ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-MOPAB245

THEORETICAL ANALYSIS OF THE CONDITIONS FOR AN ISOCHRONOUS AND CSR-IMMUNE TRIPLE-BEND ACHROMAT WITH STABLE OPTICS*

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Abstract

Transport of high-brightness beams with minimum degradation of the phase space quality is pursued in modern accelerators. For the beam transfer line which commonly consists of bending magnets, it would be desirable if the transfer line can be isochronous and coherent synchrotron radiation (CSR)-immune. For multi-pass transfer line, the achromatic cell designs with stable optics would bring great convenience. In this paper, based on the transfer matrix formalism and the CSR point-kick model, we report the detailed theoretical analysis and derive the condition for a triple-bend achromat with stable optics in which the first-order longitudinal dispersion (i.e., R_{56}) and the CSR-induced emittance growth can be eliminated. The derived condition suggests a new way of designing the bending magnet beamline that can be applied to the free-electron laser (FEL) spreader and energy recovery linac (ERL) recirculation loop.

INTRODUCTION

In modern accelerators, it is highly desirable to preserve the high-brightness beam quality, characterized with the features of low emittance, short bunch length and high peak current in the beam transfer line, which usually consists of bending magents [1–3]. However, when the bunches pass through the dipole magnets in the beam transfer line, emission of coherent synchrotron radiation (CSR) and the longitudinal dispersion can have a significant impact on the beam dynamics. CSR induces the beam energy spread to increase and causes the transverse emittance growth horizontally [4], and longitudinally leading to the microbunching instability (MBI) [5–10], thus notoriously deteriorating the beam quality. The longitudinal dispersion effect may result in unwanted bunch length variation as the particle's momentum deviation δ is correlated to the longitudinal bunch coordinate z via the longitudinal dispersion functions, i.e., $R_{56}, T_{566}, U_{5666}$, etc.

The triple-bend achromat (TBA) structure is regarded as one of the basic configurations to make an isochronous cell and a relatively simple structure, which has been widely used to transport an ultrashort, high-brightness electron beam [11–13]. It has been demonstrated [14–16] that a kind of TBA design could yield both the first-order isochronicity, i.e., $R_{56} = 0$ and the steady-state-CSR-induced emittance growth cancelled by requiring, however, somewhat extreme constrains of the transfer matrix. Nevertheless, it is found that the trace of the total transverse transfer matrix exceeds 2 in such TBA design, which means violation of optics stability for a cell with periodic stable optics. As a result, such design may be suitable for single-pass situation instead of multi-pass situation. This study sets out to investigate the requirements for a TBA design with stable optics that yields both minimal CSR-induced emittance growth and the first-order longitudinal dispersion.

DERIVATION OF THE CONDITIONS FOR A TBA

We set out to study a midpoint symmetric TBA structure that consists of three identical dipole magnets, the schematic and denotation of the transfer matrices are shown in Fig. 1.

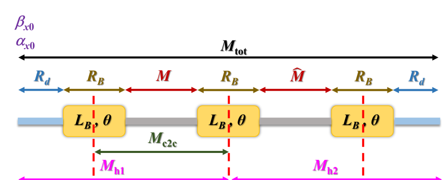

Figure 1: Schematic of the TBA structure with three identical dipole magnets of length L_B and bending angle θ. R_d and R_B represent the transfer matrix of the drift space and the dipole, respectively. The elements in the section between the adjacent dipoles are assumed as non-dipole elements, e.g., quadrupoles, sextupoles and drifts, the transfer matrices of which are expressed as M and $\hat{M}$. The total transfer matrix is denoted as M_{tot}, the transfer matrix between the first two dipoles is denoted as M_{c2c}, the transfer matrix from the entrance to the middle of the structure is denoted as M_{h1} and the second half is denoted as M_{h2}. β_0, α_0 are the Twiss functions at the entrance of the TBA.

