

# THREE APPROACHES FOR COMPLETE MEASUREMENT OF THE TRANSVERSE BEAM OPTICS ALONG THE FERMILAB MUON CAMPUS EXTRACTION LINE\*

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## Abstract

Traditionally, the process of measuring the optical parameters of a beamline has employed the use of one of two standard methods, namely the three-screen method or a quadrupole magnet scan. Both require either an area of zero dispersion to perform the measurements or knowledge of the dispersion function and momentum spread beforehand in order to provide accurate results. There is however a third method that can be used to measure the standard optical parameters, the beam parameters, the dispersion function, and the momentum spread simultaneously. This method, aptly named the six-screen method, is an extension of the more standard three-screen method. Utilizing the simulation environment of G4beamline, we simulated the 8 GeV proton beam in the M4 beamline and measured the optical and beam parameters using the two standard approaches. Those results were then used as a reference to check the viability of employing the less standard six-screen method in the M4 line. If shown to be a viable option, the six-screen method could be used to retrieve the dispersion function and momentum spread of the beam without needing to change the energy of the beam.

## INTRODUCTION

There are two standard approaches to deriving the Twiss parameters of a beam. The first approach is based on measuring the transverse beam profile at a single monitor (screen) while varying the strength of a quadrupole magnet upstream of the screen location. The second approach, referred to as the “3-screen-method”, is based on measuring the beam profile at three different screens together with the measurement of the dispersion function at those same three locations. For both methods, if the dispersion is not known then a separate measurement technique is needed to find the dispersion, or the approaches need to be employed in an area with zero dispersion.

A third, more novel approach is the aptly named “6-screen-method” and is essentially an extension of the 3-screen-method with the added benefit of providing a calculation of the transverse beam dispersion ( $D$ ) and momentum dispersion ( $\delta$ ). The 6-screen-method provides the most complete set of measurements but it is also the hardest to implement. In order to provide accurate measurements for the transverse

dispersion, the 6-screen-method has strict restrictions on the number of dipole magnets within the measurement area and the placement of screens providing profile data. This paper explores the viability of reliably using the 6-screen-method along the Fermilab Muon Campus extraction line to provide a complete measurement of the beam’s Twiss parameters.

## THE STANDARD APPROACHES

The standard approaches were both simulated in sections of the beamline that have zero dispersion or very little dispersion. Referencing Fig. 1, 75 m to 200 m along the muon cam-

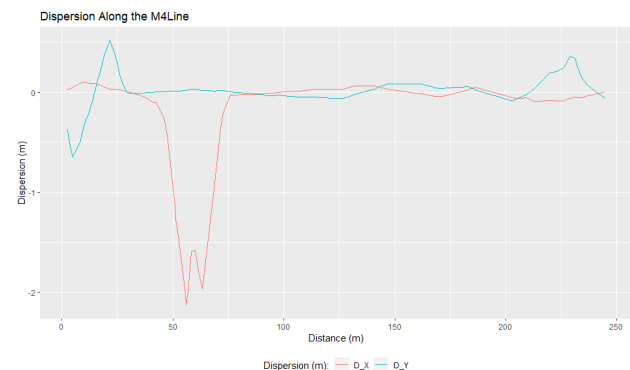


Figure 1: Tracking dispersion along the M4Line.

pus extraction line provides the necessary “nearly dispersion free” environment for the use of the standard measurement approaches.

## Quad Scans

The quad scan technique employed in this paper is a well-known method for measuring the transverse optics of particle beams and as such, its theory is well documented [1]. Figure 2 provides a visual representation for the general setup

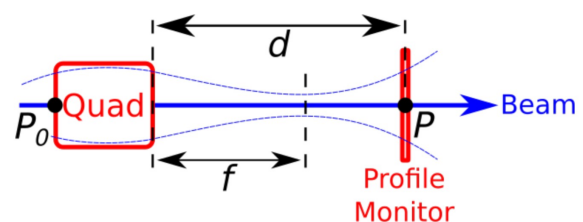


Figure 2: Visual representation showing the principle of the quad scan technique.

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of a quadrupole magnet scan. When "scanning" through different quad strengths, "F" in Fig. 2 either increases or decreases, providing several sets of transverse beam profile data. In total, five quadrupole magnets were successfully scanned, shown in Fig. 3, providing the Twiss parameters in both the horizontal and vertical planes.

