Muon *g*-2: An interplay between beam dynamics and a muon decay experiment at the precision frontier

Mike Syphers

Northern Illinois University

Fermilab

*for the Fermilab E989 Muon **g**-2 Collaboration*

International Particle Accelerator Conference
22 May 2019 Melbourne, Australia

Fundamental Particle Spin

- For a spin ½ point particle, classically the expectation is g = 1
- Stern-Gerlach and atomic spectroscopy experiments in the 1920s, became apparent $\boldsymbol{g_e = 2}$ for the electron.
- Dirac's famous equation in 1928

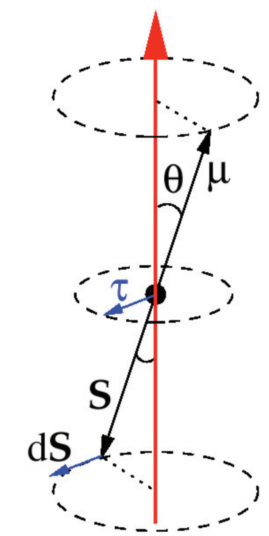
$$\left(\frac{1}{2m}(\vec{P}+e\vec{A})^2+\frac{e}{2m}\vec{\sigma}\cdot\vec{B}-eA^0\right)\psi_A=(E-m)\psi_A$$

So, for an elementary spin ½ particle in Dirac's theory, *g*=2!

- Deviations from the value g = 2 for the electron, muon, etc. accounted for by quantum field theory

spin $\vec{S}=\frac{\hbar}{2}\vec{\sigma}$

magnetic moment $\vec{\mu}=g\frac{q}{2m}\vec{S}$



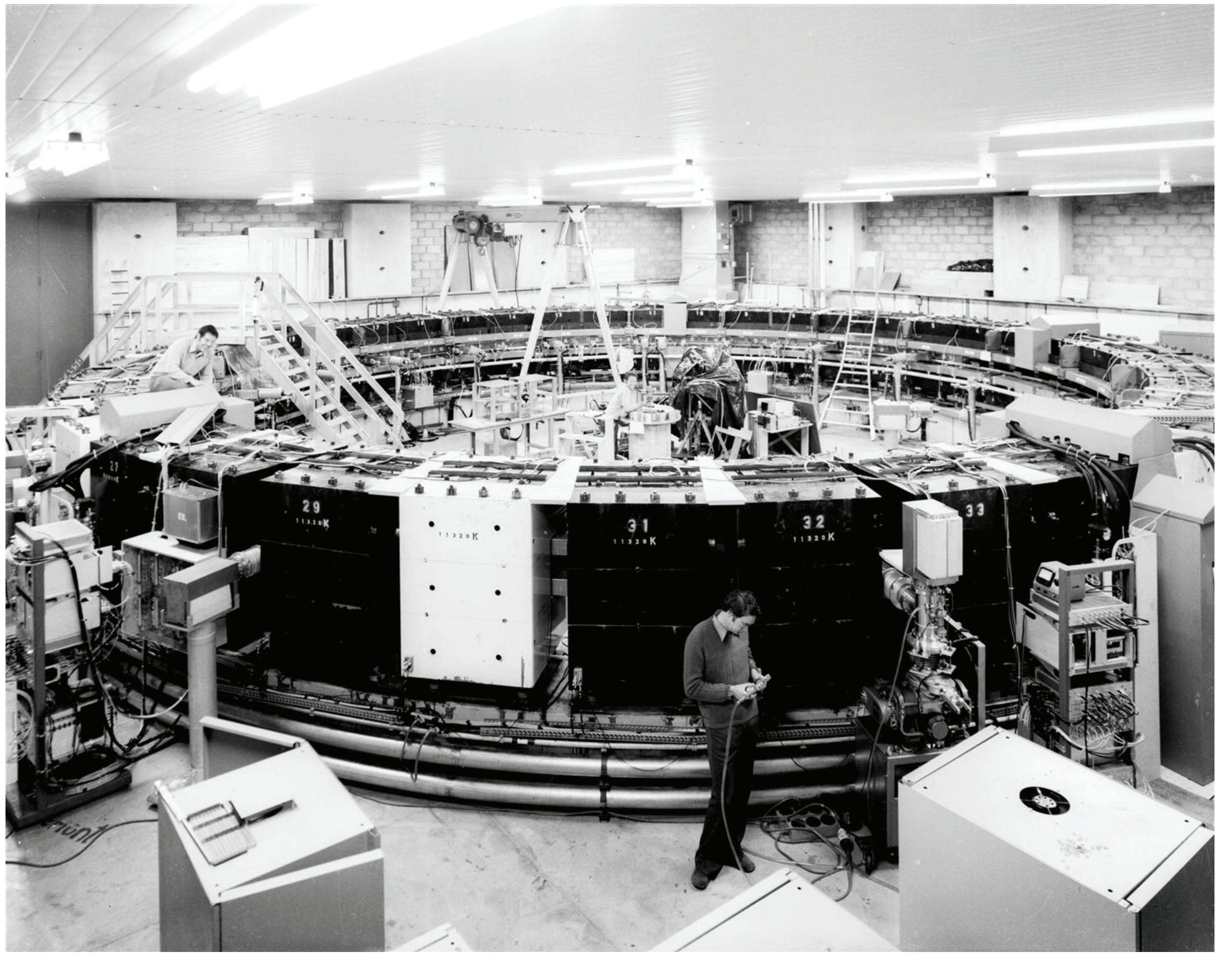
Set $g=2(1+a)$

anomaly: $a\equiv\frac{g-2}{2}$

For the muon, a is approximately 0.001166.

Brief History of Muon *g*-2

- The measurement of *a* = (*g*-2)/2 for the **muon** started out at CERN
  - 1959 (Lederman, et al.), using Synchrocyclotron — 2% result published in 1961, followed by more precise result — 0.4% error — confirming QED calculations at the time
  - 1966, using the CERN Proton Synchrotron (PS)
    - » 25x more accurate, showed inconsistency between experiment and the theory of the day
  - 1969-1979, third iteration of the experiment (still with PS) gave much more accuracy
    - » theory was confirmed to precision of 0.0007%
  - As time went on, theory continued to improve
- In 1980s, new experiment formed in U.S.
  - led to BNL g-2 Experiment E821
  - began running in 1997, final result in 2004
- Since then, theory has improved further
  - » ~3.5σ discrepancy, between E821 and SM



CERN g-2 storage ring, 1974

The Thomas BMT Equation and the Magic Momentum

- For electromagnetic fields in the lab frame, the precession of the spin vector in the rest frame of the particle is given by the Thomas-BMT eq.*:

$$\frac{d\vec{S}}{dt} = \vec{\omega}_s \times \vec{S} = -\frac{e}{\gamma m}\left[(1+a\gamma)\vec{B}_\perp + (1+a)\vec{B}_\parallel + \left(a\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right]\times\vec{S}$$

- The momentum vector of the particle will precess with

$$\frac{d\vec{p}}{dt} = \vec{\omega}_c \times \vec{p} = -\frac{e}{\gamma m}\left[\vec{B}_\perp + \frac{\gamma^2}{\gamma^2-1}\frac{\vec{E}\times\vec{\beta}}{c}\right]\times\vec{p}$$

* citations:
  — Thomas L H 1927 Philos. Mag. 3 1–22
  — Bargmann V, Michel L and Telegdi V L, 1959 Phys. Rev. Lett. 2 435–6

- For ideal condition of **purely** perpendicular magnetic field, and with electric fields:

$$\vec{\omega_a} \equiv \vec{\omega_s} - \vec{\omega_c} = -\frac{e}{m}\left[a\vec{B}_0 + (a - \frac{1}{\gamma^2-1})\frac{\vec{E}\times\vec{\beta}}{c}\right]$$

The Thomas BMT Equation and the Magic Momentum

- As we need to provide vertical focusing, if we operate at the "magic momentum" where the last term goes to zero, then can use *electrostatic* quadrupoles for this task

