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MACHINE LEARNING TECHNIQUES FOR OPTICS MEASUREMENTS AND CORRECTIONS

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Abstract

Recently, various efforts have presented Machine Learning (ML) as a powerful tool for solving accelerator problems. In the LHC a decision tree-based algorithm has been applied to detect erroneous beam position monitors demonstrating successful results in operation. Supervised regression models trained on simulations of LHC optics with quadrupole errors promise to significantly speed-up optics corrections by finding local errors in the interaction regions. The implementation details, results and future plans for these studies will be discussed following a brief introduction to ML concepts and its suitability to different problems in the domain of accelerator physics.

INTRODUCTION

Accelerator physics problems build a wide range of complex numerical and analytical tasks, e.g. modeling of different aspects of beam behavior, machine performance optimization, measurements data acquisition, and analysis. The growing complexity of modern and future accelerators provides the motivation to explore alternative techniques, which can complement traditional methods or even surpass their performance and offer opportunities to build more efficient and powerful tools. Machine Learning (ML) techniques have been introduced into numerous scientific and industrial areas demonstrating human-surpassing performance in pattern recognition, forecasting, and optimization tasks. These ML concepts can find analogies in the domain of accelerator physics as it will be shown in the following.

Considering the particular case of optics measurements and corrections, traditional techniques meet their limitation, e.g. dealing with erroneous signal artefacts that cannot be related to known patterns in the measurements data. Unsupervised ML techniques cover these limitations by learning the thresholds for anomalies detection directly from the given data as it will be shown on the example of identification of beam position monitors (BPM) faults. Another example is the optics perturbations caused by magnetic gradient field errors, which have to be corrected in order to control the beam optics. Supervised ML models built on simulations of the optics perturbed with thousands of realisations of quadrupolar magnet errors can predict the actual magnetic errors present in the machine, providing additional information for the computation of correction settings. Due to hardware and electronics issues, the signal measured at the BPMs suffers from noise that produces uncertainties in the optics functions reconstructed from the harmonic analysis of BPM turn-by-turn readings. For this problem a special kind of Neural Networks named Autoencoder has been applied as a denoising technique improving the precision of phase measurements, thus potentially leading to more precise computed corrections based on the measured optics. The following section presents a short overview on latest achievements of applying ML to different types of particle accelerators.

MACHINE LEARNING CONCEPTS IN ACCELERATOR PHYSICS

The concept of ML is known since the middle of the last century. The definition of ML is referred to computer programs and algorithms that automatically improve with experience by learning from examples with respect to some class of task and performance measures without being explicitly programmed [1]. Based on this definition we can determine a domain of accelerator tasks that can be potentially solved using ML techniques. Such tasks can be concerned by building models where analytical solutions do not exist, but the models can be "learned" from given examples instead of building them from sets of explicit rules. When building ML solutions, we should define a performance measure, e.g. accelerator performance parameter such as beam size or pulse energy. It is also important to differentiate a specific "class of task", such that ML tools are designed for particular accelerator components which can be easily tested and controlled. Currently existing ML-based methods for accelerators can be divided into virtual diagnostics, control and optimization, anomaly detection and predictive modeling. A more detailed overview for beam diagnostics can be found in [2, 3], recent advances for the field of ML for accelerators control are described in [4–7].

Most of the ML efforts in accelerator physics are being developed for automatic machine optimization, since ML methods demonstrate notable advantages compared to numerical techniques in solving control tasks for non-linear, time-varying systems with large parameter spaces. Two techniques have found an especially wide application in this domain - Bayesian optimization [8] and Reinforcement Learning [9, 10]. Control tasks can be approached in both model-based and model-independent ways, e.g. using adaptive learning techniques to implement feedback algorithms for optimizing and tuning complex noisy systems [11–13]. Predictive modeling techniques also include Gaussian Processes, which can be used to build models relating a set of parameters (e.g. quadrupole settings) to an optimization

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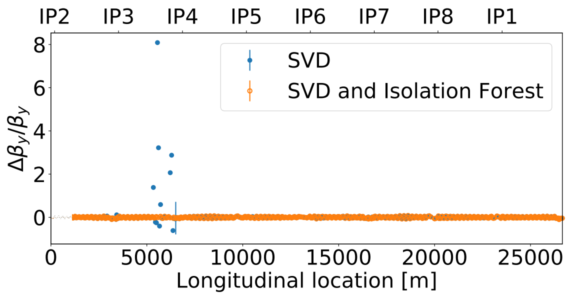

Figure 1: Comparison between β-beating measured from SVD-cleaned data and additional cleaning with IF. The data is obtained during ion commissioning in 2018.

function (e.g. pulse energy) offering the advantage to be able not only to give predictions, but also estimate uncertainty bounds [14]. ML concepts provide techniques to build virtual diagnostics tools that can assist in case a direct measurement would have negative impact on operation or in the locations where no physical instrumentation can be placed. The diagnostics of various beam properties using surrogate models has been applied at various facilities [15–19].

