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EFFECT OF ELECTROSTATIC DEFLECTORS AND FRINGE FIELDS ON SPIN FOR HADRON ELECTRIC DIPOLE MOMENT MEASUREMENTS ON STORAGE RINGS

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Abstract

The observed matter-antimatter asymmetry in the universe cannot be explained by the Standard Model (SM) of particle physics. A candidate for physics beyond the SM is a non-vanishing Electric Dipole Moment (EDM) of subatomic particles. The JEDI (Jülich Electric Dipole moment Investigations) collaboration [1] based in Jülich is preparing a direct EDM measurement of protons and deuterons first at the storage ring COSY (COoler SYnchrotron) and later at a dedicated storage ring. To achieve this, one needs a stable polarisation, i.e. around 1000 seconds for spin coherence time. One source of decoherence comes from the electrostatic deflectors, and it must be quantified. We developed an analytical model for cylindrical deflectors, including fringe fields, and the associated beam and spin transfer functions, integrated over the deflector. All boundary conditions (including zero equipotential) are taken into account, giving a realistic and accurate field map up to any order. We get universal formulas, the only adjustable parameter being the deflector gap/radius ratio, all other terms being numerical. This has been implemented in the tracking code BMAD. We present the mathematical, physical and numerical developments, as well as results for a proton storage ring.

INTRODUCTION

The matter-antimatter asymmetry problem, also known as the baryon asymmetry problem, comes from the observed difference between the quantity of matter and antimatter in the observable universe. One of the conditions to explain this asymmetry is the presence of additional $\mathcal{CP}$ violating phenomenon. A permanent Electric Dipole Moment (EDM) violates both parity and time reversal symmetries, and assuming $\mathcal{CPT}$ theorem, its also violates the $\mathcal{CP}$ symmetry. Many experiments around the world offer to give upper values for the EDM of elementary and composed particles. The JEDI collaboration proposes to use strong electric fields and a storage ring to study the EDM of charged hadrons. This experiment requires a high precision storage ring and an accurate control of the spin and of the beam.

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ELECTROSTATIC DEFLECTORS AND FRINGE FIELDS

Precession Equation and Spin Coherence Time

The dynamic of the spin of a charged particle is described by the Thomas-BMT equation [2] :

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

with Ω the precession vector :

$$\vec{\Omega} = -\frac{e}{m}(G\vec{B} + (\frac{1}{\gamma^2-1} - G)(\frac{\vec{\beta}\times\vec{E}}{c}) + \frac{\eta}{2}(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}))$$

where G is the anomalous magnetic moment : G = $\frac{g-2}{2}$ and η defined via d = $\frac{\eta e\hbar}{4mc}$ the value of the electric dipole moment. γ and β are the usual Lorentz factors and $\vec{E}$ and $\vec{B}$ the electric and magnetic fields.

In order to measure an EDM, one needs to have an extremely accurate control on the G-dependent part of this equation, which corresponds to the magnetic moment contribution to the spin rotation.

As the particles of the beam are not the reference particle, differences in term of position, impulsion and energy with respect to the reference particle will slowly lead to an horizontal depolarisation.

In order to measure an EDM with the required precision, one needs the spin coherence time (ie the time during which the spins of the particles are not scattered by more than one radian) to be as long as possible.

Deflectors and Fringe Fields

A particle with a different position, momentum or energy will undergo a different precession than the reference particle. One needs a good control and understanding on these quantities to maximise the spin coherence time. Using accurate and rigorous models for the spin dynamics in the electrostatic deflectors is essential.

Additionally, fringe fields can be a source of depolarisation by introducing new components of the field.

We developed analytical and universal models to understand the electric field, particles trajectories and spin dynamics in both the deflector and fringe fields. All the formulas work as a function of the Gap/Radius ratio of the deflector and are defined up to the second order of perturbation.

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UNIVERSAL AND ANALYTICAL MODEL FOR DEFLECTORS AND FRINGE FIELDS

Electrostatic Field

Analytical electrostatic field models can be found by using conformal transformations. A conformal mapping, or conformal transformation is a transformation of the complex plane that locally preserves the angles, i.e. the field lines and equipotential lines. An analytic function is conformal at any point where it has a nonzero derivative.

If one knows the transformation between a known system ($z = x + i \cdot y$) and the desired system ($Z = X + i \cdot Y$), then the electrostatic potentials in both systems are equal at the corresponding points : V(z) = V(Z).

We are looking for a realistic model of the fringe fields of an electrostatic deflector, including boundary conditions (V = 0) like the vacuum chamber or a diaphragm and also a realistic shape for the electrodes.

Figure 1 shows the initial and final systems we took into account for our model :

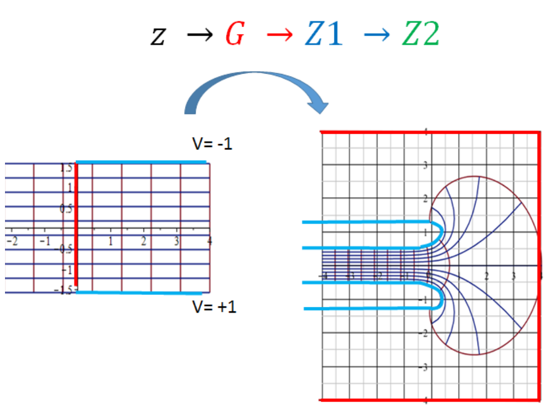

Figure 1: Left : Semi-infinite box with straight electrodes (blue) and zero equipotential boundary condition (red). The potential in this system is known (see [3]). Right : view from above of a realistic deflector with vacuum chamber and diaphragm (red) and curved electrodes (blue). The equipotential and field lines are here for illustration.