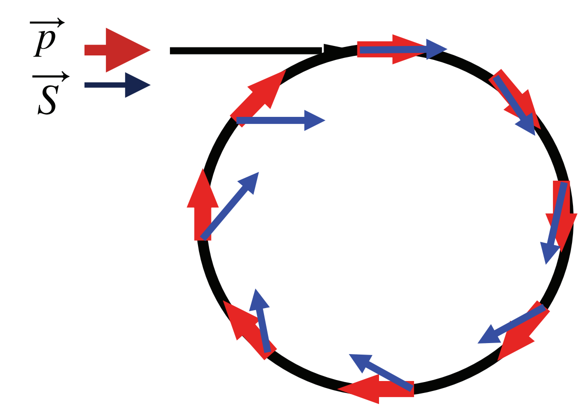
$$\vec{\omega}_a = -\frac{e}{m}\left[a\vec{B}_0 + \left(a - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right]$$

$$p_{magic} = mc/\sqrt{a}$$

~3.094 GeV/c
for the muon

- Then, ideally, rates observed at a detector at one location in the ring would contain frequency:

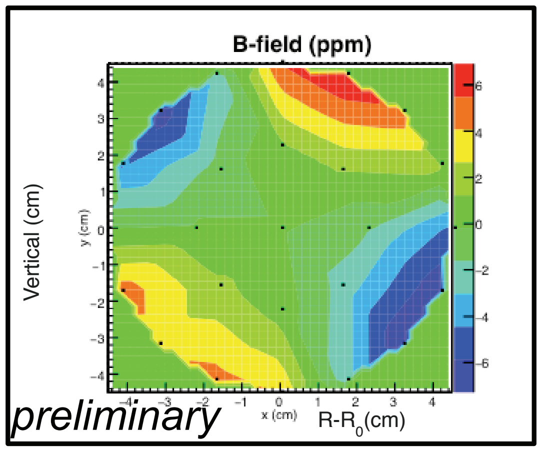
$$\omega_a = \frac{e}{m}\cdot B_0 \cdot a$$



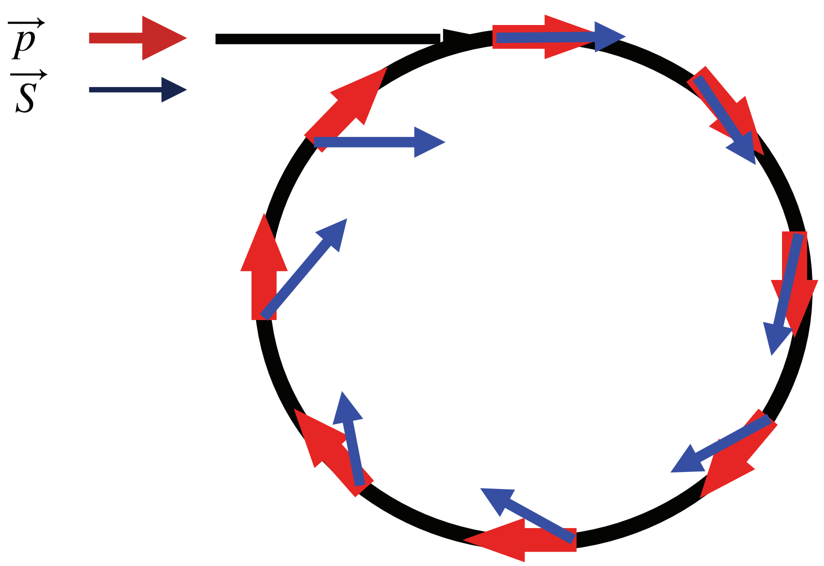
- So, send highly polarized beam of muons at the magic momentum into a highly uniform magnetic field, focused with electrostatic fields
- Detect positrons from muon decays; kinematics show those with highest energies emerge in direction of the muon's spin

Wiggle Plots

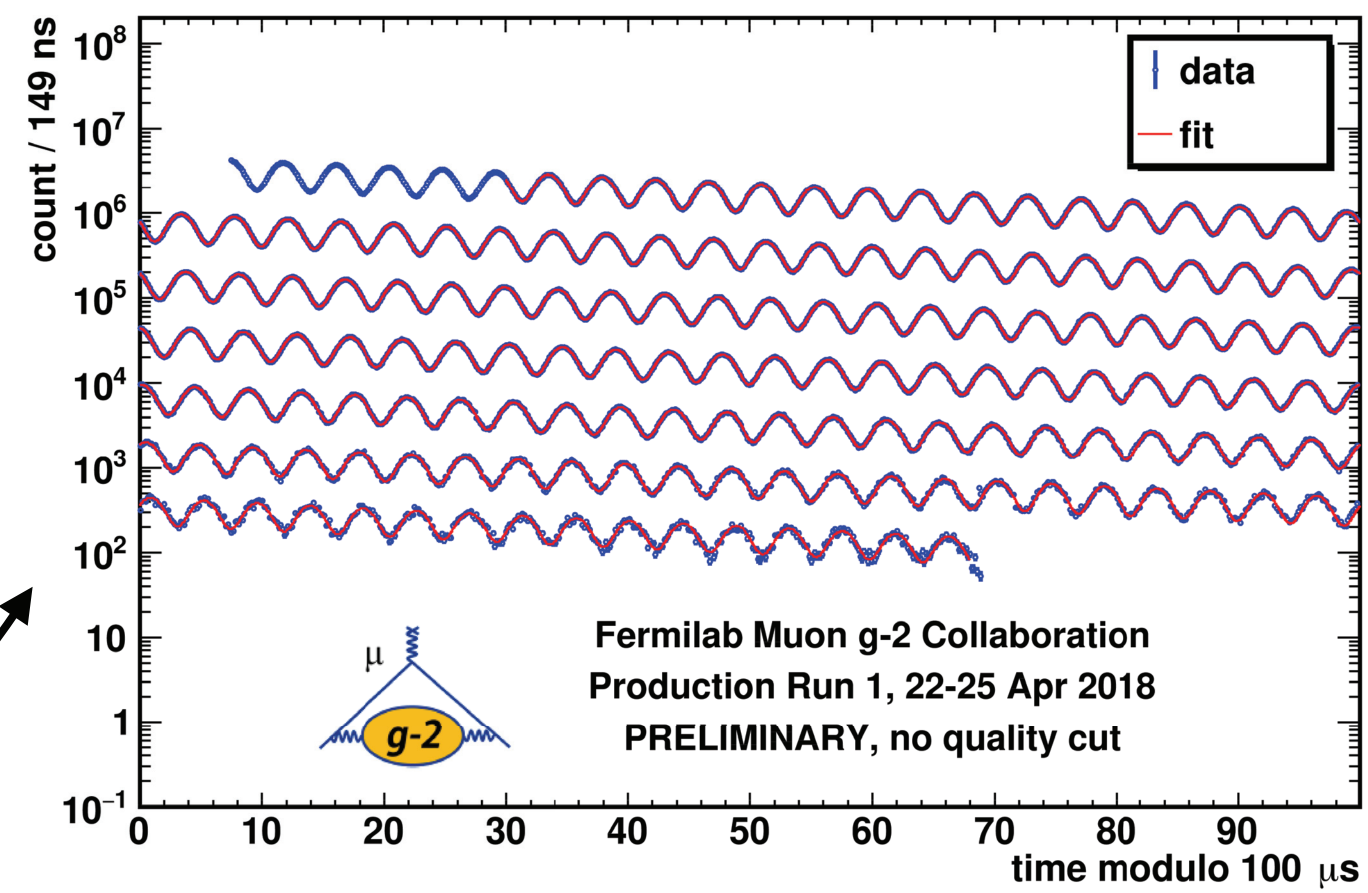
- Fixed detector in the ring would observe the rate of muon decay "wiggle" with a frequency given by $\omega_a = (e/m) \cdot B_0 \cdot a$



