IPAC19

DEVELOPMENT OF METHODS FOR CALCULATION OF BUNCH RADIATION IN PRESENCE OF DIELECTRIC OBJECTS

A. V. Tyukhtin, E. S. Belonogaya, S. N. Galyamin, V. V. Vorobev

Saint Petersburg State University

Physical Faculty

Radiophysics Department

Laboratory of Electrodynamics of Nonstationary and Microwave Processes

Supported by RSF Russian Science Foundation Grant No. 18-72-10137

Introduction

Radiation of charged particles in the presence of dielectric objects is of interest for applications in accelerator and beam physics. As a rule, complex geometry of the problem does not allow obtaining rigorous expressions for electromagnetic field.

The size of the target is frequently much more than the wavelength under consideration. This fact complicates computer calculations. However, it gives us an obvious small parameter of the problem and allows development of approximate methods of analysis.

We develop two methods which are applicable for objects having large size in comparison with wavelengths under consideration. One of them can be named "ray-optical technique", other of them can be named "aperture technique".

Ray-optical technique:

[1] E. S. Belonogaya, A. V. Tyukhtin, and S. N. Galyamin, "Approximate method for calculating the radiation from a moving charge in the presence of a complex object", Phys. Rev. E, vol. 87, p. 043201, 2013.

[2] E. S. Belonogaya, S. N. Galyamin, and A. V. Tyukhtin, "Short-wavelength radiation of a charge moving in the presence of a dielectric prism", J. Opt. Soc. Am. B, vol. 32, p. 649, 2015.

Aperture technique:

[3] S. N. Galyamin and A.V. Tyukhtin, "Dielectric concentrator for Cherenkov radiation", Phys. Rev. Lett., vol. 113, p. 064802, 2014.

[4] S. N. Galyamin, A. V. Tyukhtin, S. Antipov, and S. S. Baturin, "Terahertz radiation from an ultra-relativistic charge exiting the open end of a waveguide with a dielectric layer", Optics Express, vol. 22, p. 8902. 2014.

[5] A. V. Tyukhtin, V. V. Vorobev, E. S. Belonogaya, and S. N. Galyamin, "Radiation of a charge in presence of a dielectric object: Aperture method", Journal of Instrumentation, vol. 13, p. C02033, 2018.

[6] S. N. Galyamin, A. V. Tyukhtin and V. V. Vorobev, "Focusing the Cherenkov radiation using dielectric concentrator: simulations and comparison with theory", Journal of Instrumentation, vol. 13, p. C02029, 2018.

[7] A. V. Tyukhtin, V. V. Vorobev, S. N. Galyamin, and E.S. Belonogaya, "Radiation of a charge moving along the boundary of dielectric prism", Phys. Rev. AB, vol. 22, p. 012802, 2019.

[8] A. V. Tyukhtin, S. N. Galyamin, and V. V. Vorobev, Peculiarities of Cherenkov radiation from a charge moving through a dielectric cone, Phys. Rev. A, vol. 99, p. 023810, 2019.

Ray-optical and aperture methods

d is a size of an object

λ is a wavelength under consideration

R is a distance from the object to the observation point

The main condition for both methods: $d >> \lambda$

More exactly, we assume that:

- the size of the external boundary of the object illuminated by Cherenkov radiation (the "aperture") is much more than the wavelength;

- the main part of the aperture is far from the path of the charge (in the wavelength scale).

Ray-optical and aperture methods

The 1st and 2nd stages are the same for both methods.

1. We solve certain "etalon problem" which does not take into account "external" boundary of the target. For example, if the charge moves in the vacuum channel inside the target then we consider the channel border but do not take into account external boundaries of the object. In other words, initially we consider the problem for infinite medium with the boundary nearest to the charge trajectory and obtain the field inside the bulk of the target. This field can be called an "incident" field.

2. We select the part of the external surface of the object which is illuminated by Cherenkov radiation (the "aperture"). Using the fact that the object is much more than the wavelength under consideration we obtain asymptotic of the incident field (which is the wave field), and presents it in the form of wave of two polarizations (vertical and horizontal). Further we calculate the field at the external surface of the aperture using the Snell's and Fresnel's laws.

The 3rd stage of ray-optical technique.

Calculation of the wave field outside the object using the ray-optics laws.

The 3rd stage of aperture technique.

Calculation of the wave field outside the object using Stratton-Chu formulae ("aperture integrals").

Ray-optical and aperture methods

The 3rd stage of ray-optical technique.

Calculation of the wave field outside the object using the ray-optics laws .

Advantage: Analytical formulas obtained by this method are not laborious for further computations.

Disadvantages: essential additional limitations.

- The distance from the aperture to the observation point should not be very large, i.e. so-called “wave parameter” should be small.
- The observation point cannot be close to focuses and caustics.

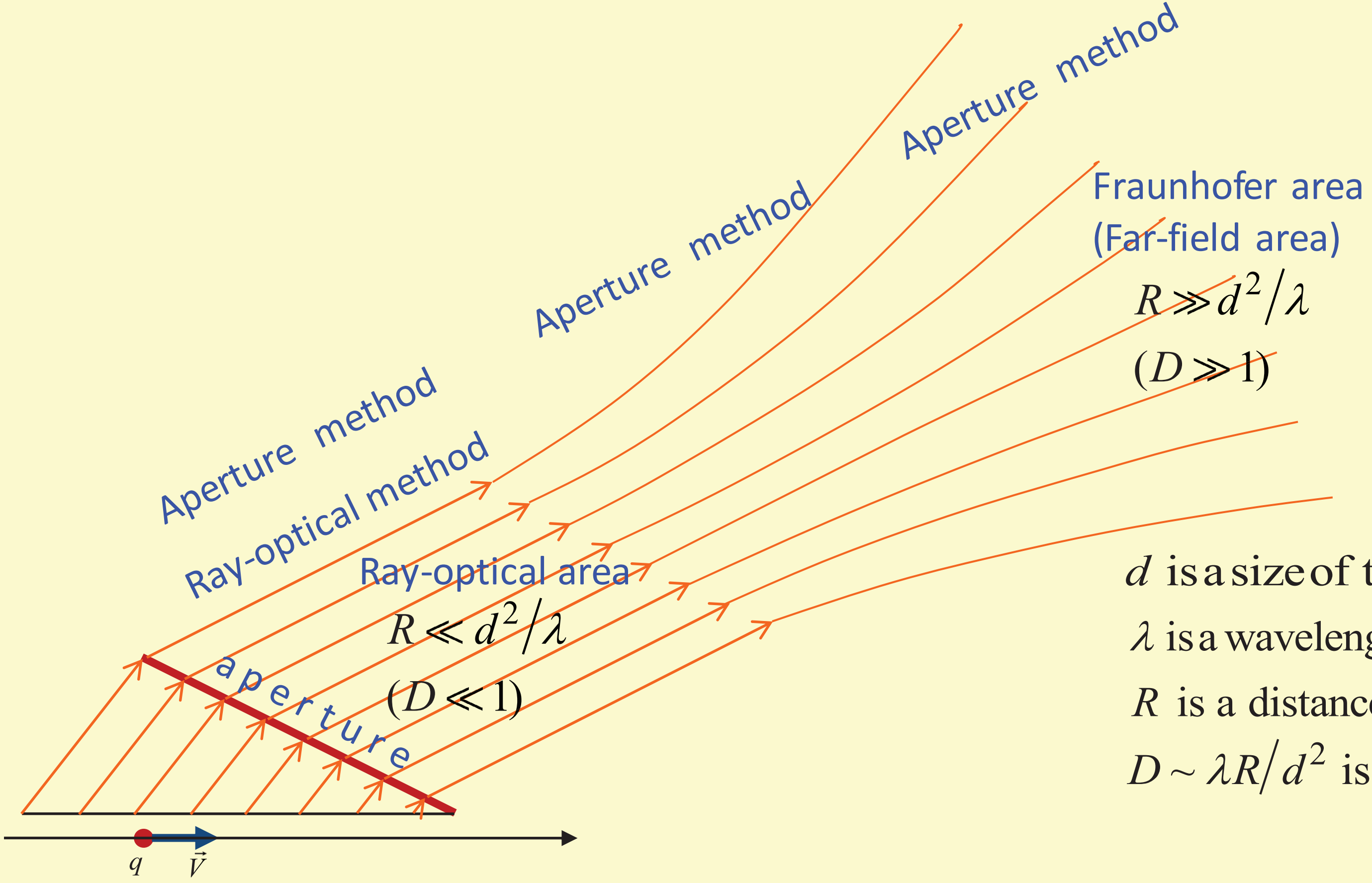
The 3rd stage of aperture technique.