ML-based tools are being developed to tune and control machine and beam behaviour [20–23]. Recently, a fully-automated collimators alignment based on beam loss spikes classification using supervised learning has become a standard tool in the LHC operation. This approach significantly reduced time and human effort needed for the the setup of the collimators system [24]. Further ML-techniques for beam dynamics studies at the LHC are presented in [25] demonstrating applications for optimisation of beam lifetime and losses, detection of collective beam instabilities and beam heating effects, as well as outlier detection in dynamic aperture simulations. Anomaly detection techniques are suitable for the detection of unusual events that do not conform to expected patterns. It can be performed using classification on labeled data (supervised learning), unsupervised learning techniques including clustering or semi-supervised learning methods such as autoencoder. One of the examples of anomaly detection at the LHC, the detection of faulty BPMs is presented in detail in the next section.

UNSUPERVISED DETECTION OF FAULTY BEAM POSITION MONITORS

In presence of faulty BPM signal, the optics functions computed from harmonic analysis of BPM readings [26, 27] are contaminated by outliers, which have to be manually removed followed by repeated optics analysis. Most of the noise and faulty signals can be removed using predefined thresholds, as well as through applying advanced signal-improvement techniques based on SVD [28]. However few nonphysical values are usually observed in the optics computed from the data cleaned with these techniques. In order to reduce the manual effort and save operational time, an anomaly detection technique called Isolation Forest (IF) [29] has been incorporated into optics measurements software infrastructure. IF is a decision-tree-based algorithm, which requires only the expected contamination rate (fraction of outliers in the data) as input parameter. This method recently became a standard part of optics measurements at LHC and has been successfully used during beam commissioning and machine developments under different optics configurations in 2018. Operational results, statistics on simulations, and comparison to clustering techniques can be found in [3, 30]. Application of IF algorithm significantly improved the reliability of the obtained optics functions and reduced the human efforts in cleaning of measurement data. We were able to identify faulty BPMs independently of the settings of previously-available cleaning tools. Determining the optimal values of the SVD settings has been shown to be crucial for the performance of the SVD-based cleaning technique [31]. However, when applying the SVD-based method with optimal settings obtained from extensive simulations studies, we could not match the results achieved using IF algorithm.

Reconstructing the optics from the harmonic-analysis data excluding the bad BPMs identified by IF prevents the appearance of outliers in the computed optical functions. Figure 1 shows an example of improving the optics computation using IF-cleaned data. It has been shown that IF is capable to identify the BPMs failing in most of the measurements, whose fault reasons could not be observed previously in the properties of the signal. Generally, we demonstrated the ability of IF technique to complement efficiently the traditional cleaning tools by removing the remaining faulty BPMs.

SUPERVISED REGRESSION MODELS FOR OPTICS CORRECTIONS

Currently, LHC optics corrections are performed in two steps, i.e. local corrections based on Segment-by-Segment technique [32] and global corrections using Response Matrix approach. Local corrections are applied around Interaction Points (IPs) where the quadrupoles are individually powered, while global corrections are performed by trimming also the circuits - quadrupoles powered in series [33,34]. These methods allow achieving unprecedentedly low β-beating [26], however the currently applied methods do not offer the possibility to estimate the entire set of actual individual magnet errors around the ring. Supervised regression models trained on a large number of LHC simulations demonstrate the potential to predict the individual quadrupole errors from the measured optics perturbations caused by these errors.

Building a Supervised Model

The general idea of applying supervised learning to optics corrections is to build regression models that use the difference between measured and design optics as input features and produce the magnet errors as output. The first preliminary approach is presented in [35] and is based on the optics perturbations introduced by quadrupoles powered in series excluding the errors in the triplet magnets around the IPs. Here we present a more realistic approach, where

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the simulated optics perturbations are introduced by single quadrupole errors around the entire ring including the IP triplets. In order to build the training set, we randomly assign errors to all quadrupoles available in the LHC according to the expected error distribution [36] and apply these errors using the settings for 2018 optics with $\beta^* = 40$ cm. We use simulated phase advance, β^*, and normalized dispersion deviations from the ideal optics as model input (3346 features in total). The output variables are the quadrupole errors used to introduce the simulated deviations from the design optics (1256 target variables). Gaussian noise generated as a random distribution with the factor $10^{-3} \times 2\pi$ and scaled by $\sqrt{\beta}$, β-function value at the BPM location, are added to the simulated phase advance measurements used as input features. The normalized dispersion is given Gaussian noise of $4 \times 10^{-3}\ \sqrt{\text{m}}$ estimated from the measurements in 2018. As it was shown in [37], applying complex models such as Orthogonal matching pursuit or convolutional neural network does not result in significantly better corrections, so we use a least-squares linear regression with weights regularization [38, 39] as baseline model for the following studies.