highly uniform B



$$N(t) = N_0 e^{-t/\tau_\mu} \left[1 + A\cos(\omega_a t + \phi_0)\right]$$

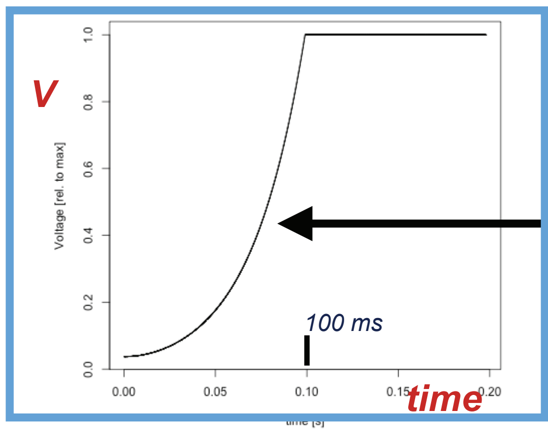


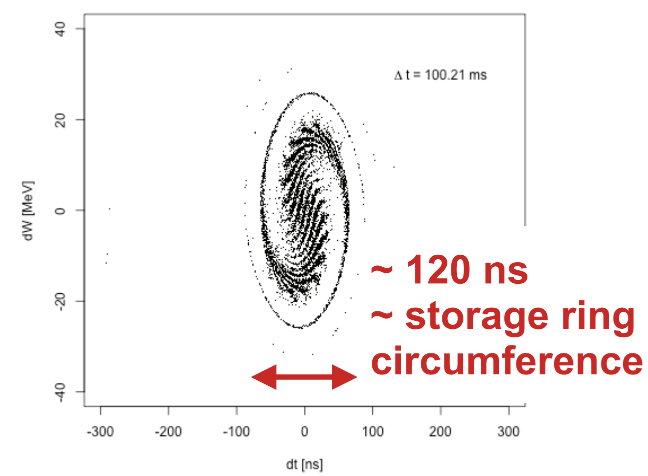
- Fermilab Muon g-2 Experiment uses 24 detector systems around the circumference, measuring positron energies, arrival times, etc.

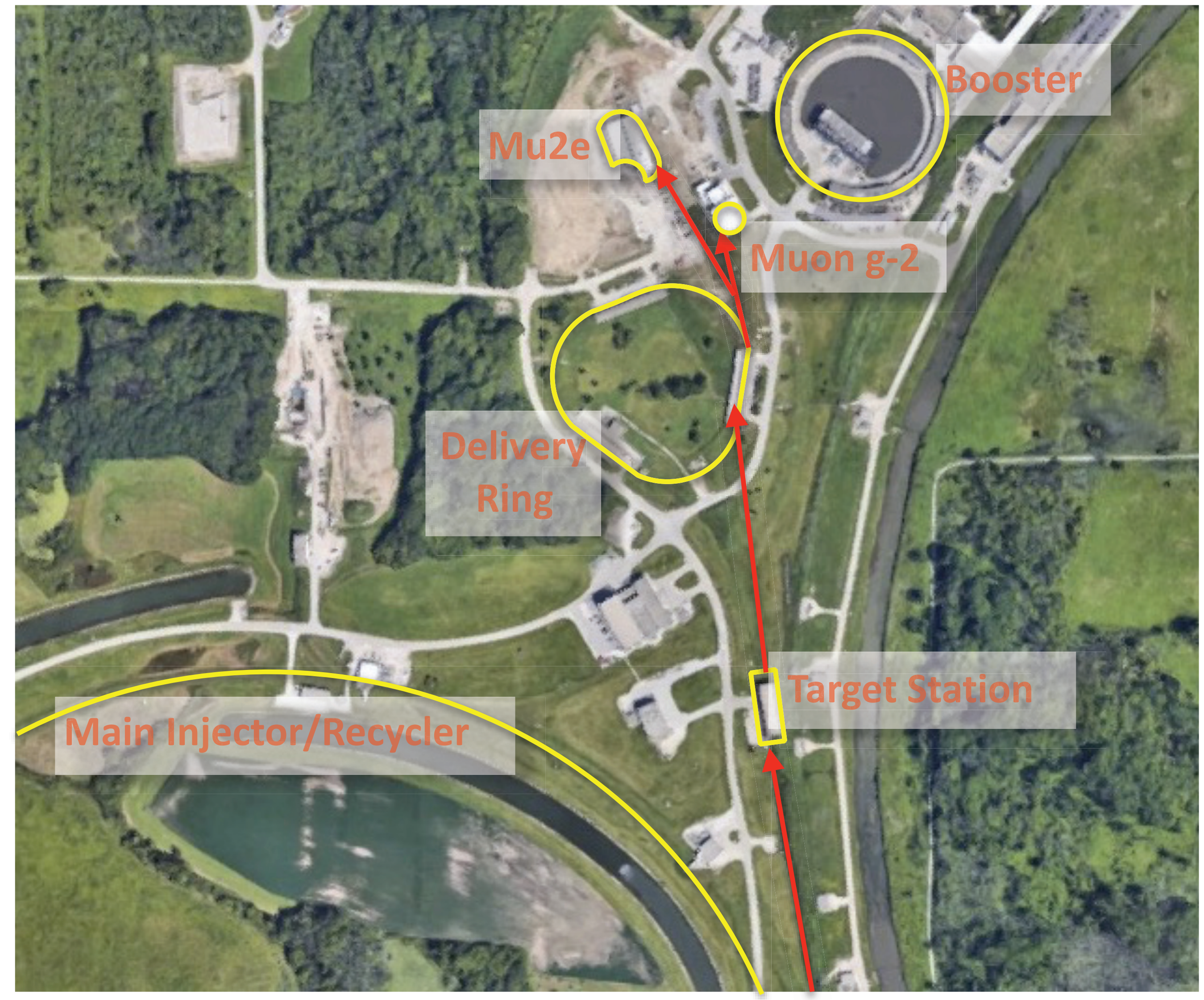
- repeat the wiggle plot millions of times…

Fermilab Implementation — E989

- Fermilab re-purposed its antiproton rings to create the The Muon Campus
- Bunch formation in the Recycler







- System delivers 16 pulses / 1.4 s
- 10^{12} protons on target / pulse
- Approx. 10^6 muons/pulse to the ring
  - $\sim 10^4$ magic muons stored / pulse
  - ~90-95% initial polarization
- Goal: **20x** the statistics of BNL E821
  - *Heavy reliance on modeling of beam production, transport, ring injection and beam storage to reduce systematic errors in the determination of anomalous magnetic moment*

please see D. Stratakis, MOZZPLM3

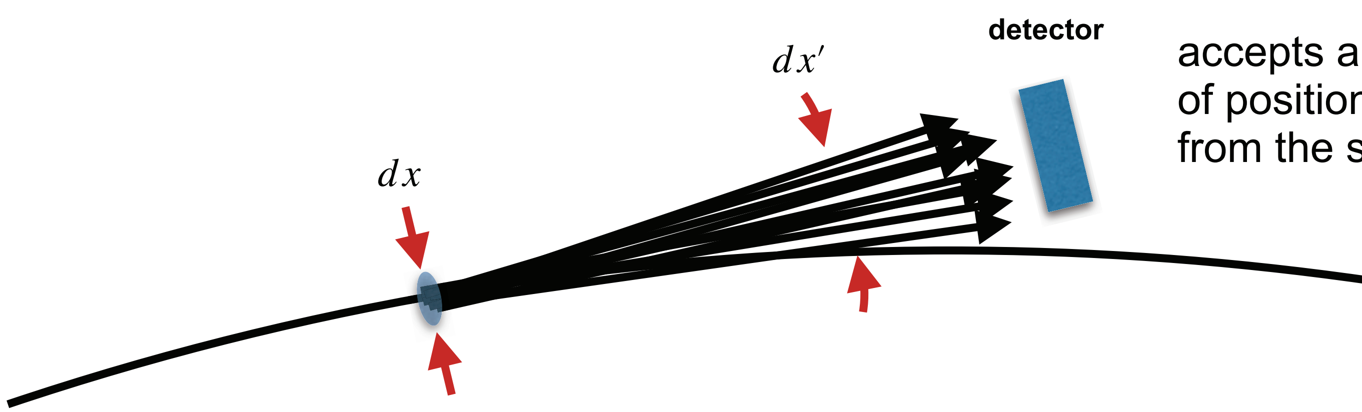
The Muon *g*-2 Storage Ring

- The storage ring is a precision-field high energy physics experiment with 24 detector stations in order to detect and identify charged particles and their time of arrival, momenta
- However, no **direct** beam measurements, *per se* — i.e., no BPMs or wire scanners or current monitors, etc.
  - all information about the beam is inferred from the detector data
- From reconstructed data, the equilibrium horizontal (momentum) distribution, vertical beam distribution and other quantities can be inferred for each injection/store.
  - these important quantities are necessary for reducing systematic errors in the final analyses of the anomalous spin frequency
- Unique opportunity for beam physics to help guide the understanding of signals and processing of data

