# ROBUST DESIGN AND CONTROL OF THE NONLINEAR DYNAMICS FOR BESSY-III*

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### *Abstract*

The design philosophy for a robust prototype lattice design for BESSY III, i.e., that is insensitive to small parameter changes, e.g. engineering tolerances – based on a *higher-order-achromat*, a la: SLS, NSLS-II, MAX IV, and SLS 2 – is outlined & presented. As usual, a well optimised design requires clear understanding of the *end-user requirements* and close collaboration between the *linear optics* designer and *nonlinear dynamics* specialist for a *systems approach*.

## INTRODUCTION

### *Requirements*

Given the science case [1] and the trade-offs for controlling the corresponding linear optics [2], a set of preliminary, self-consistent requirements are summarised in Table 1. The energy spread is limited by not degrading the performance of high-end undulators (if e.g. damping wigglers are introduced to reduce the emittance). The on-momentum DA (*dynamic aperture*) is determined by the injection system [3]. As usual, the Touschek lifetime is a challenge for medium-energy rings, i.e., it scales roughly with $\gamma^3$.

Table 1: Preliminary Requirements

| **End User** | |
|---|---|
| Circumference [m] | ~300 |
| Energy [GeV] | 2.5 |
| $\varepsilon_x$ [pm·rad] | ~100 |
| $\sigma_s$ [mm] (w/o harm. cav.) | ~2.5 |
| $\sigma_\delta$ | ~1e-3 |
| $\beta_{x,y}$ [m] mid-straight | [~2.0, ~2.0] |
| **Beam Dynamics** | |
| On-Momentum DA [mm] | [~2.0, ~1.5] |
| Off-Momentum DA [%] | 2.0+ |
| $\alpha_c$ | ~1e-4 |
| Beam Lifetime [h] | ~1.0 |

## DESIGN PHILOSOPHY & PRINCIPLES

### *Control of Linear Optics – LEGO*

In the mid-1970 Chasman & Green introduced what is now known as a (linear) *double-bend-achromat* (DBA) lattice [4]. A synchrotron optimised for *insertion devices* [IDs] for synchrotron light production. A decade later this was generalised to a *tripple-bend-achromat* (TBA) [5]; and adopted for the ALS conceptual design [6]. A 12-cell TBA.

Similarly, after a re-baselining of the initial design concept – based on an academic pursuit of the *theoretical minimum emittance* cell (TME) [7] – i.e., for an idealised lattice, not taking into consideration the impact of engineering tolerances (akin to e.g. the impact of noise for telecom systems) – the TBA cell was adopted for SLS as well but with short, medium and long straights. Hence, robust design for it required implementation of the first *higher-order-achromat* (HOA) [8].

Contrarily, MAX IV – the first *robust seven-BA* – pursued straights of the same length a priori, and (totally) ignored TME, i.e., their unit cell is a x15 (!) away from it & the resulting *chromaticity wall* [9], and obtained a HOA with 4.5% momentum aperture (a requirement) [10]. Rather than following the beaten path, they transformed the design of a state-of-the-arts synchrotron light source into an *engineering-science* problem [11]; which they resolved by *miniaturisation*. This *paradigm shift* resulted in an innovative, streamlined, cost effective solution (e.g. concrete girders) for the *inverse problem*: *to find a (robust) solution to the end users requirements*. With most components & subsystems built-to-print by local industries; akin to LEGO block approach. Also, the facility uses heat pumps to recoup the heat from the 5 MW thermal plant [12]; vs. a cooling tower to vent/waste it into the atmosphere.

Not surprisingly, MAX IV's introduction of *disruptive technology(ies)* – has prompted other facilities to upgrade; by rip-and-replace, vs. incremental upgrades.

Additionally, *reverse bends* [13] have been introduced to go beyond the TME cell; i.e., a *systems vs. reductionist approach*.

And – in hindsight, e.g. from lessons learnt [14] – a systematic approach for linear optics design has been provided [15].

Regarding the science case & linear optics design for BESSY-III, see [1, 2].