Here we only consider the transverse motion in the horizontal plane and the longitudinal motion of the particle for simplicity. The transfer matrix for the phase-space coordinate vector (x, x', z, δ) is used, which is part of the typical 6×6 transfer matrix that describes the motion of

* Work supported by NSFC(11922512, 11905073), the National Key R&D Program of China (No. 2016YFA0401900), Youth Innovation Promotion Association of Chinese Academy of Sciences (No. Y201904), the Fundamental Research Funds for the Central Universities (HUST) under Project No. 5003131049
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ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-MOPAB245

$(x, x', y, y', z, \delta)$, though we still adopt the subscripts 5 and 6 for the longitudinal dispersion functions R_{56}. The transfer matrix of a sector dipole can be expressed by

$$R_B = \begin{pmatrix} \cos\theta & \frac{\sin\theta L_B}{\theta} & 0 & \frac{(1-\cos\theta)L_B}{\theta} \\ -\frac{\theta\sin\theta}{L_B} & \cos\theta & 0 & \sin\theta \\ \sin\theta & \frac{(1-\cos\theta)L_B}{\theta} & 1 & \frac{(\theta-\sin\theta)L_B}{\theta} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

where in the following discussion L_B and θ are given quantities. Without loss of generality, the transfer matrix from the first dipole's exit to the second dipole's entrance can be expressed by

$$M = \begin{pmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

The total transfer matrix of the TBA cell can be expressed as $M_{\text{tot}} = M_{\text{h1}} M_{\text{h2}}$, where $M_{\text{h1}} = R_{B/2} M R_B R_d$ and $M_{\text{h2}} = R_d R_B \hat{M} R_{B/2}$. $R_{B/2}$ is the transfer matrix of half of the dipole and $\hat{M}$ has the same entries as M, except that m_{11} and m_{22} are exchanged.

First, the achromatic and the symplectic condition are used, thus two entries among m_{11}, m_{12}, m_{21} and m_{22} can be eliminated. The achromatic condition can be met with the imposition of $(M_{\text{h1}})_{26} = 0$. This means that the derivative of the dispersion function is vanished. As the dispersion function and its derivative is zero at the entrance of the TBA cell, for the midpoint symmetry TBA structure, $(M_{\text{h1}})_{26} = 0$ means the dispersion function and its derivative will also be zero at the exit. For the symplectic condition, it requires $(M_{\text{h1}})_{11}(M_{\text{h1}})_{22} - (M_{\text{h1}})_{12}(M_{\text{h1}})_{21} = 1$. Then only two elements, say m_{11} and m_{21}, are independent. The other two dependent elements, i.e., m_{12} and m_{22} can be expressed as

$$\begin{aligned} m_{12} &= \frac{\sec\left(\frac{\theta}{2}\right)^2 L_B\{\theta[1+m_{11}-m_{11}^2+\cos\theta(1+m_{11}^2)]+\sin\theta L_B m_{11} m_{21}\}}{2\theta\left[-L_B m_{21}+\theta m_{11}\tan\left(\frac{\theta}{2}\right)\right]} \\ m_{22} &= \frac{\sec\left(\frac{\theta}{2}\right)^2\left[\theta^2\sin\theta+\theta L_B(1-m_{11}+\cos\theta m_{11})m_{21}+\sin\theta L_B^2 m_{21}^2\right]}{2\theta\left[-L_B m_{21}+\theta m_{11}\tan\left(\frac{\theta}{2}\right)\right]}. \end{aligned} \quad (3)$$

To study the requirements of cancelling R_{56} and the CSR-induced emittance growth, $\Delta\varepsilon_n$, the two conditions will be imposed and analysed separately in the following.