Table 1: The effect of noise on the predictive power of a regression model. Regression models are trained on 60 000 samples, using only the noisy phase advances as input features, simulated for 2016 optics with $\beta^* = 40$ cm. Mean absolute error (MAE) of prediction is given in the units of absolute quadrupole errors [10^{-5}m^{-2}]. R^2 defines the coefficient of determination.

Noise [2π]	Total MAE	Triplets MAE	R^2
5×10^{-4}	1.71	1.44	0.67
1×10^{-3}	2.19	1.48	0.43
2×10^{-3}	2.5	1.52	0.25
4×10^{-3}	2.69	1.57	0.13
6×10^{-3}	2.75	1.59	0.09
8×10^{-3}	2.79	1.61	0.07
1×10^{-2}	2.82	1.61	0.05

Evaluating Regression Models

To be noted that, due to degeneracy, there are infinite possible error distributions that reproduce the same behaviour and hence, a solution to determine a unique set of quadrupole errors from the optics perturbations does not exist. However, we can validate the regression models from the ML point of view since the simulated errors used as true output values in training data are available. The typical figures of merit for regression tasks are the mean absolute error (MAE) to compare the difference between true target values and the output of the model and the coefficient of explained variance (R^2 score). In order to conclude on the learning performance, the dataset is separated into training (80%) and test (20%) sets. A big increase in the number of training samples does not necessarily result in a large increase of predictive power of the model. Considering the amount of time and storage

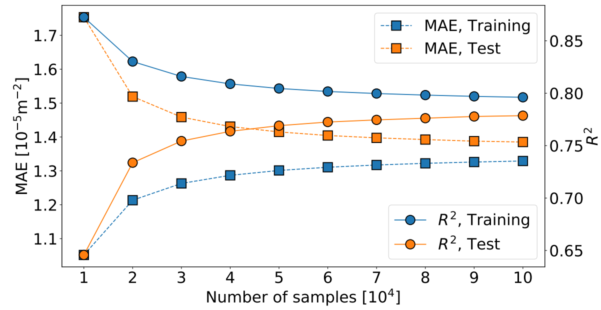

Figure 2: Model cross validation based on the loss (MAE) and R^2 coefficient depending on the number of available samples. The loss is constantly decreasing with the growing number of samples, while R^2 is increasing. This trend indicates a reasonable learning behaviour, however using datasets larger than ca. 70 000 samples does not improve the scores significantly.

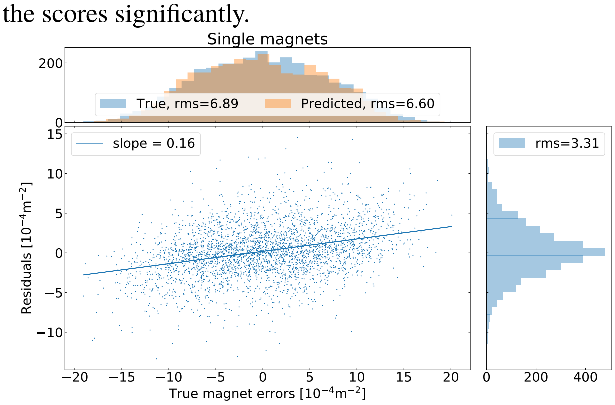

Figure 3: Results of triplet error prediction using LR model trained on 3304 input features, 100 000 samples demonstrating the relative error prediction in single quadrupoles in the triplets. The computed slope is the correlation between true values and residuals, indicating the generalization error of the model.

needed to handle the training simulation data, especially for the future online application, we need to determine the optimal training set size. The change of the model scores with respect to the number of samples (*learning curve*) also indicates the ability of the model to learn from the given data and indicates the dataset size required to achieve the optimal model performance as shown in Figure 2. In the next section we present the results from the regression model using 75 000 samples in total for training and test.

Results

The final evaluation of the model is performed on 100 independently-generated simulations. We define the correlation between the size of the simulated magnet errors and the size of residuals (difference between true and predicted values) as generalization error and compare the rms values of simulated and predicted error distributions. Figure 3 demonstrates the results of the errors prediction of the triplets quadrupoles located close to IPs and producing the largest optics perturbations.