### *Control of Nonlinear Dynamics – Symmetry*

The early synchrotrons were designed as periodic structures by introducing a FODO cell for the basic LEGO block and repeating it $N$ times; i.e., a weekly focusing cell with two sextupole families for linear chromatic correction. Hence, due to the periodicity of the ring:

1. systematic leading order resonances were supressed,
2. and the tune footprint as well; since, naively, it would scale with the number of sextupoles square vs. linearly; i.e., $(2N)^2 \rightarrow 2N$.

For a first principles approach for the SLS conceptual design in the mid-1990s, two basic strategies for robust

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* Work supported by German Bundesministerium für Bildung und Forschung, Land Berlin, and grants of Helmholtz Association.

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design of the sextupole scheme were considered, both based on symmetry [16]:

1. *-I Transformer*: introduce sextupole pairs separated by $n \cdot \pi$ phase advance in both planes.
2. *Higher-order-achromat*: introduce a *unit cell*, repeat it four or more times to generate a *super period*, and adjust the total phase advance to $n \cdot 2\pi$ in both planes.

The first approach is standard practice for collider design, e.g. ref. [17]. And has been generalised by introducing a *dispersion bump* [18]. However, because the nonlinear effects only cancel on-momentum, it tends to yield inferior momentum aperture vs. a HOA; due to systematically driven off-momentum terms.

This becomes clear by a parametric representation of the Poincaré map for the *-I transformer*

$$M = \begin{bmatrix} \cos(\mu+\xi\delta) & \beta\sin(\mu+\xi\delta) \\ -\dfrac{\sin(\mu+\xi\delta)}{\beta_x} & \cos(\mu+\xi\delta) \end{bmatrix} \to \begin{bmatrix} -1 & -\beta\xi\delta \\ \dfrac{\xi\delta}{\beta} & -1 \end{bmatrix} + (\delta^2)$$

which systematically drives $h_{11001}$ & $h_{00111}$, see poster; but this can be remedied by a *2-cell HOA* [16].

Even so, while the approach is adequate for high-energy rings, the approach tends to provide unsatisfactory performance for medium-energy rings; since Touschek lifetime scales roughly with $\gamma^3$; i.e., for a given lifetime the required momentum aperture is reduced by $\gamma^3$.

The second method originates from spectrometer design [19]. In particular, a 2nd order achromat is obtained for 4 or more cells with 2 sextupole families; i.e., all the geometric & quadratic terms to 2nd order in the phase-space coordinates $[x, p_x, y, p_y; \delta]$ are cancelled.

By using the driving terms notation $h_{ijklm}$ for the Lie generator $h$ in ref. [16] they can be interpreted as phasors. The Poincaré map for an $n$-cell super period with unit cell

$$\mathcal{M}_{\text{cell}} = \mathcal{A}^{-1}\mathcal{R}e^{:h:}\mathcal{A}$$

is

$$\begin{aligned} \mathcal{M} &= \mathcal{M}_{\text{cell}}\mathcal{M}_{\text{cell}} \cdots \mathcal{M}_{\text{cell}} \\ &= \mathcal{A}^{-1}e^{:\mathcal{R}h:}e^{:\mathcal{R}^2h:} \cdots e^{:\mathcal{R}^nh:}\mathcal{R}^n\mathcal{A} \\ &= \mathcal{A}^{-1}e^{:\mathcal{R}h+\mathcal{R}^2h+\cdots+\mathcal{R}^nh+\cdots:}\mathcal{R}^n\mathcal{A} \end{aligned}$$

To leading order, the exponent is a geometric series which for a HOA cancels for the resonance driving terms

$$(\mathcal{R}+\mathcal{R}^2+\cdots+\mathcal{R}^n)h = \mathcal{R}\sum_{k=0}^{n-1}\mathcal{R}^k\,h = \mathcal{R}\frac{I-\mathcal{R}^n}{I-\mathcal{R}} = 0$$

i.e., the phase dependent term in $h$. Akin to how the 3 phasors for a three-phase power system add up to null; a 3-cell HOA with 120° phase advance cancelling one driving term.