The First-order Isochronous Condition

The first-order isochronous condition can be met with the imposition of $(M_{\text{h1}})_{56} = 0$. This term can be analyzed directly by matrix multiplication. With another constraint imposed, one of m_{11} and m_{21} can be eliminated, and the three dependent variables can now be expressed by the remaining one free variable. We choose m_{11} as the free variable, and focus on the relation between m_{11} and m_{21}. The condition $R_{56} = 0$ requires

$$m_{21} = \frac{\theta\sin\left(\frac{\theta}{2}\right)\left[-2\sin\theta + (-3\theta + 2\sin\theta)\, m_{11}\right]}{\left[-3\theta\cos\left(\frac{\theta}{2}\right) + 3\sin\left(\frac{\theta}{2}\right) + \sin\left(\frac{3\theta}{2}\right)\right] L_B}. \quad (4)$$

The CSR-induced Emittance Growth Cancellation Condition

The CSR-induced emittance growth cancellation condition will be investigated using the CSR kick model [4, 17]. For the TBA with three identical dipoles, it has been derived in Ref. [17] [see Eq. (14) in Ref. [17]], which requires

$$(M_{\text{c2c}})_{11} - \frac{2\theta^2\sin\left(\frac{\theta}{2}\right)}{\theta\cos\left(\frac{\theta}{2}\right)L_B - 2\sin\left(\frac{\theta}{2}\right)L_B}(M_{\text{c2c}})_{12} = -\frac{1}{2}, \quad (5)$$

where $M_{\text{c2c}} = R_{B/2} M R_{B/2}$ is the the transfer matrix between the first two dipoles. Similarly, m_{21} can be eliminated and we get two sets of relations, the first is

$$\begin{aligned} m_{21} &= -\frac{\theta\cos\left(\frac{\theta}{2}\right)\csc(\theta)}{4\sqrt{2}L_B\left(\theta-2\tan\left(\frac{\theta}{2}\right)\right)} {S_1}^{1/2}, \\ S_1 &= \sec^4\left(\frac{\theta}{2}\right)\left[-2\left(7\theta^2 - 16\cos(2\theta) + 8\cos(3\theta) + 8\right)\right] \\ &+\sec^4\left(\frac{\theta}{2}\right)\{8\left[8\sin(\theta) + \sin(2\theta) - 4\sin(3\theta)\right]\}\,\theta \\ &+\sec^4\left(\frac{\theta}{2}\right)\left[-9\theta\cos(\theta) + 18\theta\cos(2\theta) + 13\theta\cos(3\theta)\right]\theta \\ &+\sec^4\left(\frac{\theta}{2}\right)\left[\left(\theta^2 - 2\right)\cos(2\theta) + 2\theta\sin(\theta) - 2\right] 16 m_{11} \\ &+\sec^4\left(\frac{\theta}{2}\right)\left\{\left[\theta^2 - 6\theta\sin(\theta) + 4\right]\cos(\theta)\right\} 16 m_{11} \\ &+\sec^4\left(\frac{\theta}{2}\right)\left[\theta\cos\left(\frac{\theta}{2}\right) - 2\sin\left(\frac{\theta}{2}\right)\right]^2 32 m_{11}^2. \end{aligned} \quad (6)$$

And the second is

$$\begin{aligned} m_{21} &= \frac{1}{8L_B\left[\theta-2\tan\left(\frac{\theta}{2}\right)\right]}\theta\csc(\theta)\{4\theta\left[\cos(\theta) - 2m_{11}\cos(\theta) - 2\right]\} \\ &+\frac{1}{8L_B\left[\theta-2\tan\left(\frac{\theta}{2}\right)\right]}\theta\csc(\theta)\left\{8\tan\left(\frac{\theta}{2}\right)\left[2m_{11}\cos(\theta) + 1\right]\right\} \\ &+\frac{1}{8L_B\left[\theta-2\tan\left(\frac{\theta}{2}\right)\right]}\theta\csc(\theta)\cos\left(\frac{\theta}{2}\right){S_2}^{1/2}, \\ S_2 &= 2\sec^4\left(\frac{\theta}{2}\right)\left\{-9\theta^2\cos(\theta) + 2\left[9\theta^2 + 16\right]\cos(2\theta)\right\} \\ &+2\sec^4\left(\frac{\theta}{2}\right)\left\{-2\left[7\theta^2 + 8\cos(3\theta) + 8\right]\right\} \\ &+2\sec^4\left(\frac{\theta}{2}\right)\{\theta\left[8\left(8\sin(\theta) + \sin(2\theta) - 4\sin(3\theta)\right) + 13\theta\cos(3\theta)\right]\} \\ &+64 m_{11}\left[\theta - 2\tan\left(\frac{\theta}{2}\right)\right]\sec^2\left(\frac{\theta}{2}\right)\left[-\theta - 2\sin(\theta) + 2\theta\cos(\theta)\right] \\ &+64 m_{11}\left[\theta - 2\tan\left(\frac{\theta}{2}\right)\right]\sec^2\left(\frac{\theta}{2}\right)\left[\theta m_{11} - 2\left(m_{11} - 1\right)\tan\left(\frac{\theta}{2}\right)\right]. \end{aligned} \quad (7)$$