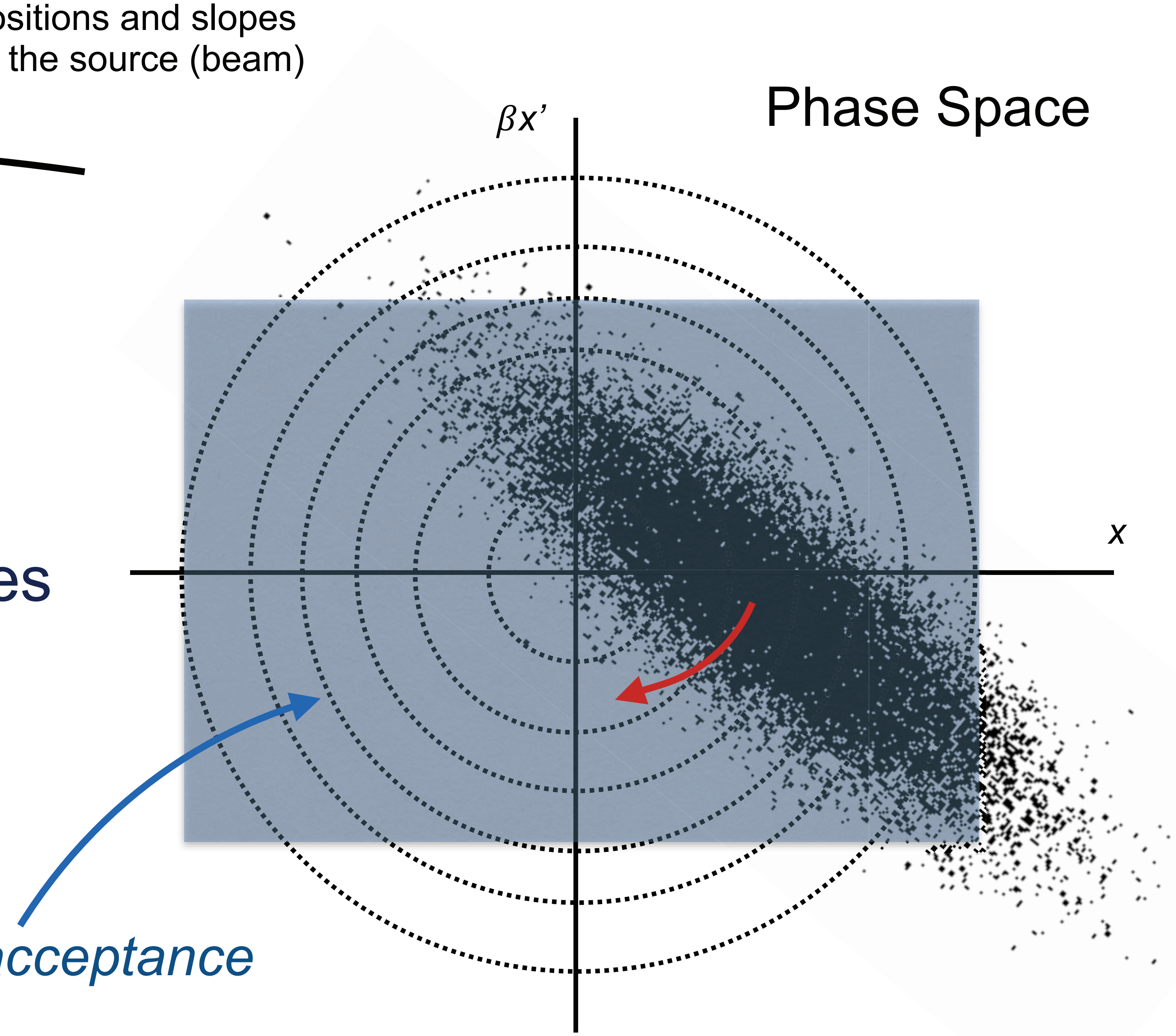
E989, Fermilab

Phase Space and Acceptance

- Detectors have a certain acceptance that is mapped onto the beam phase space; hence, due to beam dynamics, many other frequencies come into play in the particle rates other than ω_a :



accepts a certain range of positions and slopes from the source (beam)



- coherent betatron oscillation tunes
- twice those, due to quad mismatches
- tune spread from chromaticity
  - » decoherence and detuning
    - i.e., apparent frequency shifts
- *etc.*

Systematic: E-Field Contribution

- Real life: all terms come into play at some level; important to tabulate and tackle the various **systematic errors** present in the data analysis in the determination of *a*

$$\frac{d\vec{S}}{dt} = \vec{\omega}_s \times \vec{S} = -\frac{e}{\gamma m}\left[(1+a\gamma)\vec{B}_\perp + (1+a)\vec{B}_\parallel + \left(a\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right]\times\vec{S}$$

- Example: Not all (any?) muons are at the *magic momentum*, so a momentum offset or an asymmetry in the momentum distribution can generate a systematic error:

how well is this cancelled?

$$\vec{\omega_a} = -\frac{e}{m}\left[a\vec{B_0} + (a - \frac{1}{\gamma^2 - 1})\frac{\vec{E}\times\vec{\beta}}{c}\right]$$

$$x_e = D \cdot \frac{\Delta p}{p}$$

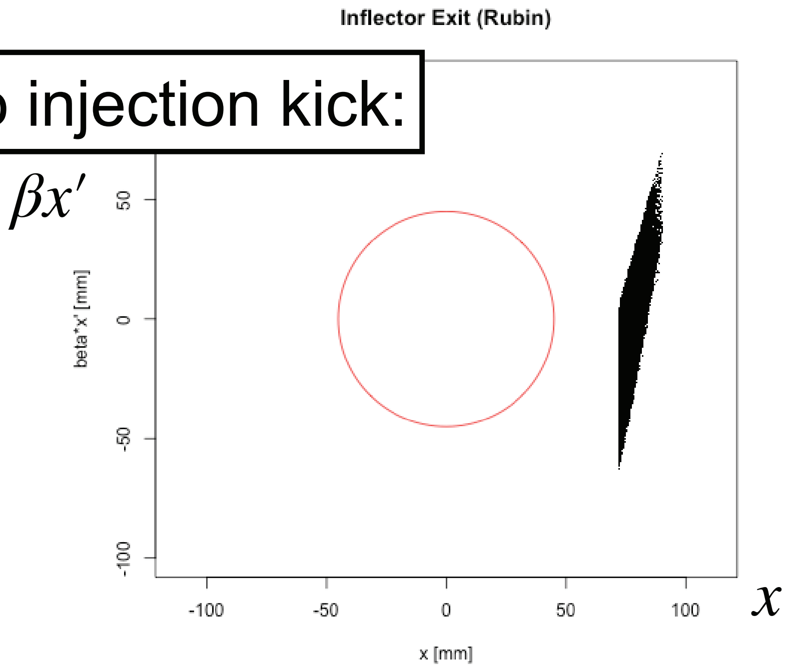
$$\frac{\Delta\omega_a}{\omega_a} = -2\frac{\langle x_e^2\rangle}{\beta_x D_x} \equiv C_E$$

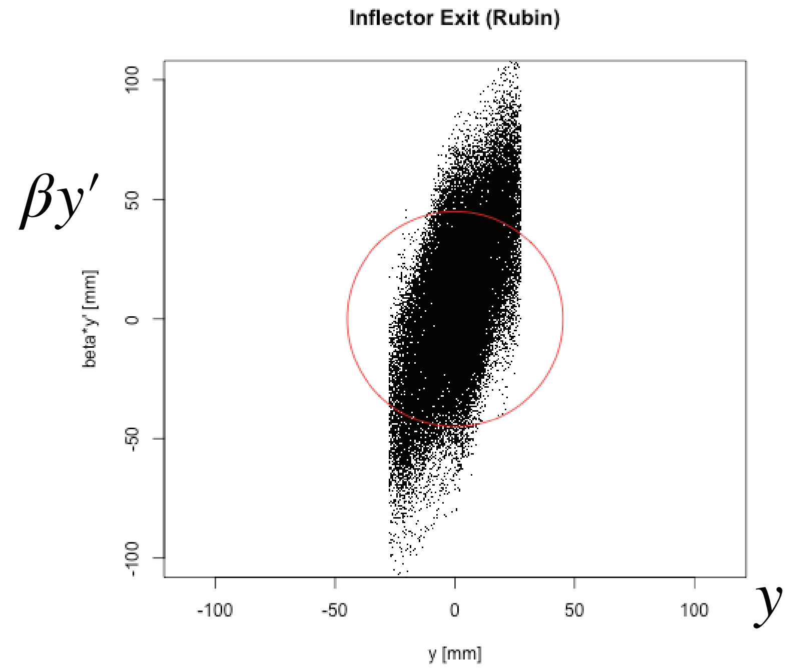
- Precise determination of the **momentum distribution** for each store will enable further improvements to this systematic correction