Calculation of the wave field outside the object using Stratton-Chu formulas (“aperture integrals”).

Advantage: No additional limitations.
The method is valid for observation point with arbitrary wave parameter, in particular, in Fraunhofer area, as well as in neighborhoods of focuses and caustics.

Disadvantage: It is necessary to compute complex double integrals.

Ray-optical and aperture methods



d is a size of the aperture

λ is a wavelength

R is a distance from the aperture

$D \sim \lambda R/d^2$ is a wave parameter

Here we focus on the aperture method.

Aperture integrals (Stratton-Chu formulae)

General form for Fourier transform of electric field

$$\vec{E}\left(\vec{R}\right)=\vec{E}^{(h)}\left(\vec{R}\right)+\vec{E}^{(e)}\left(\vec{R}\right),$$

$$\vec{E}^{(h)}\left(\vec{R}\right)=\frac{ik}{4\pi}\int_{\Sigma}\left\{\left[\vec{n}'\times\vec{H}\left(\vec{R}'\right)\right]G\left(\left|\vec{R}-\vec{R}'\right|\right)+\frac{1}{k^2}\left(\left[\vec{n}'\times\vec{H}\left(\vec{R}'\right)\right]\cdot\nabla'\right)\nabla' G\left(\left|\vec{R}-\vec{R}'\right|\right)\right\}d\Sigma',$$

$$\vec{E}^{(e)}\left(\vec{R}\right)=\frac{1}{4\pi}\int_{\Sigma}\left[\left[\vec{n}'\times\vec{E}\left(\vec{R}'\right)\right]\times\nabla' G\left(\left|\vec{R}-\vec{R}'\right|\right)\right]d\Sigma',$$

Σ is an aperture square,

$k=\omega/c$ is a wave number of the outer space,

$\vec{n}'$ is a unit normal to the aperture,

$G(R)=\dfrac{\exp(ikR)}{R}$ is a Green function,

$$\nabla'=\vec{e}_{x'}\frac{\partial}{\partial x'}+\vec{e}_{y'}\frac{\partial}{\partial y'}+\vec{e}_{z'}\frac{\partial}{\partial z'}$$

Aperture integrals (Stratton-Chu formulae)

Approximate form for Fourier transform of electric field in Fraunhofer (far-field) area

$$D \sim \lambda R / d^2 \gg 1 \quad \Rightarrow \quad R >> d \cdot d / \lambda >>> d$$

$$\vec{E}^{(h)}\left(\vec{R}\right) \approx \frac{ik\exp\left(ikR\right)}{4\pi R}\int_{\Sigma}\left[\left[\vec{e}_R \times \left[\vec{n}' \times \vec{H}\left(\vec{R}'\right)\right]\right] \times \vec{e}_R\right]\exp\left(-ik\vec{e}_R\vec{R}'\right)d\Sigma',$$

$$\vec{E}^{(e)}\left(\vec{R}\right) \approx \frac{ik\exp\left(ikR\right)}{4\pi R}\int_{\Sigma}\left[\vec{e}_R\left[\vec{n}' \times \vec{E}\left(\vec{R}'\right)\right]\right]\exp\left(-ik\vec{e}_R\vec{R}'\right)d\Sigma',$$

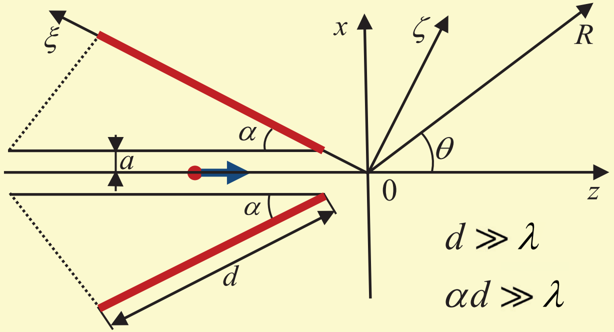
Σ is an aperture square,

$k = \omega / c$ is a wave number of the outer space,

$\vec{n}'$ is a unit normal to the aperture,

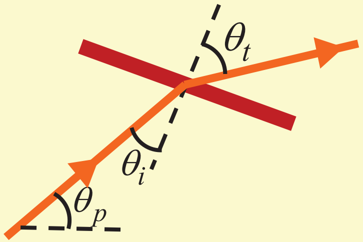
$\vec{e}_R = \vec{R} / R$.

Conical target



Charge velocity: $\vec{V} = c\beta\vec{e}_z$

Refractive index: $n = \sqrt{\varepsilon\mu}$



Cherenkov angle: $\theta_p = \arccos\left(1/(n\beta)\right)$

Angle of incidence: $\theta_i = \frac{\pi}{2} - \alpha - \theta_p$

Angle of refraction: $\theta_t = \arcsin\left(n \sin\theta_i\right)$

The wave incident on aperture (Fourier transform):

$$H_\varphi^{(i)} \approx \frac{iq}{c}\eta\sqrt{\frac{s}{2\pi r}}\exp\left\{i\left(sr + \frac{\omega}{V}z - \frac{3\pi}{4}\right)\right\}$$

$$\eta = -\frac{2i}{\pi a}\left[\kappa\frac{1-n^2\beta^2}{\varepsilon\left(1-\beta^2\right)}I_1(\kappa a)H_0^{(1)}(sa) + sI_0(\kappa a)H_1^{(1)}(sa)\right]^{-1}$$

$$s(\omega) = \frac{\omega}{V}\sqrt{n^2\beta^2 - 1} \qquad \kappa(\omega) = \frac{\omega}{V}\sqrt{1-\beta^2}$$

Fourier-transform of the field on outer surface of the aperture:

$$H_\varphi(\vec{R}) \approx \frac{T_\mathrm{v} q\eta\sqrt{s}}{c\sqrt{2\pi\xi\sin\alpha}}\exp\left\{i\frac{\omega}{V}\left(\sqrt{n^2\beta^2-1}\sin\alpha - \cos\alpha\right)\xi - \frac{i\pi}{4}\right\}$$

$$T_\mathrm{v} = 2\sqrt{\mu/\varepsilon}\cos\theta_i \Big/ \left(\sqrt{\mu/\varepsilon}\cos\theta_i + \cos\theta_t\right)$$

[5] A. V. Tyukhtin, V. V. Vorobev, E. S. Belonogaya, and S. N. Galyamin, "Radiation of a charge in presence of a dielectric object: Aperture method", Journal of Instrumentation, vol. 13, p. C02033, 2018.