The Cross Points of the Two Conditions

The expressions in Eq. (4), Eq. (6), and (7) determine two sets of requirements of m_{21} as a function of m_{11}, which corresponds to the first-order isochronous condition [Eq. (4)] and the CSR-induced emittance growth cancellation condition [Eq. (6) and (7)], respectively. The cross points of the two requirements suggest completely removal of both R_{56} the CSR-induced emittance growth.The two cross points locate at

$$\begin{aligned} m_{11} &= -1 - \cos\theta, \\ m_{11} &= \frac{P_1}{P_2}, \\ P_1 &= -9\theta^3 + \theta\left[\left(3 - 9\theta^2\right)\cos(\theta) + 26\theta\sin(\theta)\right] \\ &+\theta\left[7\theta\sin(2\theta) + 10\cos(2\theta) + \cos(3\theta)\right] \\ &-14\theta + 8\sin(\theta) - 4\sin(2\theta), \\ P_2 &= 4\left(\theta^2 + 2\right)\sin(\theta) - 10\theta - 4\sin(2\theta) \\ &+8\theta\cos(\theta) + 2\theta\cos(2\theta). \end{aligned} \quad (8)$$

12th Int. Particle Acc. Conf. IPAC2021, Campinas, SP, Brazil JACoW Publishing
ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-MOPAB245

DISCUSSION IN THE STABLE AREA

The stability criterion for periodic stable optics in the x plane, $|(M_{\mathrm{tot}})_{11} + (M_{\mathrm{tot}})_{22}| \leqslant 2$, will occupy an area in the m_{11} and m_{21} space, which will be referred to as the *stable* area in what follows.

For simplicity, We adopt a small bend-angle approximation, i.e., $\theta \ll 1$ and Taylor expand the expressions of the cross points with respect to the dipole bending angle θ. To the lowest order, the two cross points can be expressed as

$$m_{11} = \frac{7}{4} \quad m_{21} = \frac{15}{2L_B}, \text{[Cross point A]} \tag{9}$$

$$m_{11} = -2 \quad m_{21} = 0. \text{[Cross point B]} \tag{10}$$

The two points, together with the optics stable region, are shown in Fig. 2, where we take $L_B = 0.4\,\mathrm{m}$, $\theta = 4°$ and $d = 0.6\,\mathrm{m}$ as an example. The cross point A is the result derived in Ref. [16]. Here we find the other cross point B, which is exactly on the edge of the stable area and is independent of L_B, θ and d.

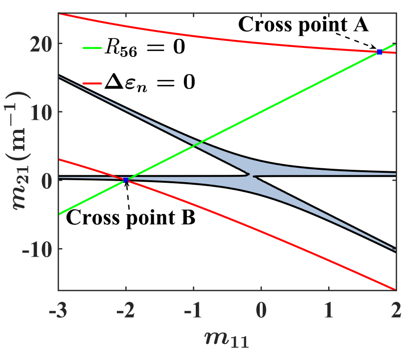

Figure 2: The stable area (shaded) and the requirements of m_{21} as a function of m_{11} derived from the imposed constraint of $R_{56} = 0$ (green line) and $\Delta\varepsilon_n = 0$ (red line). The cross points are marked (blue square).

