Abstract
In the FCC-ee pre-injector complex, a slightly modified SPS can serve as pre-booster. The baseline design foresees injecting the low-emittance electron and positron bunches off-axis into the SPS, and deploying strong wigglers to greatly enhance the radiation damping at the injection energy. We here compare the damping of large injection oscillations by means of radiation damping with the effect of other possible damping mechanisms such as a fast bunch-by-bunch feedback system and/or head-tail damping via nonzero chromaticity. As a by-product, we investigate the transverse beam stability.

OVERVIEW
When the SPS is used as a FCC-ee pre-booster ring (PBR), 6 GeV electron or positron bunches, from an S-band linac [1, 2], are injected at large transverse amplitude [3]. To damp the injection oscillations, synchrotron radiation damping can be enhanced by installing dedicated radiation wigglers [3].

In the following, we first describe our approximate optics and impedance model for the SPS PBR. We then report simulation results for the longitudinal and transverse plane, considering situations without or with radiation wigglers, for different values of linear chromaticity, and at different bunch intensities. Finally, we draw a few conclusions.

SPS PBR MODEL
For use as PBR, it is proposed to operate the SPS with an integer tune of 40 in both transverse planes [3]. This novel configuration ("Q40" optics) corresponds to a betatron phase advance of 135° per FODO cell, which minimizes the equilibrium emittance. Traditionally, the integer tune of the SPS was 26; for LHC protons also integer tunes of 20 [4, 5] and 22 [6] are being used or tested.

The electron or positron bunches for FCC-ee are injected into the SPS Q40 optics at an energy of 6 GeV. We consider an off-axis injection in the vertical plane. Rough parameters for the injection beam are compiled in Table 1. The listed energy spread is typical for the end of the SLC linac [7]; alternative values are given in [1, 2]. The emittance numbers refer to an electron beam without damping ring [3]. We also assume that between linac and PBR the bunches pass through an energy compressor, or an arc with momentum-dependent path length, where the rms bunch length increases from ∼1 mm to about 10 mm.

An approximate nonlinear optics model can be constructed from SPS beam measurements performed at several integer tunes and various beam energies during the past two decades [8–10]. Nonlinear SPS optics measurements with the Q26 optics carried out at 14 GeV in 2003 [9] provide estimates for the second and third order chromaticity:

<table>
<thead>
<tr>
<th>Variable</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy $E_b$</td>
<td>6 GeV</td>
</tr>
<tr>
<td>Geometric emittance $ε_{x,y}$</td>
<td>2.5 nm</td>
</tr>
<tr>
<td>Initial injection offset</td>
<td>12 $σ_y$</td>
</tr>
<tr>
<td>Rms momentum spread $σ_δ$</td>
<td>0.2%</td>
</tr>
<tr>
<td>Rms bunch length $σ_z$</td>
<td>10 mm</td>
</tr>
<tr>
<td>Betatron tunes $Q_{x,y}$</td>
<td>40.13, 40.18</td>
</tr>
<tr>
<td>Momentum compaction $α_C$</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

Measurements in 2016 determined the Landau octupole settings required to compensate the natural detuning with amplitude for the Q20 optics [10] (namely, the knob values $K_{LOF} = -1$ m$^{-4}$ and $K_{LOD} = 0.5$ m$^{-4}$, equivalent to $-87$ and $+44$ T/m$^4$ at 26 GeV, for eight 0.7 m long octupoles each) which correspond to a linear detuning with action variables $I_{x,y}$ of order $10^3$ per micron in either plane [10]. We multiply these octupole strengths by a factor $2$, and use them to approximately reproduce the natural machine anharmonicity, assuming that the latter is dominated by the second order contribution from the lattice sextupoles. The factor of 2 roughly takes into account the modified optical functions for the Q40 optics as compared with Q20 [11].

Traditionally, the SPS impedance, as seen by the long proton bunches, is modelled by a broad-band resonator, with a resonant frequency of 1.3 GHz, a shunt impedance of order 1010 MΩ/m and a $Q$ value of 1 [8, 12]. This model describes the coherent motion of the SPS proton beams, e.g. [8]. Here, for the shorter lepton bunches, we assume the same impedance, and take it to be circularly symmetric. We note the existence of an alternative refined SPS impedance model [13].

Adding wigglers enhances the damping [3]. The proposed SPS wigglers have a field of 5 T and a total length of 4.5 m. Table 2 compares beam parameters related to synchrotron radiation with or without wigglers.