[8] A. V. Tyukhtin, S. N. Galyamin, and V. V. Vorobev, Peculiarities of Cherenkov radiation from a charge moving through a dielectric cone, Phys. Rev. A, vol. 99, p. 023810, 2019.

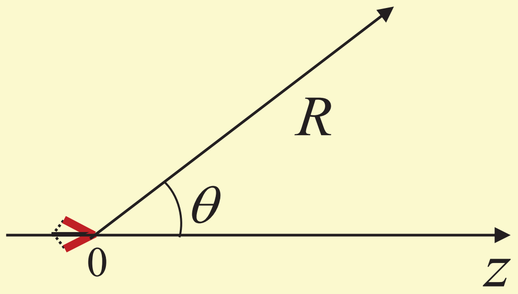
Conical target

After transformation, one can write the field in far-field zone in the form:

$$E_R = E_\varphi = H_R = H_\theta = 0$$

$$E_\theta = H_\varphi = -\frac{ke^{ikR}\sin\alpha}{2R}\int_{\xi_1}^{\xi_2}\xi' H_{\varphi'}(\vec{R}')\Big[\big(\cos\theta_t + \sin\alpha\cos\theta\big)J_1\big(k\xi'\sin\alpha\sin\theta\big) +$$

$$+i\cos\alpha\sin\theta J_0\big(k\xi'\sin\alpha\sin\theta\big)\Big]\exp\big(ik\xi'\cos\alpha\cos\theta\big)d\xi'.$$



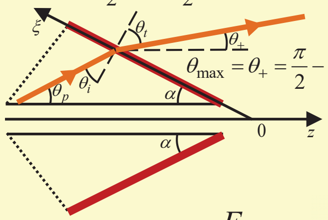
Conditions of applicability:

1) $d \gg \lambda, \quad \alpha d \gg \lambda$

2) $D \sim \lambda R / d^2 \gg 1 \;\Rightarrow\; R >> d \cdot d/\lambda >>> d$

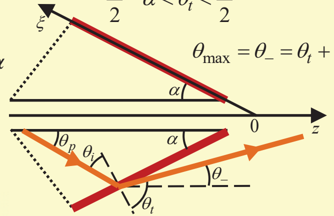
Conical target

$$-\frac{\pi}{2} < \theta_t < \frac{\pi}{2} - \alpha$$



$$\theta_{\max} = \theta_+ = \frac{\pi}{2} - \theta_t - \alpha$$

$$\frac{\pi}{2} - \alpha < \theta_t < \frac{\pi}{2}$$

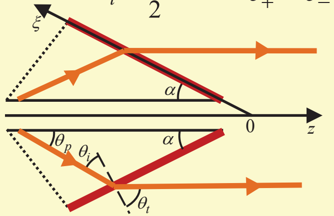


$$\theta_{\max} = \theta_- = \theta_t + \alpha - \frac{\pi}{2}$$

$$E_{\theta\max} \sim \frac{kd}{R}$$

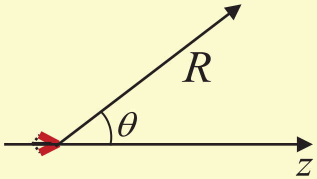
The angular half-width of the main lobe of the radiation pattern: $\delta\theta \approx \dfrac{2\pi}{k\,d\cos\theta_t}$

$$\theta_t = \frac{\pi}{2} - \alpha \qquad \theta_+ = \theta_- = 0$$

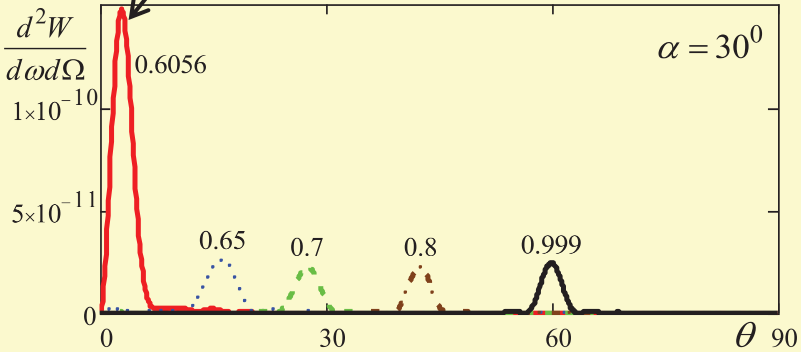


$$E_{\theta\max} \sim \frac{(kd)^{3/2}}{R}$$

“Cherenkov spotlight”



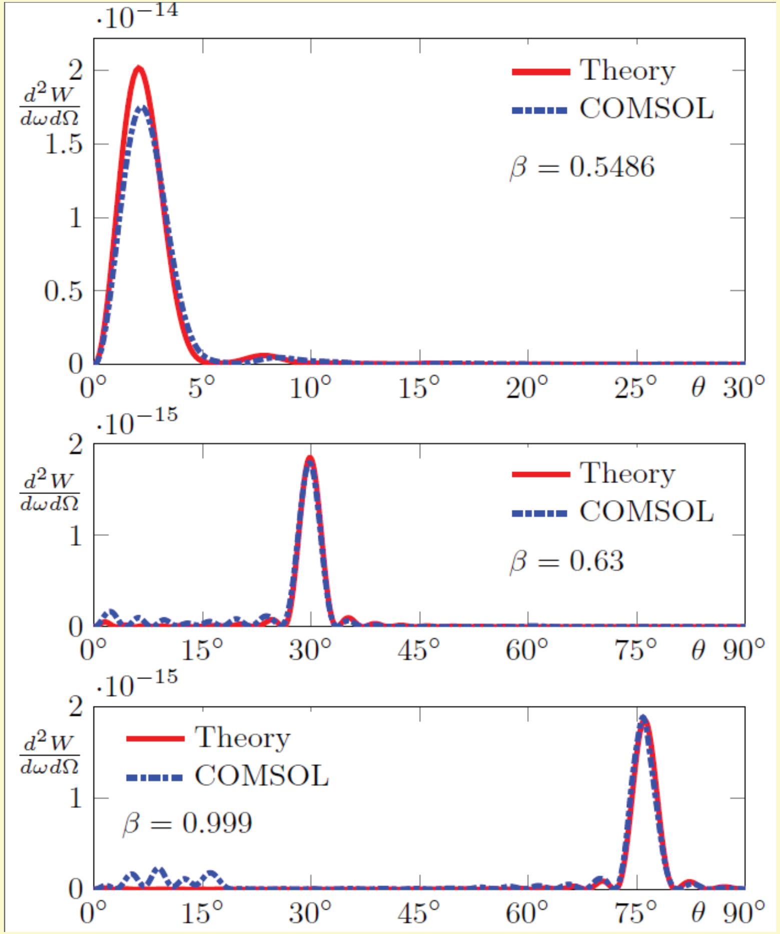
“Cherenkov spotlight”