Despite the fact that the point B satisfies the optics stability, i.e., $|(M_{\mathrm{tot}})_{11} + (M_{\mathrm{tot}})_{22}| = 2$, we find that the periodic solution of the horizontal beta function at the entrance of a TBA design, β_{x0}, will diverge at cross point B. For periodic Twiss functions, we have $\beta_{xf}=\beta_{x0}$, $\alpha_{xf}=\alpha_{x0}$ and $\gamma_{xf}=\gamma_{x0}$. Thus, one can derive that

$$\begin{aligned} &\beta_{x0}=\frac{\sqrt{(-1+2dm_{21}+L_Bm_{21})}}{2\sqrt{-m_{21}^2(2m_{11}+L_Bm_{21})}}B_0^{1/2}, \\ &B_0 = -4 + L_B^2m_{21}^2 + L_Bm_{21}\left(-1+2dm_{21}\right) \\ &+2m_{11}\left(-1+2dm_{21}+L_Bm_{21}\right), \\ &\alpha_{x0}=0, \\ &\gamma_{x0} = 1/\beta_{x0}. \end{aligned} \tag{11}$$

Here one can get that, for the corss point B derived in the above, where $m_{21} = 0$ and $m_{11} = -2$, we have $\beta_{x0} \propto 1/\sqrt{m_{21}}$ as $m_{21} \to 0^+$. It means that the β_{x0} function will indeed diverge at the corss point B.

From the above discussion, it is found that the two requirements cannot be completely satisfied in the stable area. However, when looking into in the vicinity of the cross point B using the theoretical calculation formula of R_{56} and $\Delta\varepsilon_n$, it is found that both the minimized R_{56} and $\Delta\varepsilon_n$ can be found in the vicinity of the singular point, as shown in Fig. 3 and Fig. 4.

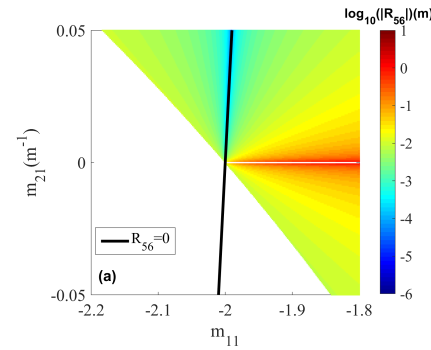

Figure 3: The value of R_{56} (The solid line shows $R_{56} = 0$).

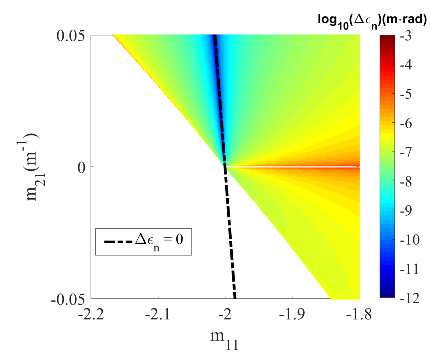

Figure 4: The value of CSR-induced emittance growth (The chain-dotted line shows $\Delta\varepsilon_n = 0$).

CONCLUSION

In this study, we theoretically analyze the first-order isochronous condition and the CSR-induced emittance growth cancellation condition for the symmetric TBA structure with three identical dipoles. A theoretical setting of a symmetric TBA cell design with periodic stable optics is identified. It is found that the first order longitudinal dispersion and CSR-induced emittance growth can be minimized simultaneously. It points a new way of designing the bending magnet beamline and is hopefully to be applied to the FEL spreader and ERL recirculation loop. For more detailed discussions, together with MBI performance on such CSR-immune TBA transport line, the interested reader shall be referred to our recent work [18].

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ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-MOPAB245

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