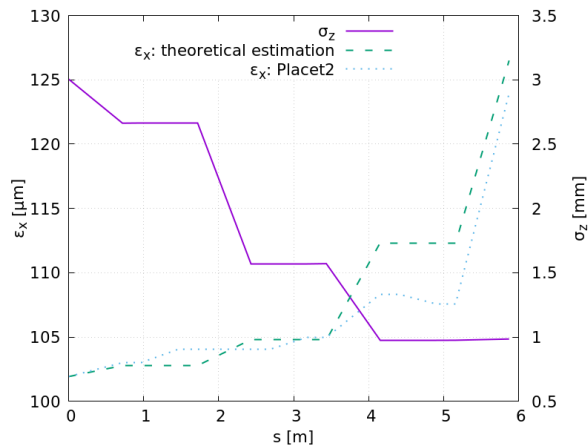


Figure 6: Emittance growth (left-side axis) and bunch length (right-side axis) along the bunch-compressor. Comparison between PLACET2 and a theoretical estimation.

and its derivative, and $\sigma_{\delta, \text{CSR}}$ is the increase in energy spread caused by CSR which, for a Gaussian electron bunch, is given by

$$\sigma_{\delta, \text{CSR}} = 0.2459 \cdot r_e \frac{N}{\gamma_{\text{rel}}} \frac{\phi_m \rho^{1/3}}{\sigma_z^{4/3}}, \quad (4)$$

where r_e represents the classical electron radius and γ_{rel} is the Lorentz factor.

CONCLUSION

We present the latest updates to PLACET2, a modern multi-bunch multi-particle tracking code developed to model the beam dynamics of any accelerator facility, regardless of the complexity of its topology. This new code allows for a multitude of studies such as multi-path optimization, tracking multiple particle species, defining non-uniform particle weights for halo studies, short and long range wakefields, and coherent and incoherent synchrotron radiation.

Amongst these features we focused on the recent developments on the effects of CSR on the bunch, presenting a new implementation for the effect, as well as its successful benchmark against known results and its application on a new design for the CLIC Drive-Beam bunch-compressor. The results of this latest study revealed an emittance growth of 25% due to CSR, indicating that the design will require mitigating measures.

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