Frequency-angular density of the radiation power $(\mathrm{W \cdot s/rad^2})$ depending on the angle θ

$q = 1\,\mathrm{nC},\ \varepsilon = 4,\ \mu = 1,\ d\omega/c = 100,\ a\omega/c = 1.$

Conical target



$\alpha = 45^0$

$q = 1\text{nC},$

$\varepsilon = 4,\ \mu = 1,$

$d\omega/c = 100,$

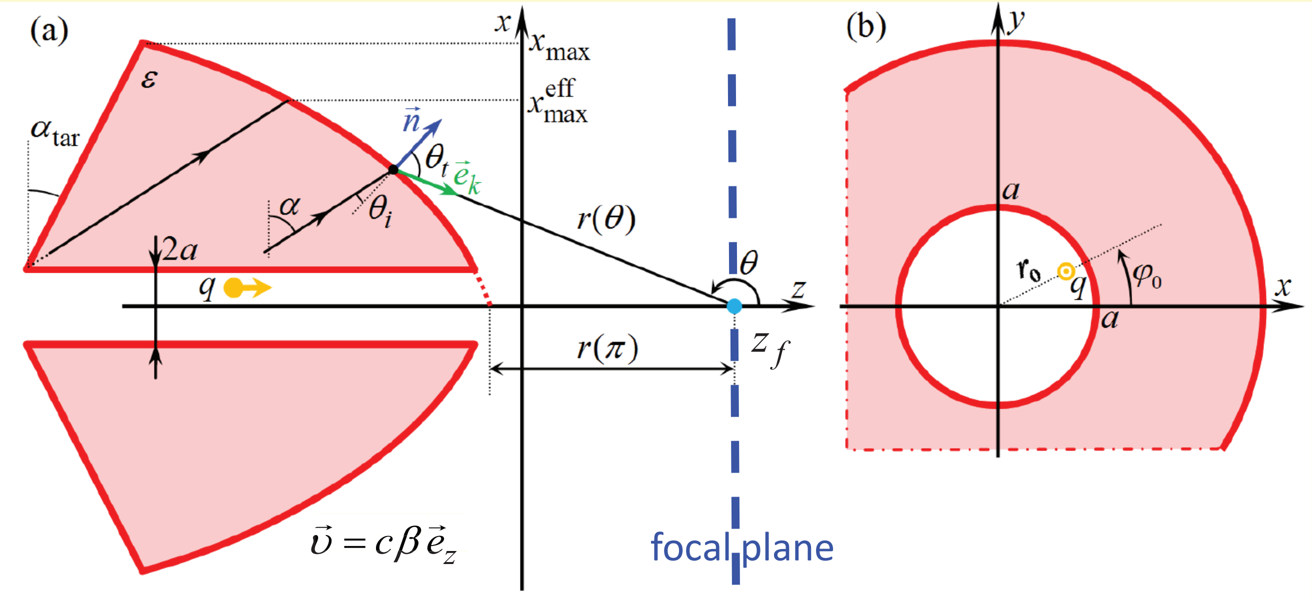
$a\omega/c = 1.$

Concentrator for Cherenkov radiation

We have offered and investigated special object which can be called a "concentrator for Cherenkov radiation". It allows concentrating radiation in small region near the focus due to the form of the outer boundary. The target looks like a cone, but the outer border is a hyperboloid. Unfortunately, focusing is possible only in small range of charge speeds, usually, close to the speed of light in the medium.

S.N. Galyamin, A.V. Tyukhtin, Phys. Rev. Lett., V. 113, P. 064802, (2014);
S.N. Galyamin et al., J. Instrum., V. 13, P. C02029, (2018).

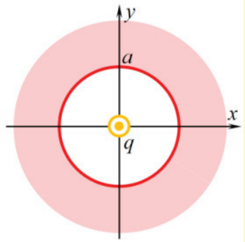
Now we consider a non-symmetrical case where charge trajectory has a shift from the structure axis.
[IPAC19: MOPGW060; arXiv: 1904.05188]

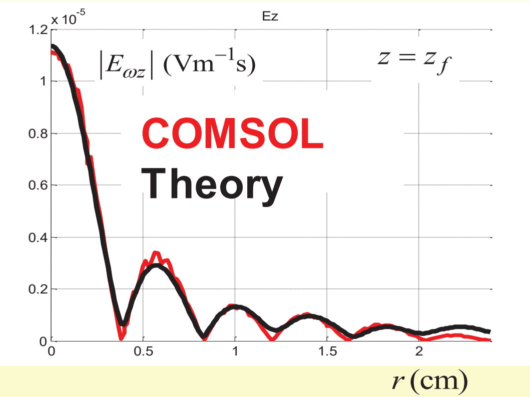


Concentrator for Cherenkov radiation

From poster MOPGW060

Comparison of theory and COMSOL simulations for the charge moving along the symmetry axis.