Modeling the Injection Process

- Incoming particle distribution, momentum distribution

prior to injection kick:





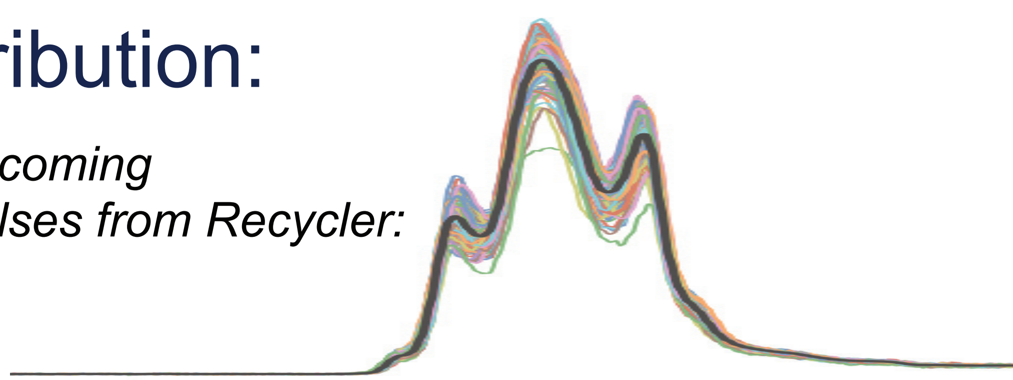
From >400,000 **simulated** particles off of the target, 54,000 arrive at ring (here, *bmad simulation)*

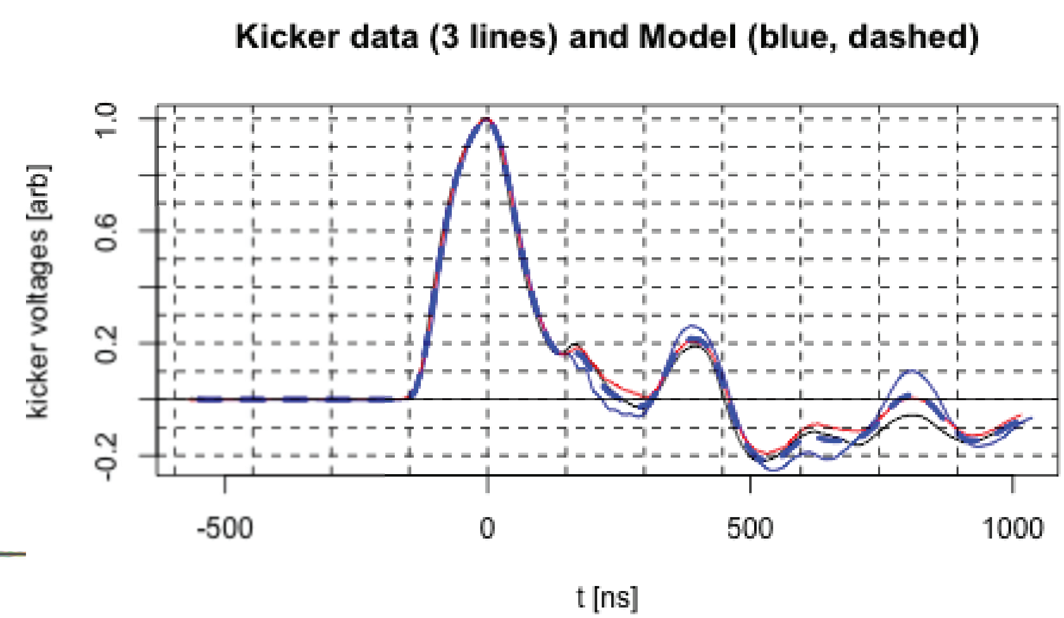
D. Stratakis, D. Rubin

Note: momentum distribution at injection is very uniform

- Injection Kicker strength and pulse shape:
- Beam time distribution:

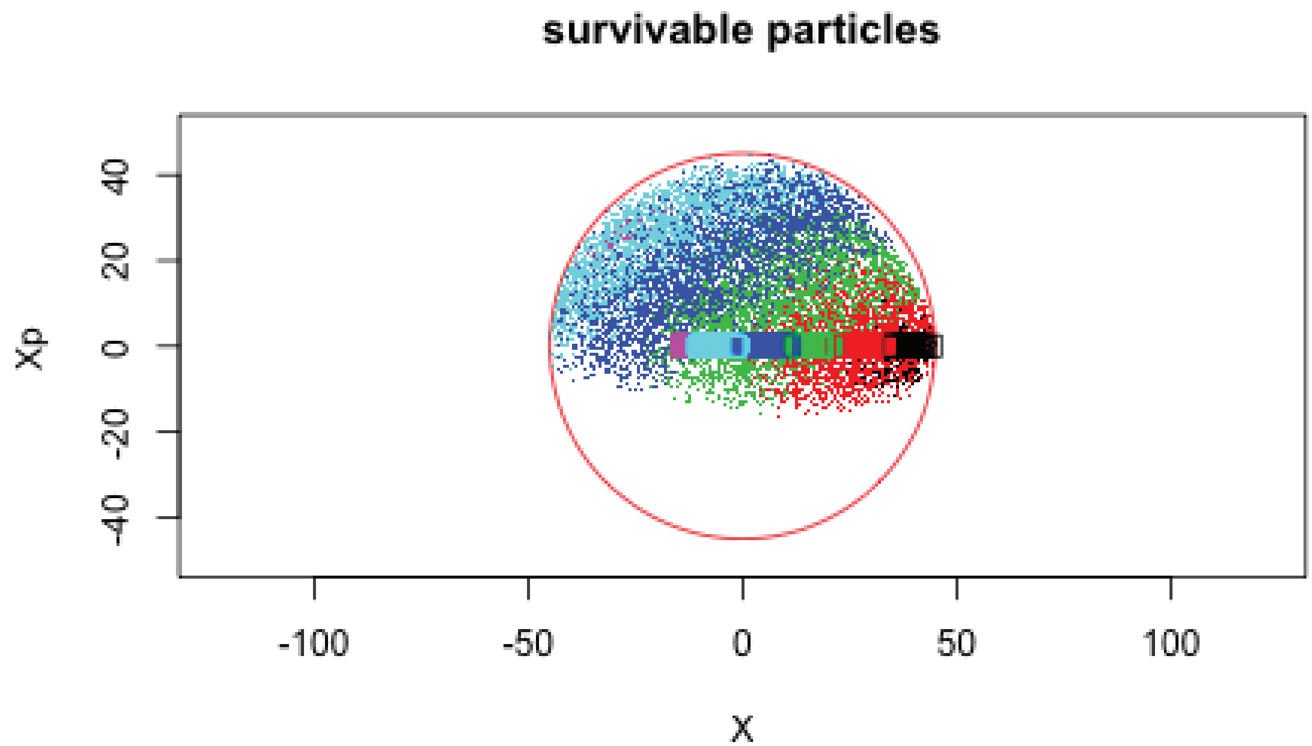
typical incoming beam pulses from Recycler:



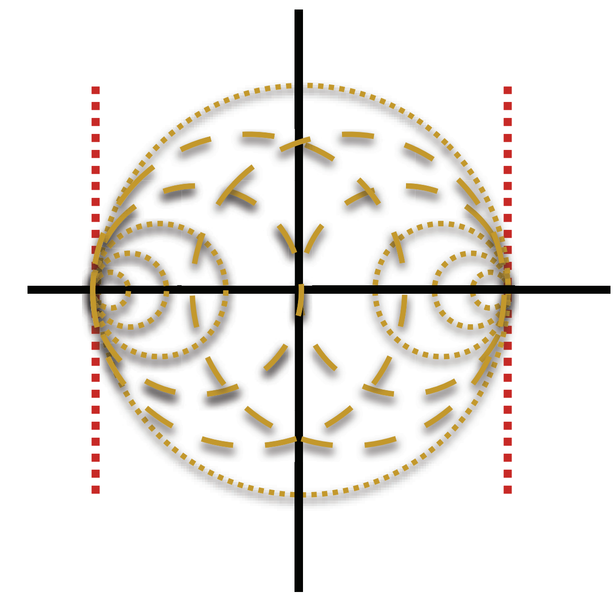


- Putting it all together…

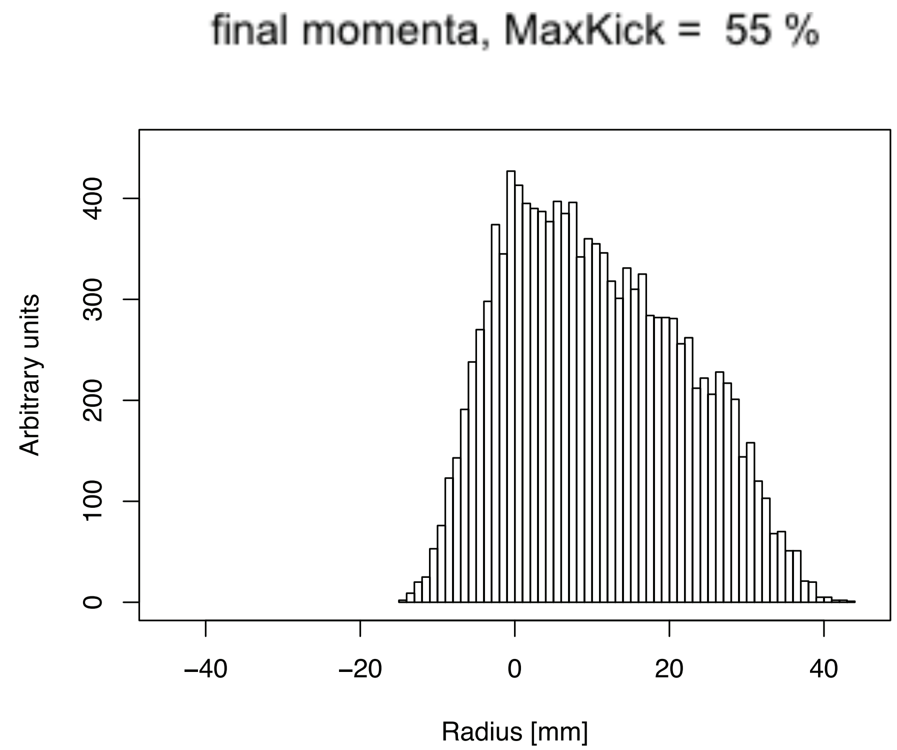
The Momentum Distribution



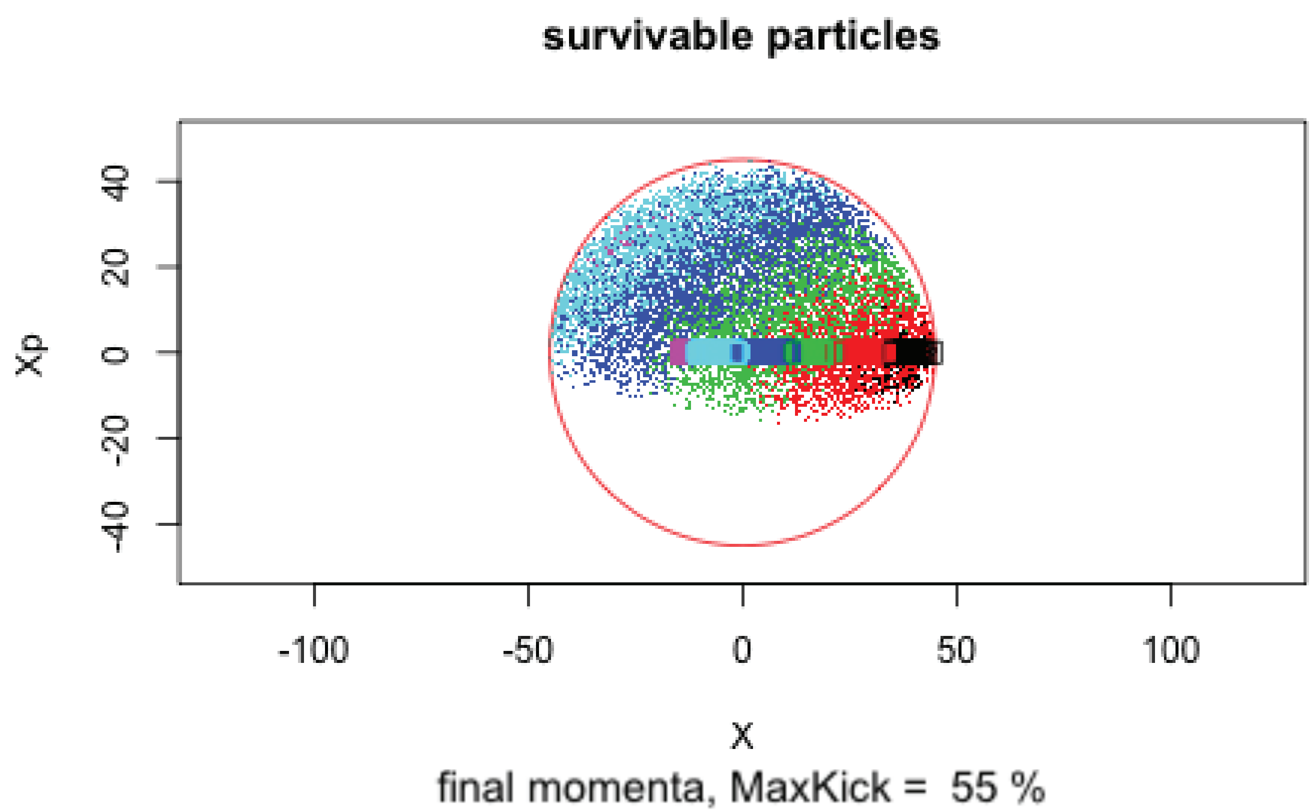
$$a_\beta < r_{ap} - D\delta$$



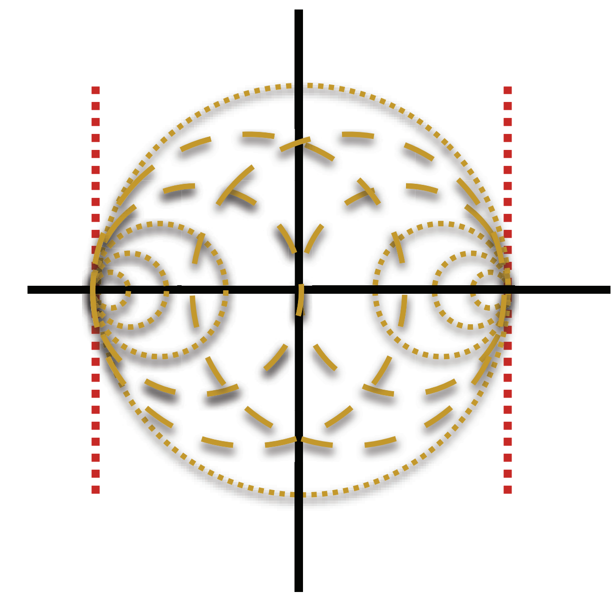
look at momentum distribution of the particles that can survive long-term:



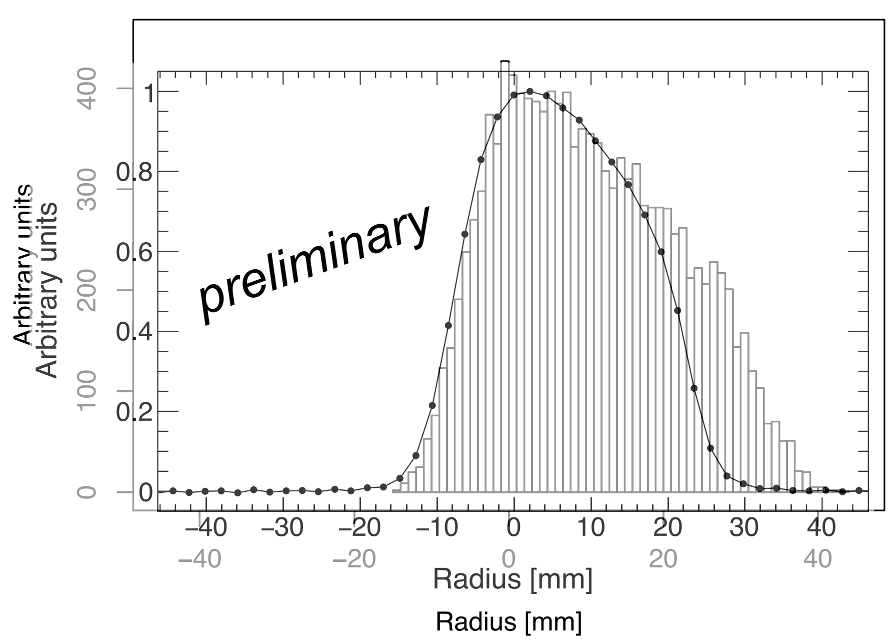
The Momentum Distribution



$a_\beta < r_{ap} - D\delta$



look at momentum distribution of the particles that can survive long-term:



a typical distribution, deduced from data analysis

see D. Rubin, IPAC18

Systematic: Pitch Correction

- Betatron oscillations lead to terms in the spin precession in which

$$\frac{d\vec{S}}{dt} = \vec{\omega}_s \times \vec{S} = -\frac{e}{\gamma m}\left[(1+a\gamma)\vec{B}_\perp + (1+a)\vec{B}_\parallel + \left(a\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right]\times\vec{S}$$

$$\vec{B}\cdot\vec{\beta} \neq 0$$ ($B_{||}$ terms)

  - in particular, the **vertical** oscillations can contribute to the spin precession in the horizontal plane

$$\vec{\omega_a} \approx -\frac{q}{m}\left[a\vec{B} - a\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}\right]$$

- Due to **vertical** betatron oscillations we would have

$$\vec{\omega}'_a \approx -\frac{q}{m}\left[a\hat{y}B_y - a\left(\frac{\gamma}{\gamma+1}\right)\beta_y B_y(\hat{s}\beta_s + \hat{y}\beta_y)\right]$$

$$\approx -\frac{q}{m}aB_y(1 - \frac{1}{2}\hat{y}'^2\cos^2\omega_y t)$$

$$\frac{\Delta\omega_a}{\omega_a} = -\frac{\langle y'^2\rangle}{4} = -\frac{\langle y^2\rangle}{4\beta_y^2} \equiv C_P$$

precise particle tracking required to accurately determine vertical distribution store-by-store

here, ring beta function…

Muon Losses Prior to Decay

- A certain fraction of the muons can get lost before they decay
  - beam-gas scattering, field fluctuations, resonance conditions, …
- So-called “lost muons” are identified as particular “hits” in the detector system that occur simultaneously on 2 or 3 consecutive detectors, assumed to be a single muon as opposed to 2 or 3 coincidental positrons
- Example: Suppose muons which can reach the aperture (halo) have a different spin distribution than those of the central core
  - perhaps they see slightly different fields, for example, due to multipoles

ϕ = polarization angle

$$\langle\phi\rangle = \frac{N_c\langle\phi_c\rangle + N_h\langle\phi_h\rangle}{N_c + N_h}$$

Want to keep $\Delta\omega_a/\omega_a \simeq$ tens of ppb or smaller,

- then, since only the halo particles get lost,

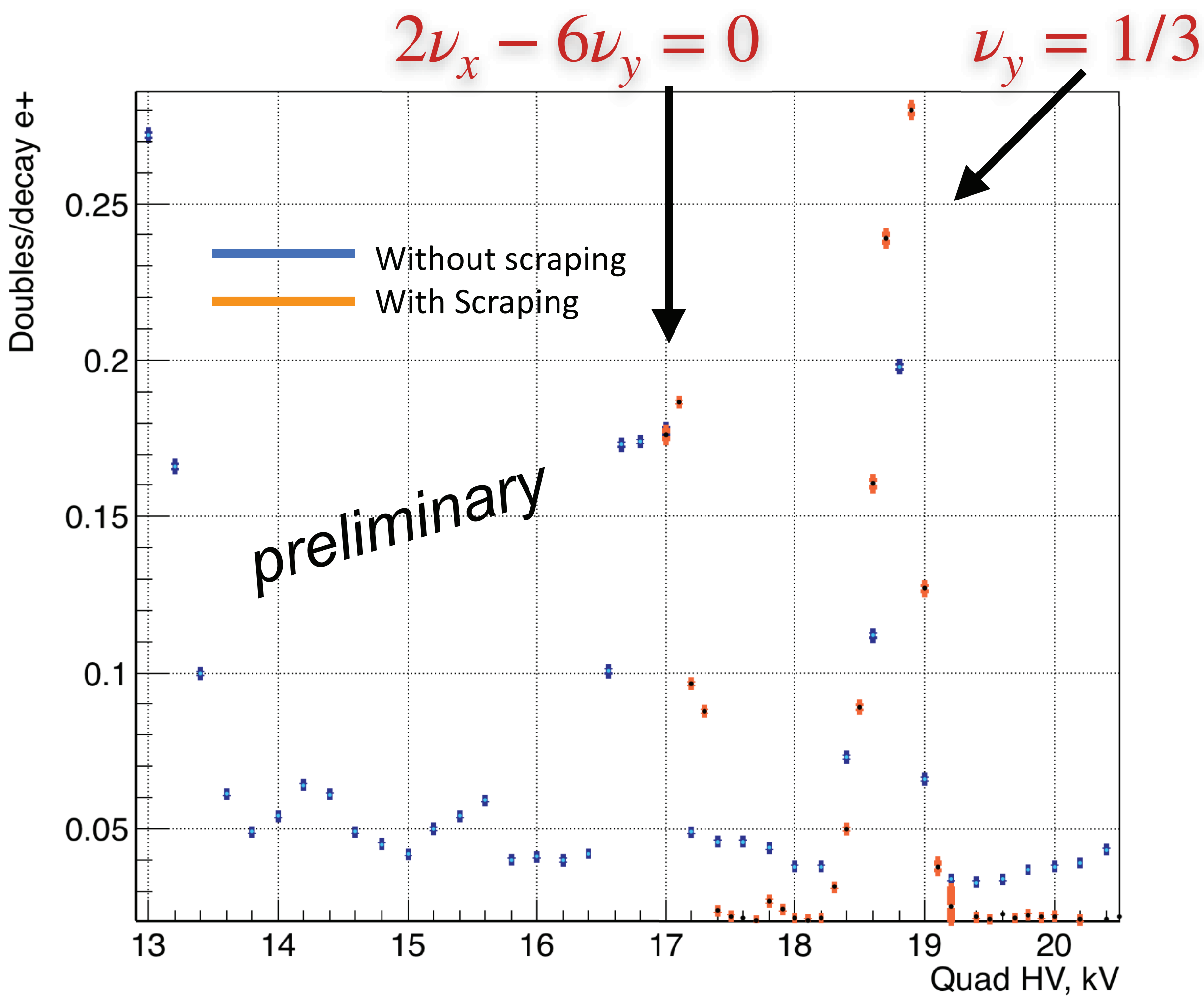
*can yield an **apparent** precession:*

$$\Delta\omega_a = \frac{d\langle\phi\rangle}{dt} = \left(\frac{N_c}{N_c + N_h}\right)\left(\frac{\dot{N}_h}{N_c + N_h}\right)(\langle\phi_h\rangle - \langle\phi_c\rangle)$$

fraction in the core — loss rate — distr. diff.