Contrarily, the phase-independent terms – which generate the tune footprint – are systematically driven. As stated in the introduction, these are controlled by the $N$-fold periodicity of the lattice with $N$ super periods.

As for control of the nonlinear dynamics for the real lattice – i.e., including the impact of engineering tolerances (mechanical misalignments and random & systematic multipole errors) – akin to e.g. noise for telecom systems [16]:

- To control the *nonlinear dynamics*, control the *linear optics*.
- To control the *linear optics*, control the *closed orbit*; in the sextupoles, or else it will generate gradient errors from feed-down).

Operating implementations of HOAs are: SLS [16], NSLS-II [20], and MAX IV [10]; and the now funded SLS 2 [14].

## APPLICATION TO BESSY-III

### Baseline Lattice

Preliminary explorations of the provided linear optics options, related constraints, and trade-offs [2] – when adding the constraints for a HOA – converged into a 16-cell 6-BA prototype lattice – with *unit cell tune* $\bar{\nu}_{\text{uc}} = [0.4, 0.1]$ & $\bar{\nu}_{\text{sp}} = [2.75, 0.75]$ for the *super period*, see Fig. 1 – which delivers on the *end user requirements*, see [1] (the current benchmark is for $\varepsilon_x = 150$ pm·rad; but is tuneable). In conclusion, that in conjunction provides good control of both the linear optics & nonlinear dynamics.

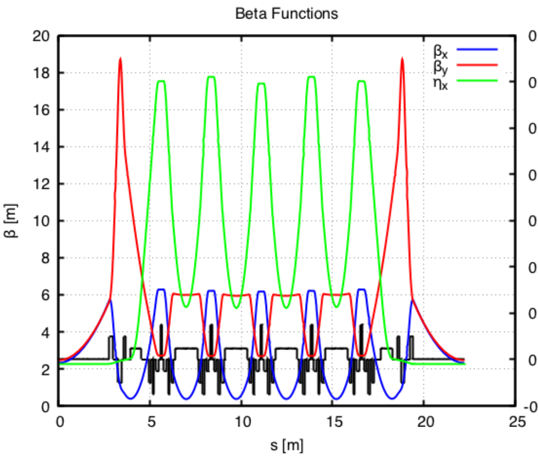

Figure 1: Linear optics for one super period.

### Benchmarks

*Standardised benchmarks* for the prototype real lattice – i.e., including the impact of engineering tolerances (mechanical misalignments and random & systematic multipole errors) comprising of:

- Linear chromatic control; good separation of the 2 chromatic sextupole families.
- Cancellation of the chromatic & geometric resonance driving terms for super period.
- Tune footprint for super period, Fig. 2.
- Control of closed orbit (100 random seeds).
- Control of linear optics; fine tuning of individual quadrupoles based on LOCO (*linear optics from closed orbits*).
- On & off-momentum frequency maps for real lattice, Fig. 3.
- On & off-momentum DA for real lattice (20 random seeds), Fig. 4.

which are included for the poster. The estimated on & off-momentum DA from the tune footprint is (for $\Delta\nu = 0.1$): $A \sim [3.0, 2.5]$ mm & $\delta = 2.0$ %; which is validated by the benchmarks for the real lattice.
Remark: the working point has not (yet) been optimised.