$n = 1.27,$

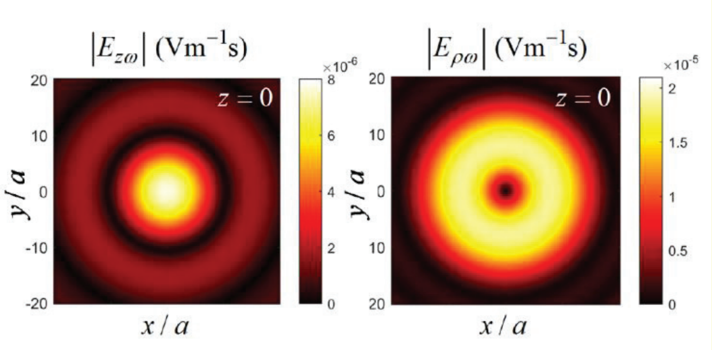
$\beta = \beta_0 = 0.8,$

$q = 1\text{nC},$

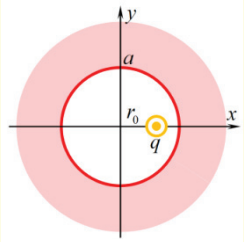
$\omega = 2\pi \cdot 100\,\text{GHz},$

$a = c/\omega = 0.047\,\text{cm}.$

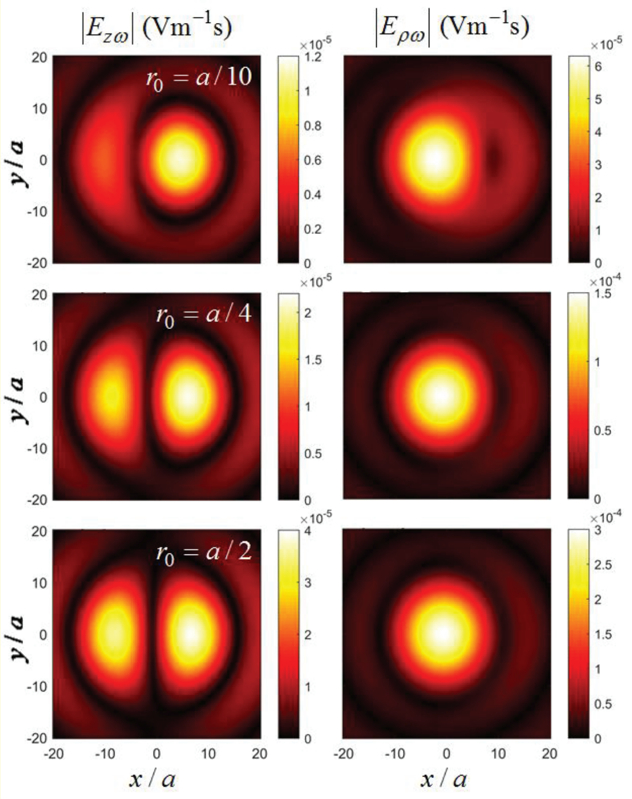
focal plane



Field distributions when the charge trajectory is shifted from the symmetry axis.

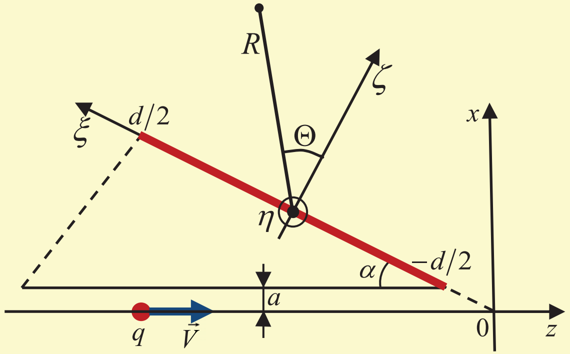


Increasing of shift r_0



focal plane

Prismatic target



Aperture has the size
d (along *ξ*) and *b* (along *η*).

$$\xi = R\sin\Theta\cos\Phi,$$
$$\eta = R\sin\Theta\sin\Phi,$$
$$\zeta = R\cos\Theta.$$

The field in the Fraunhofer (far-field) area

$$E_R = 0,$$

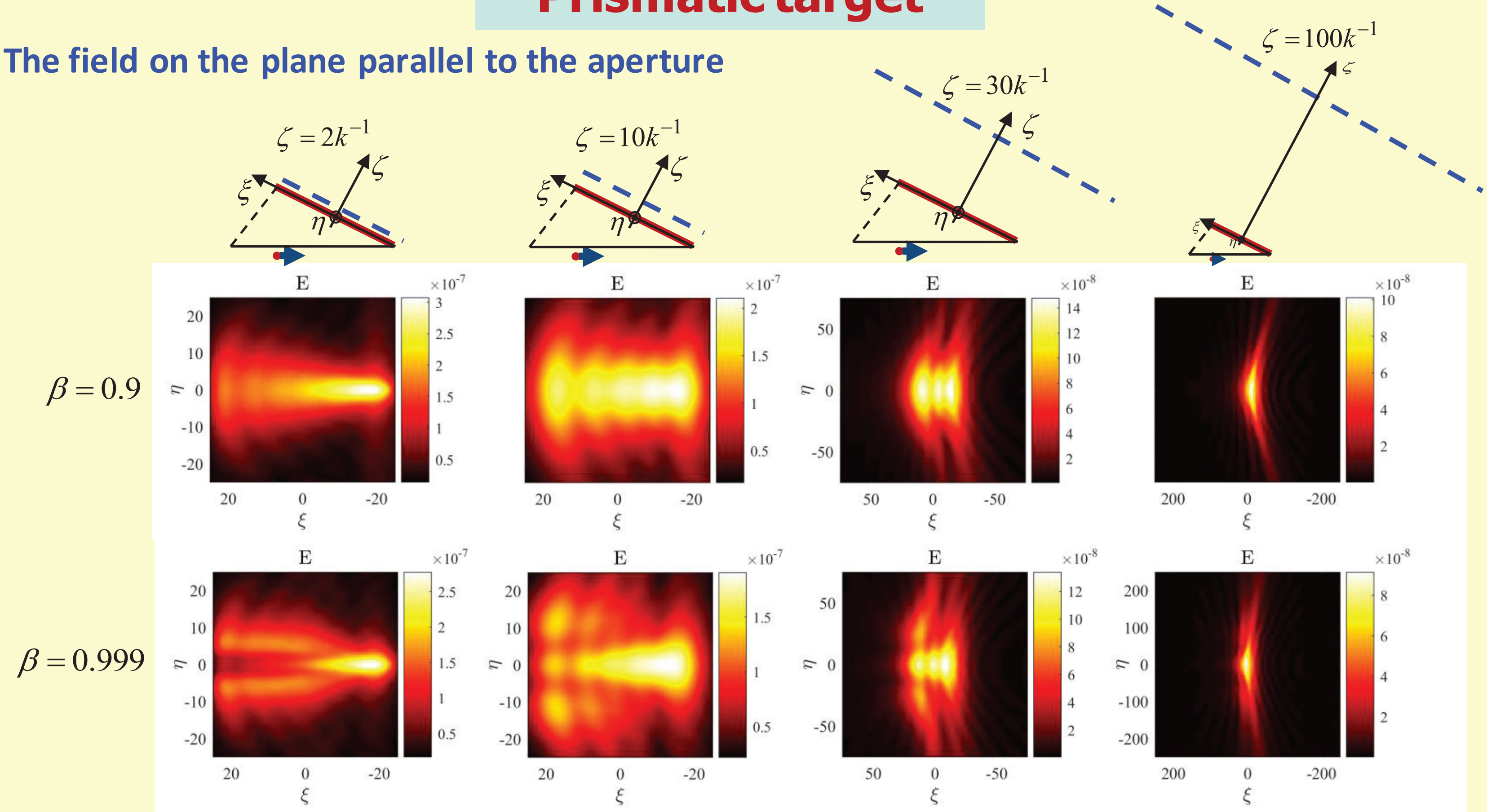
$$\begin{Bmatrix} E_\Theta \\ E_\Phi \end{Bmatrix} \approx \frac{ik\exp(ikR)}{4\pi R}\int_\Sigma \begin{Bmatrix} -E_\xi\cos\Phi - E_\eta\sin\Phi + \left[H_\xi\sin\Phi - H_\eta\cos\Phi\right]\cos\Theta \\ H_\xi\cos\Phi + H_\eta\sin\Phi + \left[E_\xi\sin\Phi - E_\eta\cos\Phi\right]\cos\Theta \end{Bmatrix}\times$$
$$\times\exp\left\{-ik\left(\xi'\cos\Phi + \eta'\sin\Phi\right)\sin\Theta\right\}d\Sigma',$$

$E_{\xi,\eta} = E_{\xi,\eta}(\vec{R}'),\ \ H_{\xi,\eta} = H_{\xi,\eta}(\vec{R}')$ are the field on the aperture.