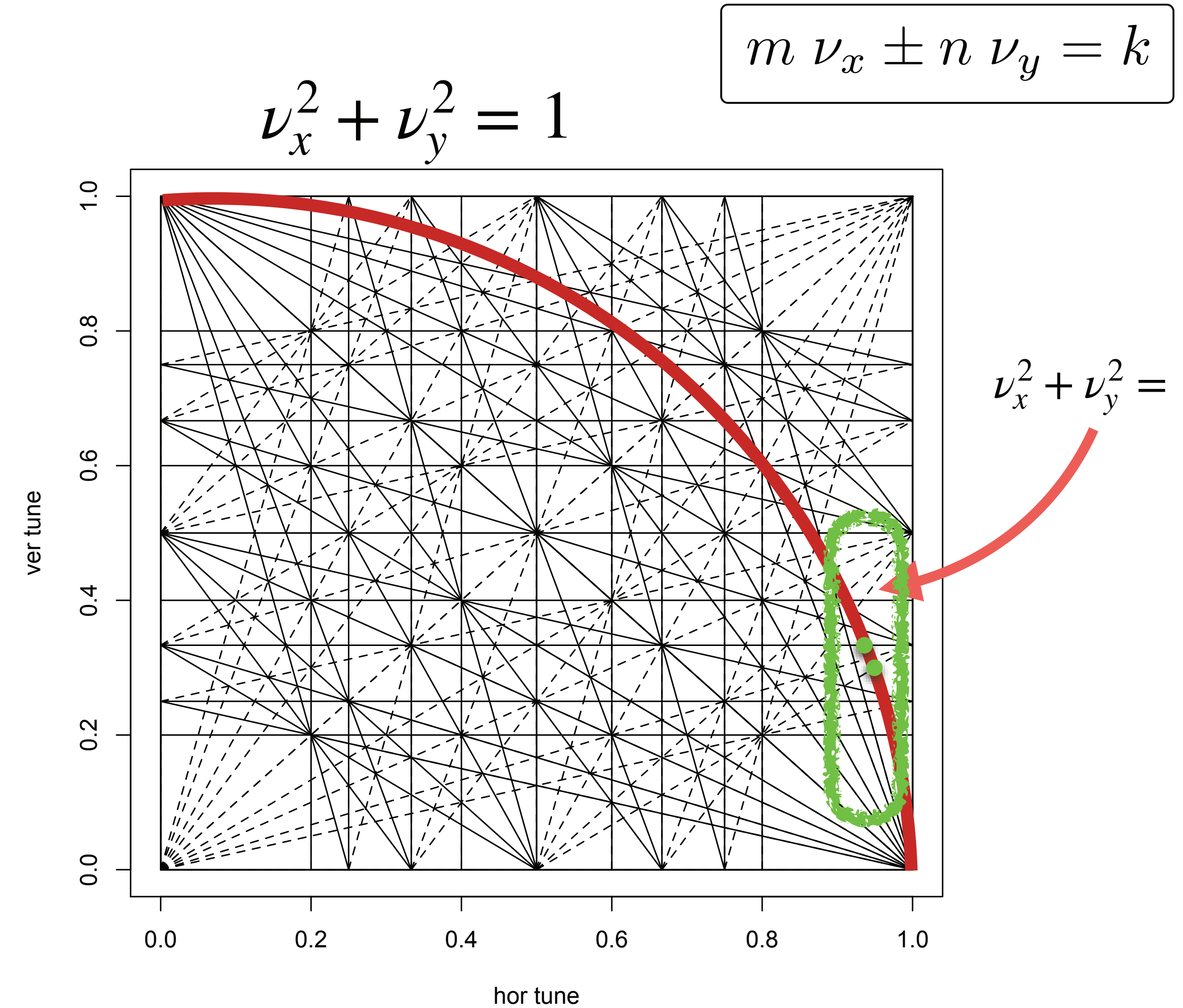
Muon Losses Prior to Decay

- Will run at both high and low vertical tune values, away from resonances, to ensure loss rates do not interfere with interpretation of ω_a
- Tune scans:



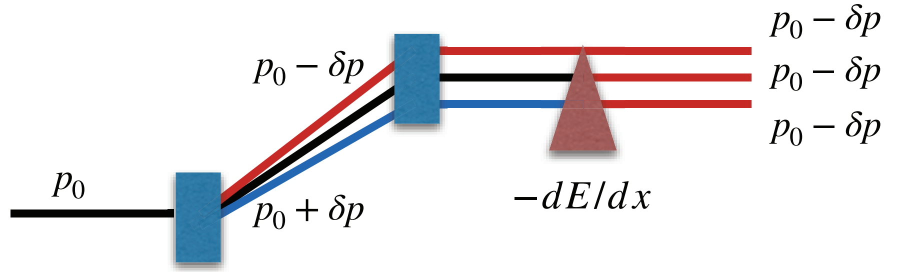
Lost muon doubles vs. quad HVPS set-points for t > 30µs after injection, with scraping and without scraping.

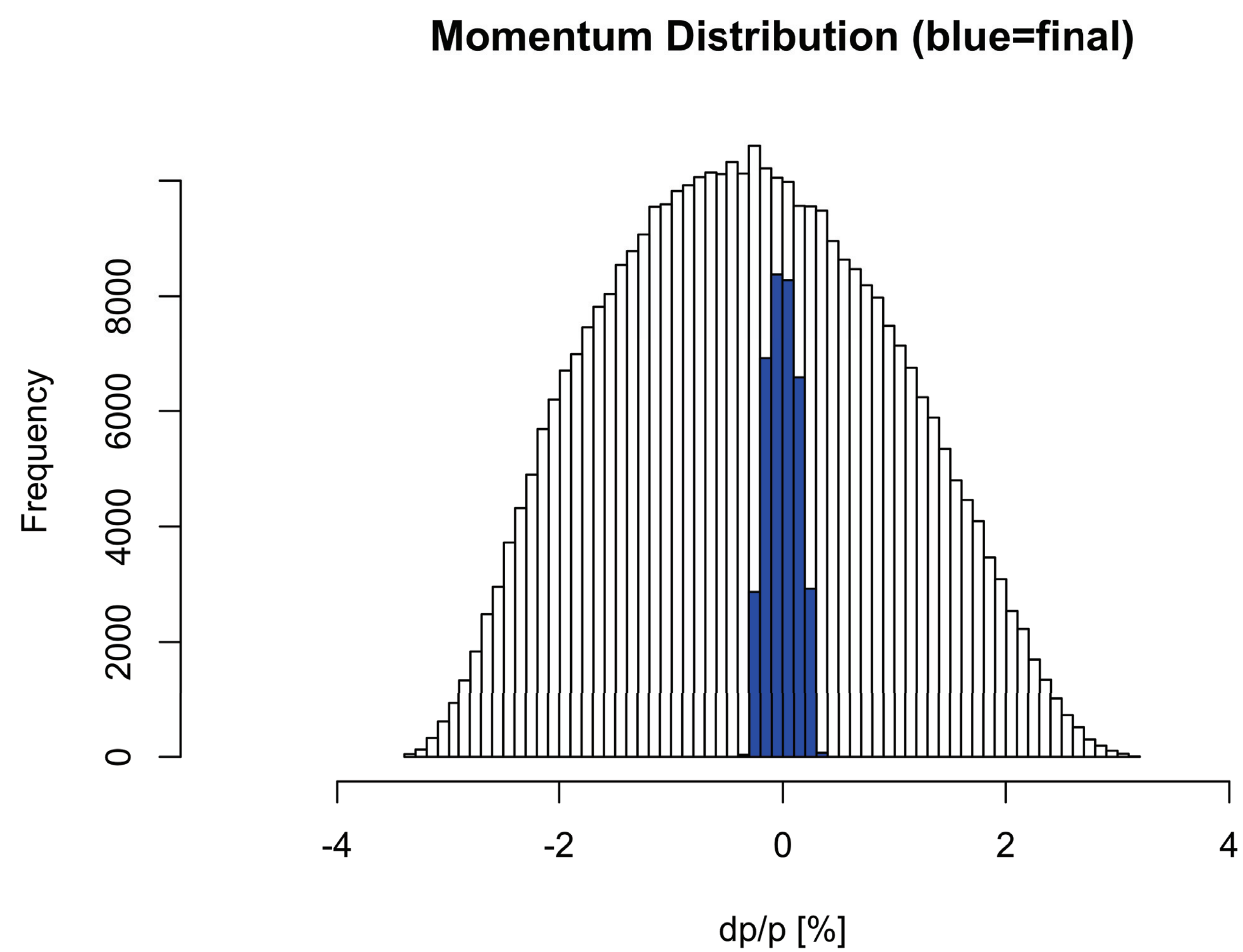
data courtesy Crnkovic, Ganguly, et al.



Outlook

- This important HEP measurement not only relies upon high flux to the apparatus, but also ***heavily*** on particle beam dynamics, including spin dynamics
  - important contribution to high-profile experiment
- Have generated ~2x BNL data set
  - looking for factor of 20 or more
  - Approx. 1-2 years more to run
- How to improve the muon flux?
  - Momentum Cooling using *wedges*





  - System in place for current run

- ***Continue running***!

Fermilab LDRD Funding