AGS DYNAMIC APERTURE AT INJECTION
OF POLARIZED PROTONS AND HELIONS
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Abstract
Polarized helions are part of the physics program for the future EIC. An AC dipole has been installed in the AGS Booster to preserve polarization as helions are accelerated to $|G\gamma|=10.5$. Extraction from the AGS Booster at $|G\gamma|=7.5$ is possible but would involve crossing the $|G\gamma|=0+\nu_y$ intrinsic resonance in the AGS, and would be the lowest rigidity beam injected into the AGS, and therefore experiences strong distortions of the optical functions because of the AGS two partial snakes. This lower rigidity would exacerbate the optical distortions from the snake, reducing the dynamic aperture. A comparison of the dynamic aperture of protons at $G\gamma=4.5$ to that of helions at $|G\gamma|=7.5$ and $|G\gamma|=10.5$ show that extraction at $|G\gamma|=10.5$ provides a larger dynamic aperture. This larger dynamic aperture would allow both betatron tunes of helion beam to be placed inside the spin tune gap generated by the two partial helices in AGS at injection.

INTRODUCTION
The goal for helions in the AGS at injection is to have both $\nu_x$ and $\nu_y$ inside the spin tune gap at injection. For protons at $G\gamma=4.5$ has proven operationally difficult due to excessive losses and tuning. With helion injection at $|G\gamma|=7.5$ being at a lower rigidity than protons at $|G\gamma|=4.5$ ($B=7.203\text{ Tm}$) with $B=6.967\text{ Tm}$, concern over both the available aperture and dynamic aperture (DA) due to these strong optical distortions of the cold snake being raised [1, 2]. DA is defined as the maximum amplitude at which a particle will not be lost from single particle dynamics and not from physical apertures [3]. The physical aperture of the AGS is the beampipe at the cold snake which is round with a 3.85 cm radius.

To quantify this, particles are tracked through only the cold snake to calculate the transport matrix. From the transport matrix, the total focusing and coupling are calculated where the focusing, FC, is defined as [4],

$$ FC = R_{12}^2 + R_{34}^2 $$

and the coupling, CP, is

$$ CP = LL + UR $$

with

$$ LL = R_{31}^2 + R_{32}^2 + R_{41}^2 + R_{42}^2 $$

$$ UR = R_{31}^2 + R_{14}^2 + R_{23}^2 + R_{24}^2. $$

As seen in Fig. 1, these optical distortions reduce exponentially with $B\rho$.

![Figure 1: Coupling and focusing from the AGS cold snake as a function of $B\rho$ and comparison to an exponential function.](image)

The partial snake magnet assembly also contains a solenoid magnet for coupling correction. The nominal current used is $I_{\text{sol}}=220\text{ A}$ which corresponds to a field of $B_{\text{sol}}=0.2\text{ T}$. There is an engineering limit of $I_{\text{sol, max}}=235\text{ A}$, although the magnet has been tested up to 300 A. From this, there are only marginal improvements that can be made on correcting the coupling so these simulations will use the $I_{\text{sol}}=220\text{ A}$. An example of the coupling from $B_{\text{sol}}=0.5\text{ T}$ field is shown in Fig. 2, when the coupling at low energy is improved but is larger at higher energy.

DYNAMIC APERTURE CALCULATION
The DA is determined numerically at a given optics configuration ($\nu_x, \nu_y$ location) through particle tracking for many turns to determine at what excursion the particle is lost. Figure 3 shows the stable range of $x$ and $y$ coordinates for $x=8.77$ and $y=8.88$.

The DA is calculated with various tune configurations to compare working points at injection. The methodology of the DA calculation follows:

1. Fit model to tunes and find closed orbit,
2. Find $\pm x$ limit where beam survives,
3. Populate range \([X_L, X_U]\) with 20 particles separated by \(dx\), and find \(Y_M\) (maximum stable \(Y\)) at each \(X\) coordinate.

