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AN ANALYTIC APPROACH TO EMITTANCE GROWTH FROM THE BEAM-BEAM EFFECT WITH APPLICATIONS TO THE LHeC*

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Abstract

In colliders with asymmetric rigidity such as the proposed Large Hadron electron Collider, jitter in the weaker beam can cause emittance growth via coherent beam-beam interactions. The LHeC in this case would collide 7 TeV protons on 60 GeV electrons, which can be modeled using a weak-strong model. In this work we estimate the proton beam emittance growth by separating out the longitudinal angular kicks from an off-center bunch interaction and produce an analytic expression for the emittance growth per turn in systems like the LHeC.

INTRODUCTION

The beam-beam effect is the term given to the mutual lensing action that each beam in a collider causes on the other. In the proposed Large Hadron electron Collider (LHeC) the colliding beams would have an asymmetric collision between a 7 TeV proton beam, and a 60 GeV electron beam from a dedicated recirculating linac [1]. Due to the asymmetric rigidities in these beams, the beam-beam tune shift is 9.6×10^{-5} for the proton beam and 0.75 for the electron beam, it is the coherent effects that will drive emittance growth. These coherent effects will add different transverse momentum changes to different parts of the beam, increasing emittance. Since this is a linac-ring system, the offset jitter that drives this increase will not reach an equilibrium, since the linac will continuously add a new beam with new jitter [2].

GROWTH MECHANISM

Due to the asymmetric rigidities, the proton beam can pull the electron beam in and through the proton beam. This action will add transverse kicks in a manner that is coupled with the longitudinal position of the beam. An example of the kicks given are shown in Fig. 1. Using the definition of a change in emittance for a given transverse dimension, $\sqrt{\langle x^2\rangle\langle(p_x+\Delta p_x)^2\rangle - \langle xp_x\rangle^2}$, we find that we can isolate the Δp_x term as $\varepsilon_0\sqrt{1+\langle\Delta p_x^2\rangle\langle x^2\rangle/\varepsilon_0^2}$. If we collect terms and realize that $\langle x^2\rangle=\varepsilon_0\beta^*$, then we can estimate,

$$\Delta\varepsilon_n = \frac{1}{2}(\beta\gamma)\beta^* < \Delta p_x{}^2 > \frac{\sigma_{jitter}^2}{\sigma_x^2}. \quad (1)$$

Where $\Delta\varepsilon_n$ is the change per interaction of the normalized emittance $(\beta\gamma)$ are the relativistic quantities, β^* is the β function at IP, σ_{jitter} is the offset jitter, and σ_x is the transverse beamsize. Finding $<\Delta p_x^2>$ is our main challenge [3] since it depends on the path of the electron beam through the proton beam.

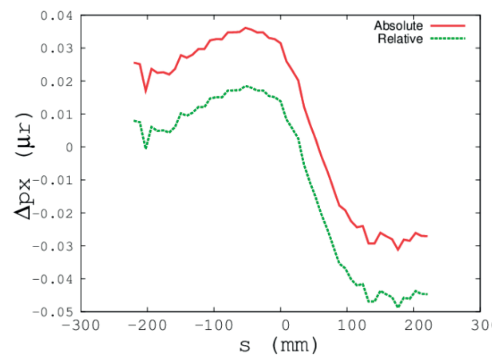

Figure 1: This figure shows the angular offset of the proton beam as a function of longitudinal position caused by an electron beam moving through the proton beam with an offset of 0.2 σ_x. Absolute is the total kick re-ceived, while relative is the kick with the average sub-tracted out.

DETERMINING $<\Delta P_X^2>$

The simplest method of determining the $<\Delta p_x^2>$ would be to integrate the kicks received by the proton beam based on the relative position of the electron beam as they collide. The kicks are modeled using the Basetti-Erskine formula [4], and we have started out with three methods of determining the path of the electron beam through the proton beam. One simple way is to model the system in a beam-beam code such as GUINEA-PIG [5], another is to directly integrate using the equations of motion, and finally an attempt at a polynomial ansatz was made. The paths these methods make through the LHeC proton beam are shown in Fig. 2.