A. V. Tyukhtin, V. V. Vorobev, S. N. Galyamin, and E.S. Belonogaya, “Radiation of a charge moving along the boundary of dielectric prism”, Phys. Rev. AB, vol. 22, p. 012802, 2019.

Prismatic target

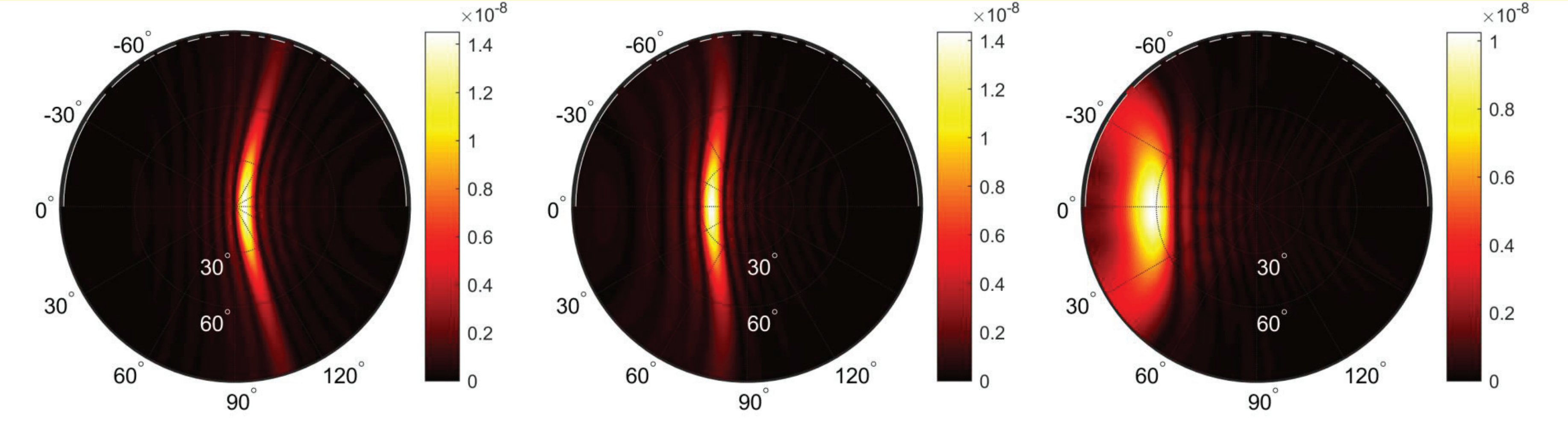
The field on the plane parallel to the aperture



Electric field magnitude $|E|$ in the plane ξ, η (parallel to the aperture) for the following parameters: $\varepsilon = 4, \ \mu = 1, \ a = 1, \ d = b = 50, \ \alpha = 30^0$.

Prismatic target

Radiation patterns in the far-field (Fraunhofer) area



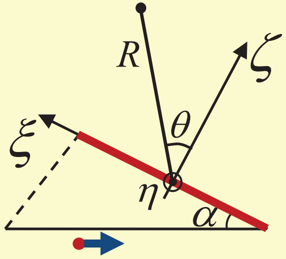
$\alpha = 30^0$ $\alpha = 45^0$ $\alpha = 60^0$

Electric field magnitude $\left|\vec{E}\right|$ in the far field zone.

Spherical coordinate system is used (with respect to normal ζ).

Radial axis is θ, polar axis is φ.

Parameters: $\varepsilon = 4, \;\; \mu = 1, \;\; a = 1, \;\; d = b = 50, \;\; \beta = 0.9$



Spherical target

Dielectric ball with radius R_0 having the cylindrical vacuum channel with radius a.

Charge velocity: $\vec{V} = c\beta\vec{e}_z$

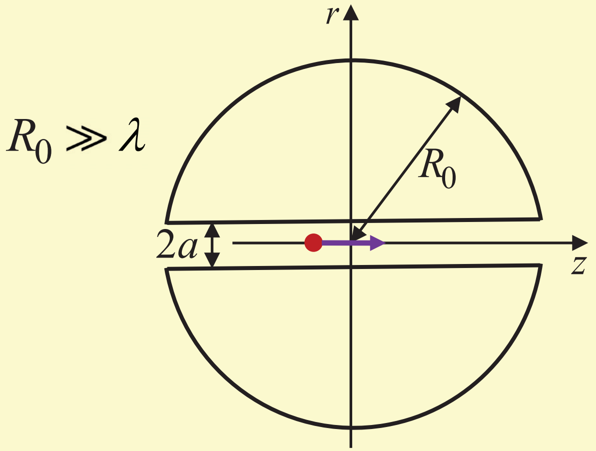
Refractive index: $n = \sqrt{\varepsilon\mu}$

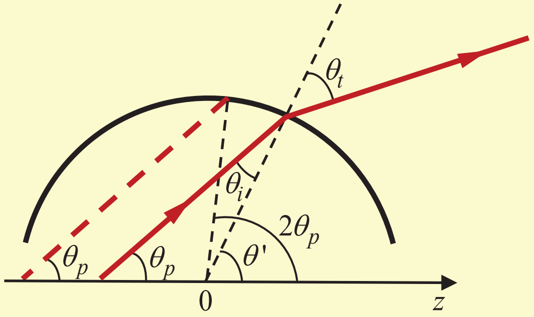
Cherenkov angle: $\theta_p = \arccos\left(1/(n\beta)\right)$

Angle of incidence: $\theta_i(\theta') = \theta' - \theta_p$

Angle of refraction: $\theta_t(\theta') = \arcsin\left(n\sin\theta_i(\theta')\right)$

