SYNCHROTRON RADIATION MODULE IN OCELOT TOOLKIT

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Abstract

Synchrotron radiation (SR) sources based on single-pass accelerators (e.g. linacs, plasma accelerators) have to cope with electron beams with a rather complicated phase space distribution. In this case, the convolution method usually employed to calculate radiation properties can give poor accuracy or be not applicable at all. Moreover, dynamical effects can also play a role in the emission mechanism. This happens when the beam parameters (e.g. beam current) significantly change during the passage through the undulator. In this work, we present a dedicated SR module of the OCELOT toolkit [1,2], which is well suited to deal with these situations.

INTRODUCTION

Recently, light sources based on single-pass accelerators have become widespread (FEL, THz sources). In contrast with storage ring based light sources, single-pass based sources have better electron-beam slice quality, but a more complicated phase space, e.g. correlated energy spread (chirp), non-gaussian current profile, etc, which impact on radiation generation. Furthermore, magnetic systems used for generating radiation might be rather complicated, including segmented undulators with focusing elements in between, or combinations of undulator and a dipole magnets. This complexity leads in turn to non-trivial calculation issues, when it comes to the determination of SR properties. Other calculation issues arise from dynamical changes of the electron beam properties during the process of radiation generation. For example, dynamical effects can take place in the case of insertion devices with large K-value, when the electron beam is chirped in energy [3], or when quantum fluctuations and energy losses in long undulators need to be accounted for. In the most general case, a Monte-Carlo method can be used to overcome these issues.

The multiphysics simulation toolkit OCELOT focuses, in the first place, on the analysis of electron beam properties pertaining to the generation of radiation by Free-Electron Lasers (FELs). OCELOT adequately describes the beam dynamics, including collective effects [1,2,4] and is able to perform FEL simulations interfacing with open FEL code Genesis [5].

The OCELOT SR module is capable of calculating spectrum and spatial distribution of spontaneous radiation from a single electron in a magnetic field defined by file data (field on an insertion device axis or 2D map in the plane of the electron motion) or using standard elements as Undulator with arbitrary defined period, length and K. It has shown good agreement with experiments, see e.g. [6].

In this work we focus on new capabilities of the OCELOT SR module.

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SR SOLVER

The OCELOT SR module consists of two independent blocks. One is a trajectory solver based on Runge-Kutta integrator. The trajectory solver is part of the beam dynamics module and can be used for the beam tracking through arbitrary magnetic fields. The second block is a SR solver.

The SR solver has been developed based on [7,8]. The electric field of the radiation observed at the point \( \bar{x} = \{x, y, z\} \) and time \( t \) is (in Gaussian units):

\[
\tilde{E}(\tau, \bar{x}) = \frac{e}{c|x - \bar{r}(t)|} \left[ \hat{n} \times (\hat{n} - \hat{\beta}(t) \times \dot{\hat{\beta}}(t)) \right] \frac{\exp(i \omega \tau)}{(1 - \hat{n} \cdot \hat{\beta}(t))^3} + \frac{\exp(i \omega \tau)}{(1 - \hat{n} \cdot \hat{\beta}(t))^3} \frac{\hat{n} \cdot \dot{\hat{\beta}}(t)}{c |x - \bar{r}(t)|}
\]

where \( \hat{n} = \frac{\bar{x} - \bar{r}(t)}{|\bar{x} - \bar{r}(t)|} \) is the unit vector pointing from the particle to the observer, \( e \) is the electron charge, \( \hat{\beta} \) is the normalized velocity vector, \( \tau(t) = t + \frac{1}{c} |\bar{x} - \bar{r}(t)| \), \( t \) is the emission time.

The SR solver works in the frequency domain and the Fourier transformed electric field is

\[
\tilde{E}(\omega, \bar{x}) = \int_{-\infty}^{\infty} \exp(i \omega \tau) \tilde{E}(\tau, \bar{x}) d\tau
\]

Remembering the important relation between the increment of the time \( \tau \) at the observer and time \( t \) of emission

\[
d\tau = \left(1 - \hat{n} \cdot \hat{\beta}(t)\right) dt
\]

we get

\[
\tilde{E}(\omega, \bar{x}) = \int_{-\infty}^{\infty} \exp(i \omega \tau(t)) \tilde{E}(\tau(t), \bar{x}) (1 - \hat{n} \cdot \hat{\beta}(t)) d\tau
\]

To integrate the last expression numerically we make transformation \( t \rightarrow z \) to use the longitudinal coordinate as evolution variable. Both trajectory and SR solvers are implemented in Python. The code architecture includes the possibility of parallelization with mpi4py [9].