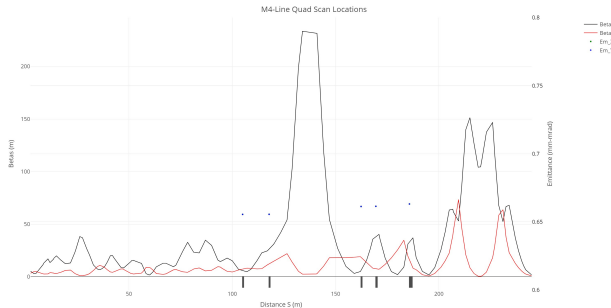


Figure 3: Tracking the beta functions along the muon campus extraction line. The dots represent the location and the emittance of the five quads that were scanned. The black rectangles at the bottom of the figure represent the relative size and location of the quadrupole magnets.

### Three Screen Method

The 3-screen-method, like the quad scan method, requires a dispersion free or nearly dispersion free zone. The idea

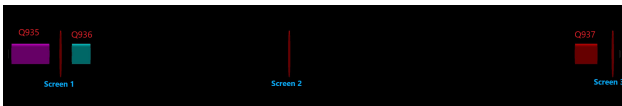


Figure 4: The 3-screen-method setup in G4beamline for extracting the Twiss parameters at quad Q935.

is that as long as we know the transfer matrices for each element of the beam between the quadrupole magnet we want to measure and the third screen we are using, then we can extract the Twiss parameters from the three different beam profiles that are collected. Looking at Fig. 4, we need the transfer matrices for each magnetic element and each drift space. As with the quad scan technique, the math behind the 3-screen-method is well known [2].

An advantage of the 3-screen-method over the quad scan technique is that it is much less restrictive in terms of where it can be applied in the beamline. As long as you have the transfer matrices for each element in the beam between the screens then this method can be applied.

### Standard Methods Results

To show that both the quad scan technique as well as the 3-screen-method are valid measurement approaches, the results from taking data at quadrupole magnet Q935 were analyzed and compared.

Looking at Fig. 5, you will notice that overall, the quadrupole scanning technique gives better results for both planes in phase space. In the horizontal plane, both standard

	Alpha	Beta (m)	Gamma (1/m)	Emittance (mm-mrad)
<b>X-Direction</b>				
From G4beamline	-11.647	46.996	2.908	0.661
From Quad Scan	-11.704	50.543	2.730	0.685
% Error	0.483	7.547	6.123	3.692
From 3-Screen	-12.029	49.716	2.931	0.656
% Error	3.227	5.284	0.779	0.743
% Difference	2.742	1.649	7.091	4.370
<b>Y-Direction</b>				
From G4beamline	11.981	34.083	4.241	0.252
From Quad Scan	11.679	32.652	4.208	0.259
% Error	2.524	4.107	0.768	2.702
From 3-Screen	17.563	41.962	7.375	0.253
% Error	46.589	23.118	73.889	0.404
% Difference	40.246	24.955	54.686	2.263

Figure 5: Comparison of results for Twiss parameters at Q935.

methods provided results that agreed with each other.

One of the issues with trying to determine where to apply either of the standard methods is worrying about dispersion. In general, a separate measurement is needed to measure the dispersion and the momentum spread of the beam. This idea is why the next method is so alluring.

## THE SIX SCREEN METHOD

The 6-screen-method is essentially an extension to the 3-screen-method. With three more screens worth of transverse beam profile data, three more unknowns can be solved for. The math for this method is quite complex and I will be deferring to [2] for the derivations of the underlying theory. The main advantage for the use of this technique is that you are no longer required to locate a dispersion free zone of the beamline. The 6-screen-method can be used to derive not only the standard Twiss parameters ( $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\epsilon$ ) but also the dispersion function ( $D$ ), the slope of the dispersion function ( $D'$ ) as well as the momentum spread ( $\delta$ ).

In order for the method to be valid however, new restrictions arise in the form of how many screens are available to provide profile data and the location of the bending elements in relation to the screens. Recall that when working with magnetic bending elements, a rectangular dipole for example, the transfer matrix becomes the extended, 3X3 matrix given by

$$M_{extended} = \begin{pmatrix} C_i & S_i & \xi_i \\ C'_i & S'_i & \xi'_i \\ 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

where the last column captures the effects of the bending. Note that the last column in Eq. (1) is only populated if the bending magnet acts in the plane where the calculations are taking place. So, if you are looking in the horizontal plane, only horizontally bending dipoles contribute to the last column, otherwise it is treated as a drift space. With those restrictions in mind, we are again limited to only a few locations in the extraction line where this method could be applied. The method was first applied to a small test section simulated around the second set of horizontally bending dipoles shown in Fig. 6.