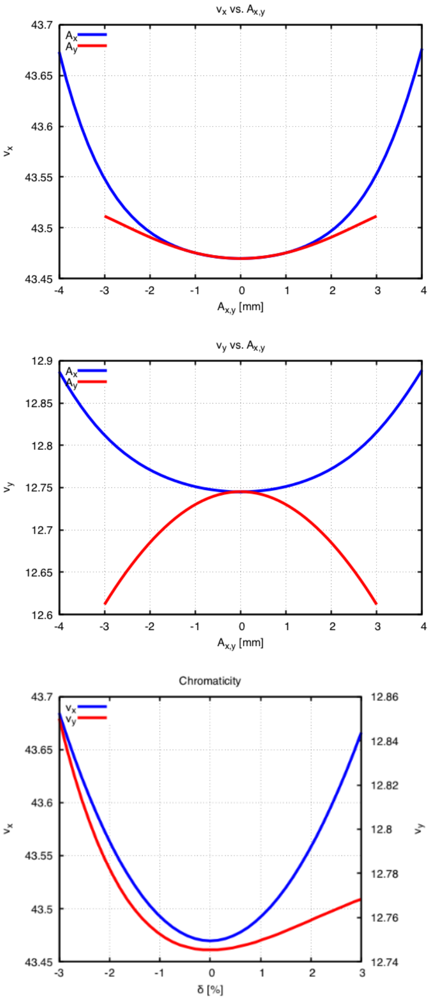

Figure 2: Control of on & off-momentum tune footprint.

In conclusion, a robust baseline lattice design has been established; with only two (chromatic) sextupole families, so far. Hence, the next step is to introduce two chromatic octupole families to reduce the off-momentum tune footprint.

## CONCLUSIONS

The design philosophy for a robust prototype lattice design for BESSY III, based on a *higher-order-achromat* has been outlined & presented. The design has been validated by standardised benchmarks which includes the impact of engineering tolerances (mechanical misalignments and random & systematic multipole errors) that meets the stated requirements.

As usual, a well optimised design requires a clear statement of the *end-user requirements* and close collaboration between the *linear optics* designer and *nonlinear dynamics* specialist for a *systems approach*.

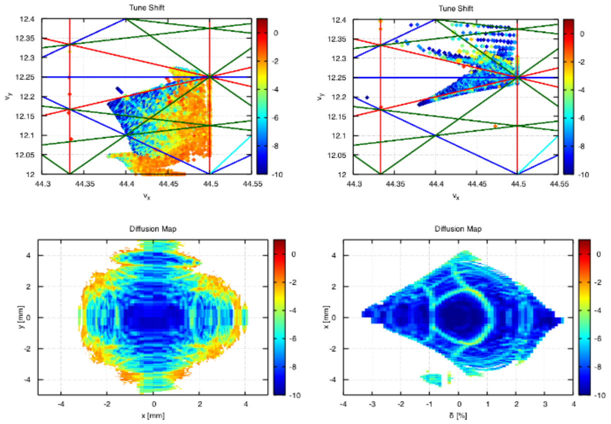

Figure 3: On and off momentum frequency maps for real lattice (see poster).

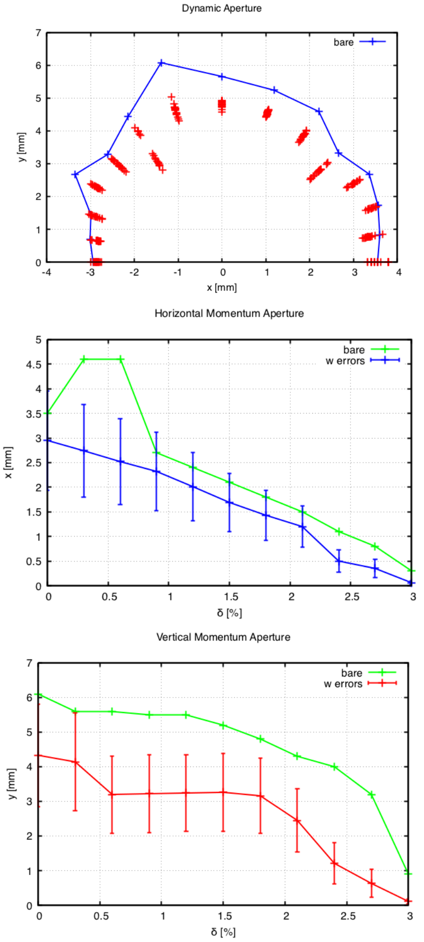

Figure 4: Control of on & off-momentum dynamic aperture, $\beta = [2.3, 2.5]$ m (20 seeds).

## ACKNOWLEDGEMENTS

Although it has been a while I would like to acknowledge the advice I got from Gottfried Mülhaupt when the Swiss Light Source project got funded and I was moving on to the private sector: *There is going to be pressure on you to cut corners. Resist that!*

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