Section of the ball with channel





Spherical target

Aperture integrals for spherical target

$$\vec{E} = \vec{E}^{(h)} + \vec{E}^{(e)} = \vec{E}^{(h1)} + \vec{E}^{(h2)} + \vec{E}^{(e)},$$

$$\begin{Bmatrix} E_r^{(h1)} \\ E_z^{(h1)} \end{Bmatrix} = \frac{ikR_0^2}{4\pi} \int_{\theta_1}^{\theta_2} d\theta' \int_0^{2\pi} d\varphi' \begin{Bmatrix} -\cos\theta'\cos\varphi' \\ \sin\theta' \end{Bmatrix} \sin\theta' \frac{\exp\{ik\tilde{R}\}}{\tilde{R}} H_{\varphi'}^{(t)}(\theta'),$$

$$\begin{Bmatrix} E_r^{(h2)} \\ E_z^{(h2)} \end{Bmatrix} = \frac{ikR_0^2 R}{4\pi} \int_{\theta_1}^{\theta_2} d\theta' \int_0^{2\pi} d\varphi' \begin{Bmatrix} R_0 \sin\theta'\cos\varphi' - R\sin\theta \\ R_0\cos\theta' - R\cos\theta \end{Bmatrix} \sin\theta' \left(\cos\theta\sin\theta' - \sin\theta\cos\theta'\cos\varphi'\right) H_{\varphi'}^{(t)}(\theta') \frac{\exp\{ik\tilde{R}\}}{\tilde{R}^3},$$

$$\begin{Bmatrix} E_r^{(e)} \\ E_z^{(e)} \end{Bmatrix} = \frac{ikR_0^2}{4\pi} \int_{\theta_1}^{\theta_2} d\theta' \int_0^{2\pi} d\varphi' \begin{Bmatrix} \left(R_0\cos\theta' - R\cos\theta\right)\cos\varphi' \\ -R_0\sin\theta' + R\sin\theta\cos\varphi' \end{Bmatrix} \sin\theta' \frac{\exp\{ik\tilde{R}\}}{\tilde{R}^2} E_{\theta'}^{(t)}(\theta'),$$

$$\theta_1 = \max\left\{\theta_p - \theta_*, \arcsin(\mathrm{a}/\mathrm{R}_0), 0\right\}$$

$$\theta_2 = \min\left\{\theta_p + \theta_*, 2\theta_p\right\}$$

$$\tilde{R} = \sqrt{R_0^2 + R^2 - 2RR_0\left(\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos\varphi'\right)}$$

Tangential components of the field
on the external surface of the aperture: $H_{\varphi'}^{(t)} = T_v H_{\varphi'}^{(i)}, \quad E_{\theta'}^{(t)} = H_{\varphi'}^{(t)} \cos\theta_t$

Transmission coefficient: $T_v(\theta') = \dfrac{2\cos\theta_i(\theta')}{\cos\theta_i(\theta') + \sqrt{\varepsilon/\mu}\cos\theta_t(\theta')}$

$H_{\varphi'}^{(i)}$ is magnetic component of “incident” (i.e. the field of Cherenkov radiation) on the aperture.

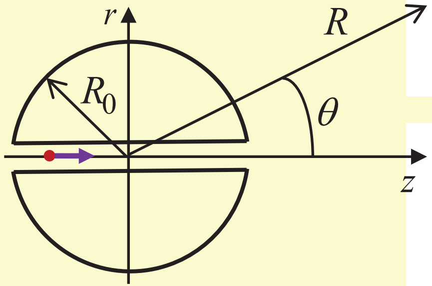
Spherical target

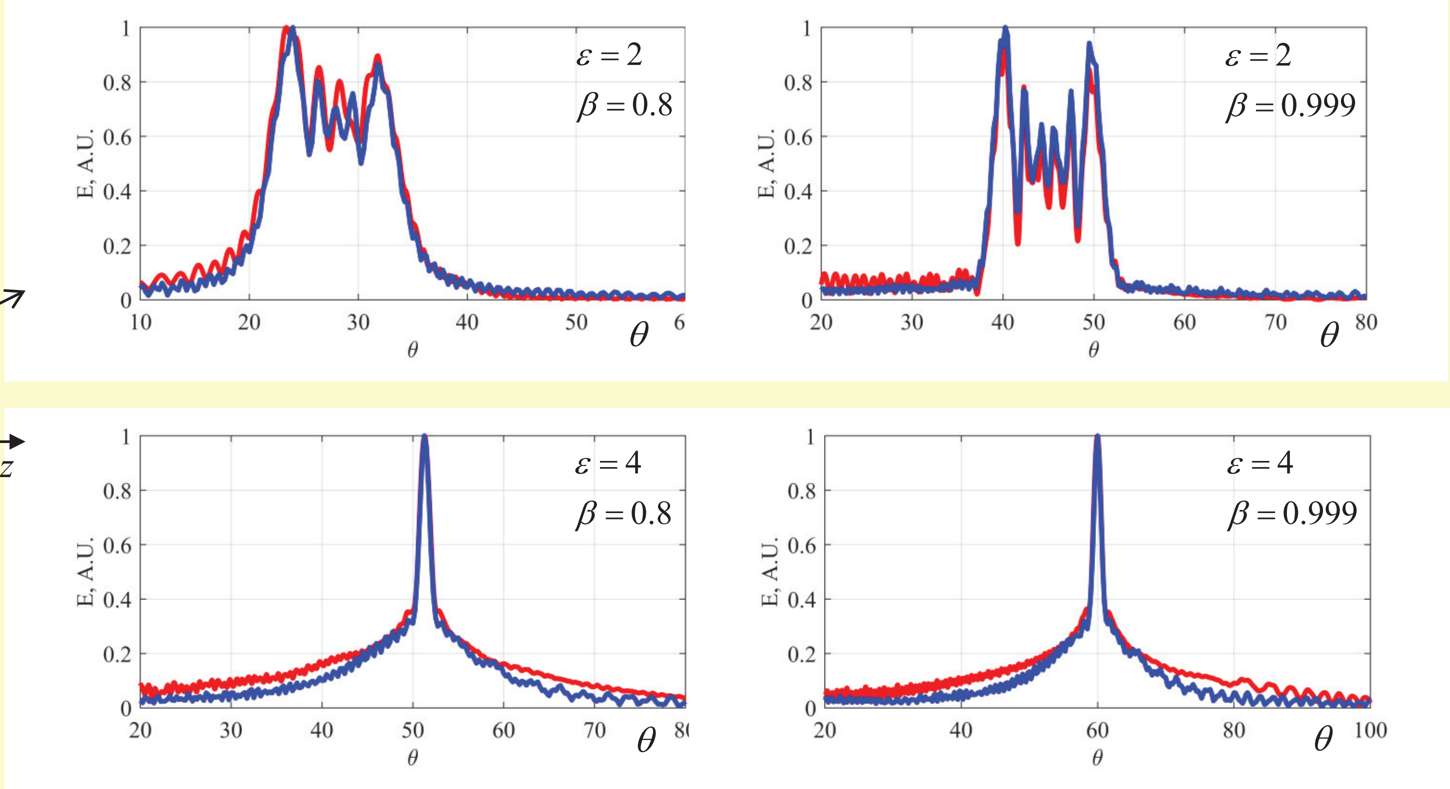
Analytical results (red) and COMSOL Multiphysics results (blue) for Fourier-transform of electric field