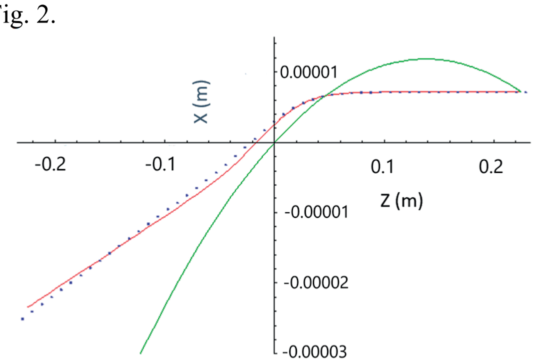

Figure 2: This figure shows the comparison of the three methods used to calculate the path of the electron beam through the proton beam. The dotted line shows the path as calculated using GUINEA-PIG, the red line is the

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path directly integrated from the equations of motion, and the green path represents our polynomial ansatz.

Using GUINEA-PIG and a simplified map of the LHC lattice, assuming linear 6D motion with no dispersion, we can simulate the beam-beam interactions and how they affect the circulating proton beam. The data from multiple random number seeds, as well as the growth rates predicted using the three methods described are shown in Fig. 3.

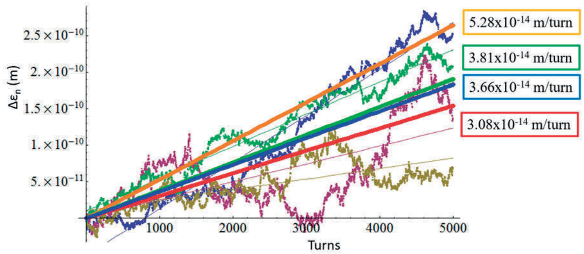

Figure 3: The solid orange line is the rate predicted by GUINEA-PIG, the solid green line is the rate predicted by the ansatz, the solid blue line is the average of the four simulation rates, and the solid red line is the directly integrated path. The point data is the simulated emittances of four different random seeds sent through the same set of kicks (0.2 σ_x) in our model system.

TOWARDS AN ANALYTIC MODEL

While it is possible to estimate the emittance growth rate in a system like the LHeC by either using a beam-beam code like GUINEA-PIG or to directly integrate the equations of motion, it would be far simpler if we could use a formula to get a quick estimation.

We can begin to model these systems analytically by making a series of assumptions;

- One beam's motion can be considered constant (Reference) and one beam can be considered as moving (Colliding). (i.e. a weak strong system)
- Both beams are round at impact, and have a gaussian profile so that the round beam Basetti-Erskine approximation can be used.
- The colliding beam can be considered a gaussian disk of charge moving through the reference beam.

These assumptions prove to be very good for the LHeC system since the electron bunch is much less rigid than the proton bunch, and much shorter. In the following notation we will use subscripts to denote the quantity a beam "sees" so for instance D_C would be the disruption parameter "seen" by the colliding beam. One other issue that could confound this system, is the fact that the beams experience an hourglass effect as they go through the Interaction Point (IP). In order to keep this method useful, we need to keep the parameters dimensionless. Thus, we create the dimensionless quantity $e_C=\sigma_z/\beta^*$. A similar term e_R can be made for the reference beam, but due to the assumptions made that will have a negligible effect on our system since the bunch length of the colliding beam is so short. This is included in the dimensionless equations by changing the σ_r term which has been normalized to one, and recast it as $(1+(z_0e_C)^2)$. If we wish to find a non-hourglass system we set e_C to zero. If we start with the equations of motion for the colliding beam, assuming that the offset is only in one dimension,

$$r''(z) + \frac{2Nr_{particle}}{\gamma\sqrt{2}\sigma_z}\frac{e^{\frac{-r(z)^2}{2\sigma_r^2}}-1}{r(z)}e^{-\left(\frac{z}{\sigma_z}\right)^2} = 0, \quad (2)$$

where $r_{particle}$ is the classical radius of colliding beam particle, γ is the relativistic quantity, r and z are the variables, and σ_r and σ_z are their rms sizes. We can cast this into dimensionless coordinates ($z_0=z/\sigma_z$), ($r_0=r/\sigma_r$) and make use of the beam-beam disruption parameter,

$$D_{x,y} = \frac{2Nr_{particle}\sigma_z}{\gamma\sigma_{x,y}(\sigma_x+\sigma_y)}, \quad (3)$$

to reduce the equations of motion, with D_C and e_C held as constants to,

$$r_0''(z_0, D_C, e_C) + \frac{2D_C}{\sqrt{2}}\frac{e^{\frac{-r_0(z_0,D_C,e_C)^2}{2(1+(z_0e_C)^2)}}-1}{r_0(z_0,D_C,e_C)}e^{-z_0^2} = 0. \quad (4)$$