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M4Line Dispersion with Dipole and Screen Locations

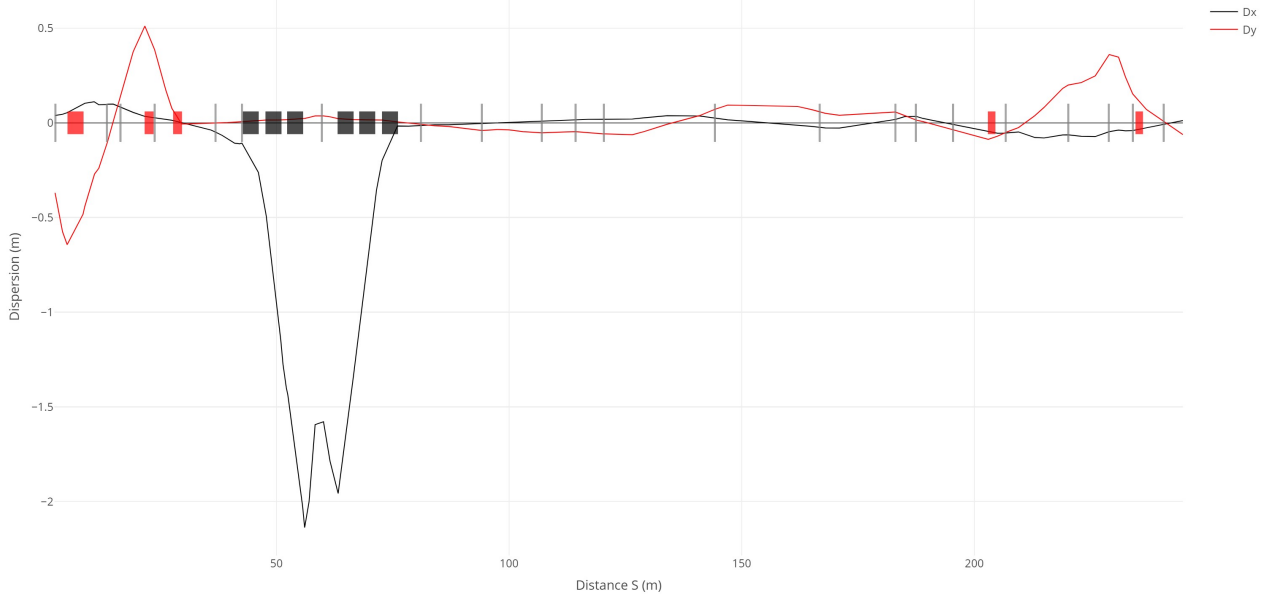


Figure 6: This plot tracks the dispersion along the Muon Campus extraction line. The red rectangles represent vertically bending dipoles, the black rectangles represent horizontally bending dipoles and the grey lines represent profile monitors.

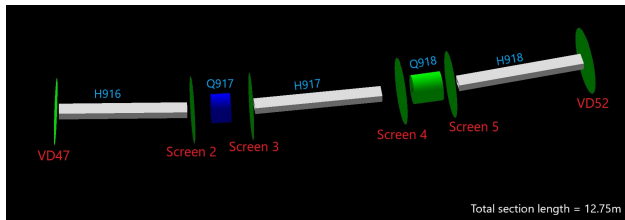


Figure 7: Simulated test section for the 6-screen-method. "VDs" and "screens" are simulated profile monitors. The long grey rectangles are dipoles and the cylinders are quadrupole magnets.

I should note that the small test section shown in Fig. 7 contains virtual screens added in with the intent of testing the functionality of the method itself. Looking at Fig. 8, the

	Alpha	Beta (m)	Gamma (1/m)	Emitance (mm-mrad)	Dispersion (m)	D-Prime (m)
X-Direction						
From G4beamline	-0.332	9.423	0.118	0.653	-1.963	0.256
From 6-Screen Method	-0.462	8.424	0.144	0.530	-2.025	0.255
%Error	39.199	10.605	22.037	194.290	3.128	0.327

Figure 8: Results for the horizontal plane using the 6-screen-method.

results from the test section are promising. While not providing the same level of accuracy as a quadrupole magnet scan alone, the 6-screen-method has provided a measurement of the standard Twiss parameters along with the dispersion function, which was the goal.

## CONCLUSION

With some fine tuning of the math behind the 6-screen-method and further testing its limits, it has the potential to provide accurate and complete measurements of all of the beam's optical parameters without the need of a separate measurement technique for the dispersion and momentum spread. The use of machine learning is being explored with the possibility of performing these measurements in real time as the beam is in operation.

Further work on implementing the 6-screen-method along the extraction line of the Muon Campus at Fermilab is currently underway. I would like to sincerely thank Diktys Stratakis and Michael Syphers for their continued support and encouragement.

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