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The previously described results are obtained from simulations. We also investigate the ability of regression models to compute magnet errors to correct the β-beating in the virgin machine, using LHC data from 2016 commissioning, measured for $\beta^* = 40$ cm before any corrections. Since normalized dispersion and β^* are not available in this measurement set, we need to train the model using only the phase advance deviations from ideal optics as input features. In this case, the model achieves significantly smaller training and test scores than regression model trained on larger amount of features ($R^2 = 0.45$ compared to 0.78), demonstrating the importance of normalized dispersion and β^* for the magnet errors reconstruction. Since the actual magnet errors generating the measured optics perturbations in the uncorrected machine are unknown, we cannot evaluate the model prediction as in the case of simulations. Instead, we reconstruct the β-beating from the predicted quadrupole errors and compare it to the measurement. The difference is then the expected remaining optics errors after applying the predicted strengths as corrections. According to the residual β-beating obtained by comparing the measured and the reconstructed optics using the predicted magnetic errors, the absolute β-beating in Beam 1 can be potentially reduced from rms values of 12% and 54% to 2% and 7% in horizontal and vertical planes, respectively. For Beam 2, the rms β-beating decreases from 49% to 9% in the horizontal and from 12% to 3% in the vertical plane. The obtained regression-based corrections can be potentially improved by training a more powerful model including the sextupoles misalignments and non-linear effects. In case non-linearities are added, a Neural Network (NN) regression model will be potentially needed in order to resolve non-linear correlations using the hidden layers. The application of NN can be also advantageous for the training procedure. After training a NN-model for a specific optics setting, we can avoid fully re-training a new model for a different optics. Instead, only the last layer will have to be re-trained on additional data for the new optics. This can reduce the amount of training data and time needed to create predictive regression models.

We also investigated the effect of the noise on predictive power of the model. The comparison of prediction errors between models trained on the input data given different noise factors is shown in Table 1. Loss values indicate that the accuracy of the triplet errors prediction is less concerned by the noise than the rest of the magnets. The study shows how important is to keep the measurements noise level as low as possible. Next section focuses specifically on this problem and its possible ML-based solution.

RECONSTRUCTION AND DENOISING OF PHASE MEASUREMENTS

As shown in Table 1, reducing the noise in the phase-advance measurement used as input for quadrupole errors prediction models can potentially improve the accuracy of the prediction. Moreover, the presence of the noise enforces acquisition of several turn-by-turn measurements for each beam in order to obtain statistically significant error bars in the optics functions caused by the uncertainties due to the noise in BPM signal. A possible ML-based solution to reduce the noise in the phase measurements is the application of autoencoder [40]. We trained an autoencoder network on a set of noisy phase measurements simulated as described in the previous section as well as the originally simulated phase measurements. During the training, the autoencoder aims to minimize the difference between true output, i.e simulated phase advances without noise, and the output produced by the network from the noisy input data. To perform the denoising and produce the original phase as output, the model needs to extract features that capture relevant information in the data. Applying an autoecoder trained on 10 000 simulated phase advance measurement sets demonstrates the reduction of the simulated phase noise by a factor of 2.

Another potential application of autoencoder is the reconstruction of missing BPM signal. We trained an autoencoder using simulated phase advance measurements set where 10% data points have been replaced by 0 indicating a missing value, e.g. if a BPM has been identified as faulty and removed in previous analysis stages. As training output we provide the original set of phase advances without missing values, such that autoencoder output can be compared to this original output. The training target is to minimize the difference between original phase advances and autoencoder output. The MAE computed for 100 validation samples is $0.93 \times 10^{-3} [2\pi]$. This method can be applied in order to reconstruct the missing values to provide the input to quadrupole errors prediction regression models trained on simulations.

SUMMARY

Although ML techniques have found their first applications in accelerator physics just a few years ago, they already have been proven as powerful tools for various control, optimization and automation tasks. We presented several applications developed for optics measurements and corrections at the LHC. Operational results of the application of decision tree based technique for faulty BPMs detection show its effectiveness and advantages compared to the cleaning using the traditional techniques only.

The application of regression models allows to gain knowledge about quadrupole errors in the LHC obtaining the entire set of errors around the ring in one step as demonstrated by simulating the LHC optics. It was possible since the the trained model was able to relate the optics deviations from ideal model to magnets errors that caused these perturbations. This has been shown on simulations of 2018 optics as well as on LHC measurements from 2016 commissioning. The quality of phase measurements which is the fundamental part of optics and corrections computation can be potentially improved by applying autoencoder network in order to perform denoising of the measured data and reconstruct the missing BPM signal.

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