This has the advantage that a given disruption parameter will define a unique path for the colliding beam through the reference beam. We assume a 1 σ_r initial offset and assume a linear scaling with momentum. Meaning that using our dimensionless coordinates we go from,

$$\langle\Delta p_r^2\rangle = \left(\frac{2Nr_{particle}}{\gamma}\right)^2\frac{\int\left(\left(e^{-r^2/2\sigma_r}-1\right)/r\right)^2 e^{-\frac{2z^2}{\sigma_z^2}}dz}{\int e^{-\frac{2z^2}{\sigma_z^2}}dz}, \quad (5)$$

to,

$$\langle\Delta p_r^2\rangle = 4D_R^2\left(\frac{\sigma_{r_R}}{\sigma_{z_R}}\right)^2 N(D_C, e_C), \quad (6)$$

where,

$$N(D_C, e_C) = \frac{\int\left(\left(e^{-\frac{r_0(z_0,D_C,e_C)^2}{2}}-1\right)/r_0(z_0,D_C,e_C)\right)^2 e^{-2z_0^2}dz_0}{\int e^{-2z_0^2}dz_0}. \quad (7)$$

$N(D_C,e_C)$ is the average momentum change squared, however if we assume that the bunches are damped well in our circulator ring, then the overall kick that the whole reference bunch receives ($<\Delta p_r>^2$) will need to be removed. This is accomplished by subtracting the quantity,

$$I(D_C, e_C) = \frac{\left(\int\left(\left(e^{-\frac{r_0(z_0,D_C,e_C)^2}{2}}-1\right)/r_0(z_0,D_C,e_C)\right)e^{-2z_0^2}dz_0\right)^2}{\int e^{-2z_0^2}dz_0}. \quad (8)$$

Eq. 7, and Eq. 8 can be best understood when viewed graphically, as is shown in Fig. 4.

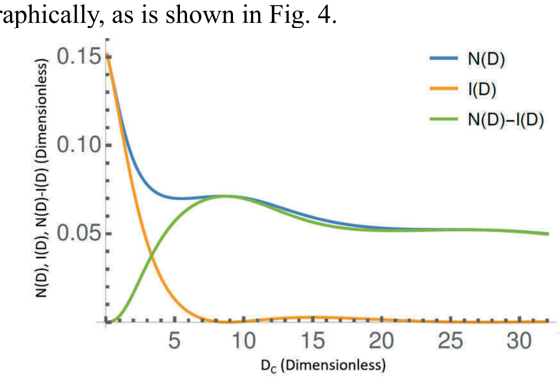

Figure 4: This plot shows $N(D_C,0)$, $I(D_C,0)$, and $N(D_C,0)-I(D_C,0)$ in blue, orange, and green respectively.

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In Fig. 4 we see that the green line, which is a system with perfect damping of offsets in the recirculating beam will have a maximum growth rate at a D_C of 8.89. This is the point where the colliding beam has equal paths on both sides of the reference beam. This can be seen in Fig. 5.

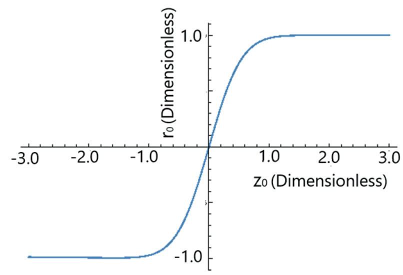

Figure 5: This is the path of the colliding beam through the reference beam at the point of maximum emittance growth.

Thus, half of the beam receives a kick in one direction, and half receives a kick in the other direction. This is also why the N(D_C,0) and N(D_C,0)-I(D_C,0) lines are equal at that point. Above this value both lines are very close, this is because the reference beam has such a strong pull that it will "suck" the colliding beam into it and keep it there for the remainder of the interaction. Examples of N(D_C,e_C) and N(D_C,e_C)-I(D_C,e_C), where the hourglass effect of the reference beam is included, are shown in Figs. 6 and 7 respectively.