$R_0 = 300 \cdot c/\omega,$

$a = c/\omega,$

$R = 600 \cdot c/\omega.$

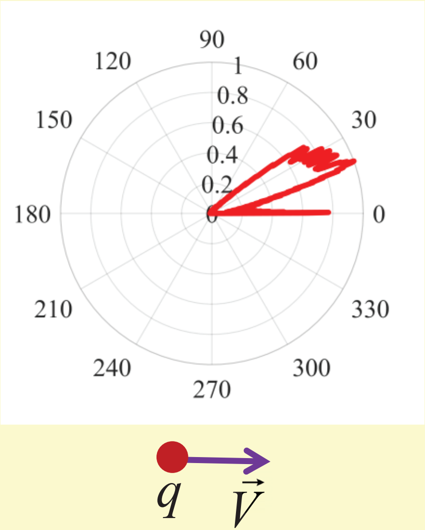




Spherical target

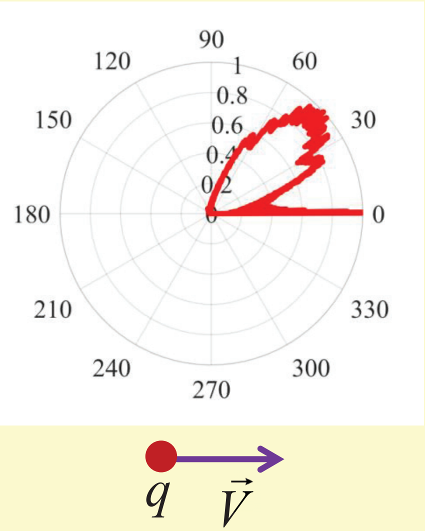
Radiation patterns for electric component in far-field (Fraunhofer) area

$$R_0 = 300 \cdot c/\omega, \quad a = c/\omega$$



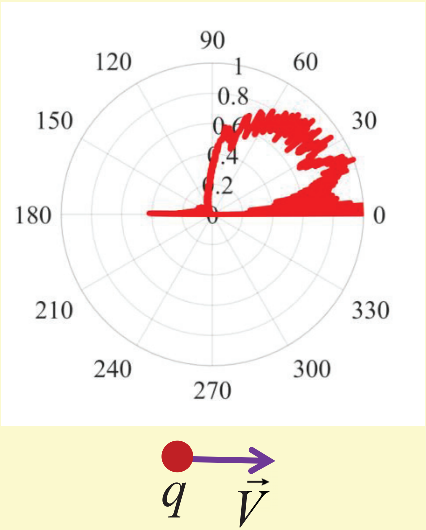
$\varepsilon = 2$

$\beta = 0.8$



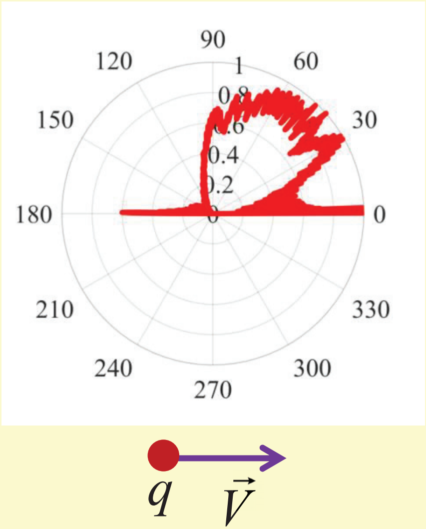
$\varepsilon = 2$

$\beta = 0.999$



$\varepsilon = 4$

$\beta = 0.8$



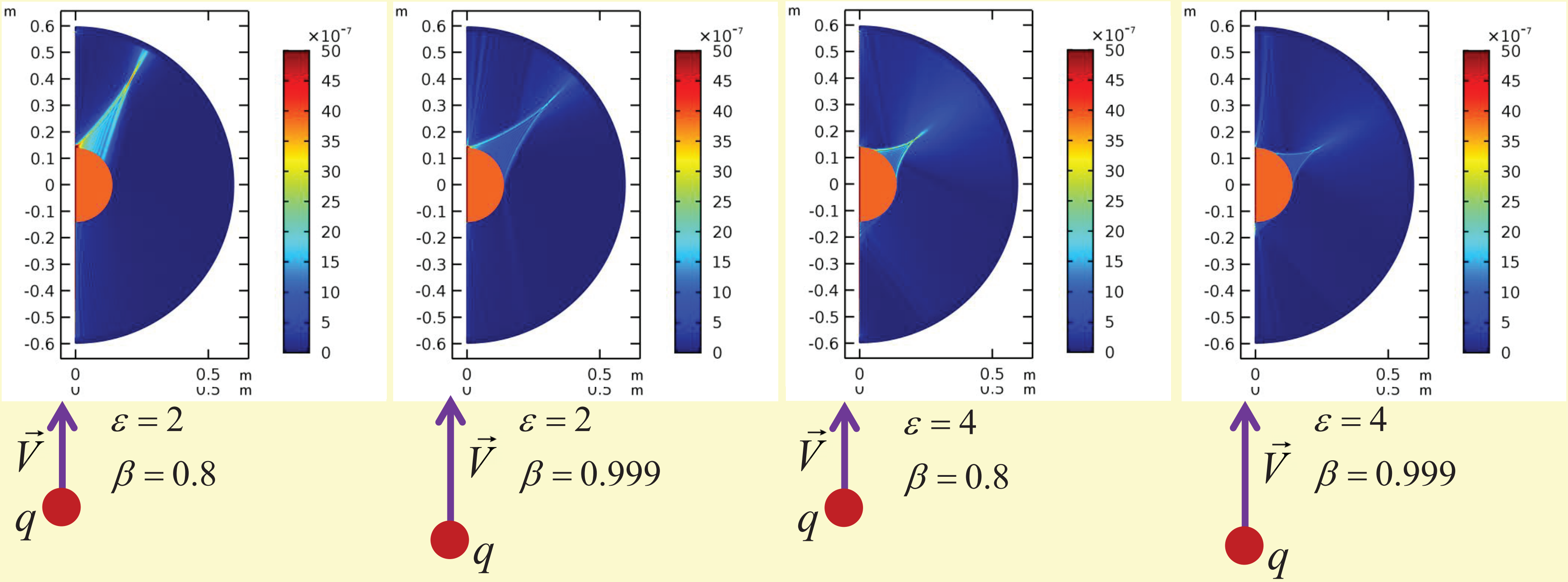
$\varepsilon = 4$

$\beta = 0.999$

Spherical target

Distribution of electric field outside the dielectric ball

$$R_0 = 300 \cdot c/\omega, \quad a = c/\omega, \quad q = 1\,\mathrm{nC}, \quad \omega = 2\pi \cdot 100\,\mathrm{GHz}$$



Conclusion

We have developed two methods of calculation of radiation from bunches in the presence of relatively large dielectric objects. One of them can be named “ray-optical technique”, and the other can be named “aperture technique”. The last one is analogues to Kirchhoff method which has the widest distribution in optics, radiophysics and other “wave sciences”.

The aperture method has been tested with use of Comsol simulations for series of objects. It has been confirmed that this technique can by applied for dielectric objects with the size of several wavelengths or more. Unlike ray-optical method, the aperture method has no limits concerning the observation point.

Analytical und numerical investigation of radiation in the cases of a cone, a prism, a ball, and a concentrator of Cherenkov radiation have been performed. The characteristic distributions of the radiation field have been obtained. Nonordinary physical phenomena have been described. Among them, concentration of Cherenkov radiation from different objects, Cherenkov spotlight for the conical target et al.

Thank you for attention!