SIMULATION RESULTS

Dynamical Effects on Coherent THz Radiation

Coherent emission occurs when ultra-relativistic electron bunches are shorter than the wavelength of the emitted radiation. In this case, the different electron contributions to the emitted field sum up in phase and the output intensity scales as the square number of electrons in the bunch. Linear accelerators can produce electron bunches with a length of order of tens of \( \mu \)m, which corresponds to the THz range. Therefore, such accelerator based THz sources can produce very bright light due to coherent emission.

An electron bunch at the entrance of a THz undulator setup has typically an energy chirp, because of the necessity to compress it in magnetic chicanes. Then, the chirped electron bunch evolves passing through a highly dispersive
THz undulator with a large magnetic field amplitude, and the shape of its longitudinal phase space changes.

In order to simulate this scenario with a realistic choice of simulation parameters we limited ourselves to the case of the FLASH THz undulator with 1.2 T magnetic field, 0.4 m period and 9 periods \[10,11\] and typical beam energy chirp \( E' = 80 \text{ MeV/mm} \) \[12\] or \( \alpha = \frac{E'}{E_0} = 0.13 \text{ mm}^{-1} \), where \( E_0 = 600 \text{ MeV} \). In these simulations we used a "cold" beam approximation and we assumed zero transverse emittance, and zero uncorrelated energy spread.

In Fig. 1 we show the energy distributions of the electron beam with different energy chirps (positive, zero, and negative) as well as the beam current. Note the average energy of all electron beams is 600 MeV.

As was mentioned above, during propagation of the energy chirped beam through THz undulator the longitudinal beam profile evolves due to strong dispersion. Using the OCELOT trajectory solver, we tracked the beams though undulator setup. The results, in terms of final current profiles, are shown in Fig. 2. They are in good agreement with analytical estimations of the compression factor \( C \).

As we can see, the chirped beam is compressed or decompressed depending on the sign of the energy chirp. At the same time, the unchirped beam remains unchanged. The difference between the maximal current with and without chirp is more than 30%, which obviously cannot be neglected. In order to simulate radiation emission from such setup we use the OCELOT SR module relying on a Monte-Carlo method. Simulations were run in parallel on a cluster for 30000 macroparticles covering the phase-space distribution, which was enough for obtaining converging results. The contributions to the field from all electrons sum up in phase. The result of simulations is shown in Fig. 3.

An analytical approach to describe the influence of the energy chirped beam on coherent radiation was developed and cross-checking of the simulation results with theory can be found in [3].

### Quantum Fluctuations and Energy Loss Effects

Travelling down the undulator, the electron transfers part of its energy to the light wave and consequently decreases its kinetic energy. The energy radiated by an electron in the magnetic field is defined by the well-known formula:

\[
U_0 = \frac{C_r}{2 \pi} E^4 I_2
\]

where \( C_r = \frac{4 \pi}{3} \frac{r_x}{m c^2} = 8.846 \times 10^{-14} \text{ m/MeV}^3 \), \( E \) is the electron energy, \( I_2 = \int \frac{1}{\rho^2} d\zeta \) is second radiation integral.

The beam energy decreases with the undulator distance \( z \). This fact leads to changes in the resonance condition for different undulator segments. The shape of the undulator spectral lines is thus considerably changed, they become much wider. This effect was analyzed numerically for the case of the SASE1 undulator (35 undulators with 125 periods each, \( K = 4 \)) of the European XFEL, and results are shown in Fig. 4. The beam energy is 17.5 GeV.
The implementation of quantum diffusion of energy spread in the OCELOT SR module code was done using the Monte Carlo method and by adding a correction to the energy spread of the electron beam according to [13]:

$$\langle d(\delta\gamma)^2 \rangle \frac{d}{d\gamma} = \frac{56\pi^3}{15} \frac{\hbar}{mc^2} \gamma^4 \frac{K^3 f(K)}{\lambda_w}$$

where $K$ is the undulator deflection parameter, $\lambda_w$ is the undulator period, and for a planar undulator

$$f(K) = 1.2 + \frac{1}{K + 1.33K^2 + 0.40K^3}$$

Fig. 5 shows spectra of the spontaneous radiation from 15 undulator segments with and without quantum fluctuations. Simulation was done with 1000 macro particles.

Figure 4: Normalized undulator radiation intensity with and without energy loss.

Figure 5: Normalized undulator radiation intensity with and without quantum fluctuations. 15 undulators with number of periods 125 each.

CONCLUSION

The OCELOT SR module is part of the OCELOT multiphysics simulation toolkit. It aims at simulating spontaneous radiation from a single particle or an electron beam (modelled as a collection of macroparticles). In this work, presented simulation results which exploit new capabilities of the module such as quantum fluctuations, electron energy losses through radiation as well as dynamical effects.

The OCELOT toolkit with the SR module is capable of simulating complex problems including beam dynamics effects in an accelerator and spontaneous radiation simulation from an electron beam. It allows for the optimization of accelerator parameters, in order to obtain given light properties.

REFERENCES