4. Simulations with 169 points \((x = (0.69 + 0.02k)k \in \{1, 2, ..., 13\})\) and \(y = (0.85 + 0.02k)k \in \{1, 2, ..., 13\})\).

![Figure 2: Figure showing the coupling error of the cold snake with a 0.2 T and a 0.5 T solenoid field with respect to \(B_\rho\).](image)

![Figure 3: Example dynamic aperture in the AGS with \(\nu_x = 8.77\) and \(\nu_y = 8.88\).](image)

When edge searching, the algorithm moves in steps of 0.5 \(mm\) from closed orbit until edge is found and then performs a binary search. The binary search continues to a resolution of ±0.008 \(mm\). This process is shown for one set of \(\nu_x\) and \(\nu_y\) in Fig. 3. This implementation uses a combination of zgoubi and pyzgoubi where:

- zgoubi handles all the tracking and optics computations,
- pyzgoubi handles particle coordinates and fitting algorithms described above,
- pyzgoubi creates a thread for each \(y\) and \(x\) configuration.

![Figure 4: Comparison of DA for helions at \(|G_\gamma| = 7.5\) in Fig. 4. Comparison of Fig. 4a with Fig. 4b shows that in the absence of the snakes, the DA is larger than the limiting aperture of the machine. Comparison of Fig. 4c with Fig. 4d shows that in the presence of snakes, the strong optical defects cause a further reduction in the DA when the limiting aperture is introduced.](image)

**DYNAMIC APERTURE RESULTS**

A comparison of the DA is made for the AGS in the absence and presence of snakes, in addition to the absence and presence of the physical limiting aperture. This is shown for helions at \(|G_\gamma| = 7.5\) in Fig. 4. Comparison of Fig. 4a with Fig. 4b shows that in the absence of the snakes, the DA is larger than the limiting aperture of the machine. Comparison of Fig. 4c with Fig. 4d shows that in the presence of snakes, the strong optical defects cause a further reduction in the DA when the limiting aperture is introduced.

Simulations are performed at \(B_\rho = 6.968\) \(Tm\), \(B_\rho = 7.203\) \(Tm\), and \(B_\rho = 10.780\) \(Tm\) as seen in Fig. 5.

From Fig. 5, one observes that there are subtle differences in the DA between helions at \(B_\rho = 6.968\) \(Tm\) and protons at \(B_\rho = 7.203\) \(Tm\) although a factor of 2 gain is observed when comparing to the \(B_\rho = 10.780\) \(Tm\) case.
Figure 5: Dynamic aperture of AGS sorted by rigidity: a) helions at $|G_G| = 7.5$, $B\rho = 6.968$ Tm; b) protons at $|G_G| = 4.5$, helions $B\rho = 7.203$ Tm; c) $|G_G| = 10.5$, $B\rho = 10.780$ Tm.

The DA calculations are done for 1,000 turns to minimize computing time. Ideally the DA tracking would be for a number of turns equal to the time the particles are at injection energy. Computationally this proves problematic since the time for simulations is linearly proportional to the number of turns.

Note that there is virtually zero DA available in the region of interest with $[Q_x, Q_y]=[8.9, 8.9]$ for helions at $|G_G| = 7.5$ and protons at $|G_G| = 4.5$. This further supports extraction of helions at $|G_G|=10.5$ as the available DA is substantially larger. This is also observed in Fig. 6 where the three configurations are compared at a fixed tune of $[Q_x, Q_y]=[0.75, 0.91]$.

CONCLUSION

Simulations showed that the available DA increases with magnetic rigidity due to reduced optical defects from the AGS snakes. Extraction for helions at $|G_G| = 10.5$ provide a larger dynamic aperture that would allow the tunes to be put within the spin tune gap at injection. This extraction energy also allows the $|G_G| = 0 + \nu_y$ resonance to be avoided in the AGS.

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REFERENCES