For the simulation, we use the code PyHEADTAIL [14], an extended version of HEADTAIL [15, 16] written in Python. We track $2.5 \times 10^4$ macroparticles over $2 \times 10^4$ turns, which corresponds to half a longitudinal radiation
Table 2: Parameters Related to Synchrotron Radiation at a Beam Energy of 6 GeV without and with Wigglers

<table>
<thead>
<tr>
<th>Variable</th>
<th>bare SPS</th>
<th>w. wigglers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. emittance $\varepsilon_{eq}$ [nm]$^2$</td>
<td>2.43</td>
<td>0.13</td>
</tr>
<tr>
<td>Eq. energy spread $\sigma_d$ [%]</td>
<td>0.018</td>
<td>0.30</td>
</tr>
<tr>
<td>Hor. damping time [s]</td>
<td>1.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Hor. damping time [turns]</td>
<td>80,000</td>
<td>4,400</td>
</tr>
<tr>
<td>Energy loss / turn $U_0$ [MeV]</td>
<td>0.15</td>
<td>2.7</td>
</tr>
<tr>
<td>RF voltage [MV]</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Eq. bunch length $\sigma_z,eq$ [mm]</td>
<td>3</td>
<td>33</td>
</tr>
<tr>
<td>Synchrotron tune $Q_s$</td>
<td>$6 \times 10^{-5}$</td>
<td>$8 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

LONGITUDINAL BEHAVIOUR

Figures 1 and 2 present the simulated evolutions of bunch length and energy spread after injection into the SPS, without and with wigglers, respectively. After the initial residual mismatch has rapidly filamented, within $\sim 200$ turns, the effect of synchrotron radiation becomes apparent. While in the case without wigglers bunch length and momentum spread decrease (Fig. 1), the wigglers increase the equilibrium momentum spread and bunch length so that both of them are larger than the values at injection (Fig. 2).

TRANSVERSE BEHAVIOUR

With a nominal bunch population of $N_b = 2 \times 10^{10}$ and $Q'_{x,y} = +1$, about 3000 turns after injection the bunch becomes vertically unstable, as is shown in Fig. 3. The signal from a simulated head-tail monitor, in Fig. 4, reveals that the instability is dominated by the $l = -1$ head-tail mode. The analytical model of DELPHI [17], based on the Sacherer theory, predicts an instability rise time of about 1500 turns for $N_b = 1.5 \times 10^{10}$ due to the transverse-mode-coupling instability [12, 18, 19].

Lowering the intensity to $N_b \approx 5 \times 10^9$ the centroid motion appears almost stable. However, without wigglers, a weak residual $l = -1$ mode instability still drives significant vertical emittance growth; see Fig. 5. On the other hand, with wigglers added, the beam is stable at least up to $N_b = 10^{10}$, half the design bunch intensity; in this case, after an initial growth due to filamentation, the vertical emittance shrinks due to radiation damping, as is shown in Fig. 6.

In our simulation, the transverse instability can be fully suppressed, even at $N_b = 2 \times 10^{10}$, by choosing a large negative chromaticity, e.g. $Q'_{y} = -5$ in conjunction with a transverse damper (50 turn damping time) that acts on the bunch centroid motion. Beam stabilization by negative chromaticity and transverse feedback was already proposed and experimentally tested [20, 21]. We note the fast damping of the initial injection oscillation in Fig. 7, in a time much shorter than the transverse radiation damping time even when including the wigglers. Figure 8 shows a fast restoration of the vertical emittance after off-axis injection.
CONCLUSIONS

Our simulations suggest that operating the bare SPS Q40 optics as PBR for leptons with a fast turn-by-turn damper at negative chromaticity will ensure both a fast damping of injection oscillations and transverse beam stability. Additional SPS wigglers would strongly enhance radiation ramping and allow for stable beam operation at small positive chromaticity up to about half the design intensity. In the future, further simulations will be carried out, for a more accurate SPS transverse impedance model [13].

The effect of the longitudinal impedance has not yet been studied. Historical experience [19] indicates, however, that the SPS PBR is likely to operate close to, or above, the longitudinal microwave threshold.

ACKNOWLEDGEMENT

We warmly thank H. Bartosik, N. Biancacci, L. Mether, E. Métral, and C. Zannini for helpful discussions.
REFERENCES