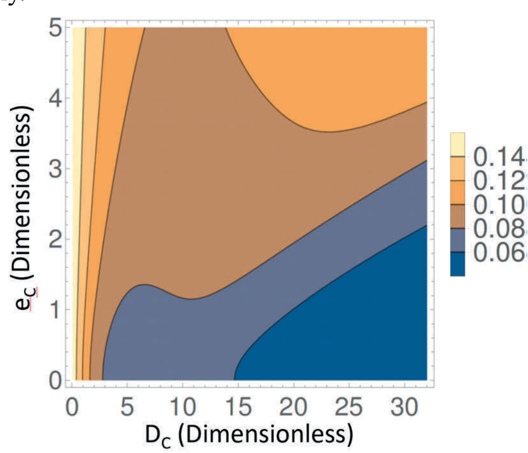

Figure 6: This is a plot of N(D_C,e_C) over a range of both numbers.

If we pull together all of the information so far, then we can show that for systems that follow our basic assumptions, we can calculate the per-turn growth rate as,

$$\Delta\epsilon_n = 2\beta^*(\beta\gamma)D_R^2\left(\frac{\sigma_{rR}}{\sigma_{zR}}\right)^2\left(\frac{\sigma_{jitter}}{\sigma_{rR}}\right)^2 E(D_c, e_c), \quad (9)$$

where,

$$E(D_c, e_c) = \mathrm{N}(D_c, e_c)\text{-}k\mathrm{I}(D_c, e_c). \quad (10)$$

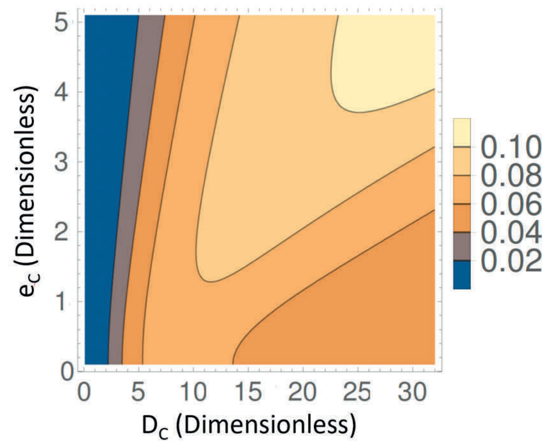

Figure 7: This is a plot of N(D_C,e_C)-I(D_C,e_C) over values of both D_C and e_C.

Where k is 0 if we assume no offset corrections in the recirculating beam, and 1 if we assume there are. e_C can be used as appropriate, but is 0 if not needed. Eq. 9 and 10 when combined with either graphs such as Figs. 4 6 and 7, or when interpolating from lookup tables can provide a rapid method of estimating per-turn emittance growth rates in asymmetric systems like those found in the LHeC. Examples of these new methods, are shown in Fig. 8.

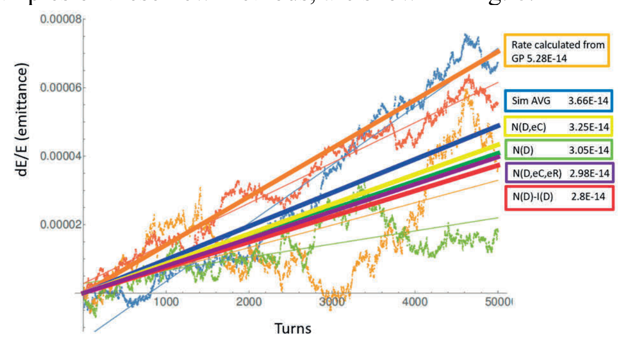

Figure 8: In this figure we see a variety of analytic methods shown in this work as compared to the same growth data shown in Fig. 3.

The analytic method that most closely matches the average of the simulations is N(D_C,e_C). Since these simulations include hourglass, but do not correct for offset errors in the recirculation, this was to be expected.

CONCLUSIONS AND FURTHER WORK

This work has made strides in creating an analytic formula to estimate the emittance growth rates for the beam-beam effect. There is however more work to be done, both in expanding from the current focusing interactions (ep collisions) to defocusing interactions (pp collisions), and also in moving away from looking up numbers on a curve to a true formula. Though one interesting insight gained from this study is the fact that for high D_C the growth rate doesn't increase anymore.

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