This profile can be obtained by applying successively three transformations :

$$G(z) = 2.68 - \frac{11}{4} \cdot e^{\frac{4}{11} \cdot z} \cdot H_{\frac{2}{11},\frac{2}{11},\frac{13}{11}}(-e^{2z})$$

$$Z_1 = [1 + G(z) + e^{G(z)}]$$

$$Z_2 = \frac{1}{\pi} \cdot [1.376 + Z_1 + e^{Z_1}]$$

where the H function is a hyper-geometrical function.

This model can be extended for a cylindrical deflector as an universal field function depending only on the ratio Gap/Radius of the deflector.

This allows us to compute at will the multipolar components of the electrostatic field by successive derivations of the potential function, as well as the longitudinal component.

Trajectories

The equations of motion are computed from the perturbed Hamiltonian [4] and by using the variation of parameters method :

$$\mathcal{H} = -\left(1 + \frac{x}{\rho_0}\right) \cdot \sqrt{\frac{\gamma^2 - 1}{\gamma_0^2 \cdot \beta_0^2} - (p_x^2 + p_y^2)} = \mathcal{H}_{lin} + \tilde{\mathcal{H}}$$

In the case of the fringe fields, we used the field computed by conformal mapping in the previous part, including transverse and longitudinal dynamics.

The final result is universal and analytical transfer functions up to order 2.

Spin Dynamics

The spin dynamics is computed from the Thomas-BMT equation. The Ω precession function is separated in two distinct part : a constant part and a perturbative part.

The principal (constant part) rotation, corresponding to a fully perpendicular electric field and fully longitudinal momentum of the beam can be solved analytically with a direct integration of the rotation vector.

The perturbative part (considered small) has for solution, up to order two :

$$R = I + \sum_{1}^{\infty} \int_0^{t_n} U(t_n) dt_n \int_0^{t_{n-1}} U(t_{n-1}) dt_{n-1} \ldots \int_0^{t_0} U(t_0) dt_0$$

These integrals are solved fully analytically in the case of the inner part of the deflector.

For the fringe fields, these are functions of different integrals of the field and motion equations computed previously. All these integrals have already been computed once and for all and stored in a separate file.

They can be used for any electrostatic deflector with only input parameter the ratio Gap/Radius of the deflector.

Note : At the end, all the rotation matrices have the form :

$$R(\theta) = \begin{bmatrix} cos\left(\frac{\theta}{2}\right) - iu_z.sin\left(\frac{\theta}{2}\right) & -(iu_x + u_y).sin\left(\frac{\theta}{2}\right) \\ (-iu_x + u_y).sin\left(\frac{\theta}{2}\right) & cos\left(\frac{\theta}{2}\right) + iu_z.sin\left(\frac{\theta}{2}\right) \end{bmatrix}$$

with θ the total rotation angle, (u_x, u_y, u_z) the components of the rotation axis.

The real and complex components of this matrix are function of the initial phase-space vector $(x_0, p_{x0}, y_0, p_{y0}, dz, \Delta P/P_0)$ before entering the fringe field or the deflector.

This method allows us to study independently the effects of the components (x,p_x...) in order to better understand the spin dynamics and beam depolarisation and study their relative amplitudes.

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IMPLEMENTATION IN BMAD

This model is currently fully implemented in the BMAD [6] code. It is now under test and benchmark on a fully electrostatic ring (see fig. 2) :

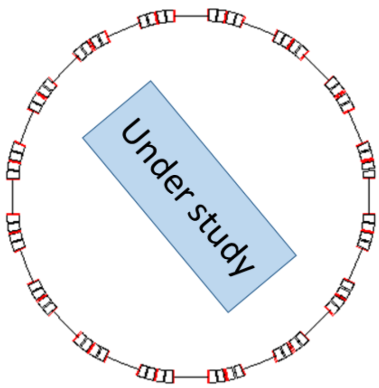

Figure 2: Simplified lattice currently under study with the developed model. This is a proton frozen spin lattice with an energy of ≈ 230 MeV. It is composed of 64 electrostatic deflectors + fringe fields with a radius of 32 m and an electric field of ≈ 13 MV/m. The focusing is done with magnetic quadrupoles, with no effect on the spin.

The first simulations show a good agreement with known spin motion effects.

The spin oscillations are stable in a quasi-frozen spin lattice, where the spin is supposed to oscillates around an equilibrium position.

Also, first order effects as the mixing in energy due to synchrotron oscillations have been validated (see Figure 3).

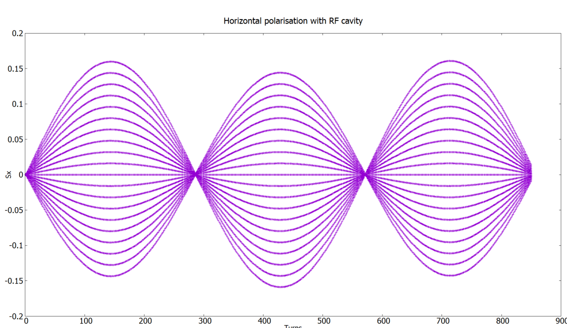

Figure 3: Evolution of the horizontal polarisation of an off-momentum particle. Due to the RF cavity, the average value of the energy is zero.

Synchrotron oscillations allow the spin coherence time to be much longer.

Focus is now on second order effects, and more analysis is needed.

CONCLUSION

In order to achieve a high spin coherence time, one needs to have a great control on the spin dynamics. In this way, we developed a (semi-)analytic model integrated as far as possible. The remaining integrals have been computed numerically and are universal values depending only on the gap/radius ratio of the deflector.

The fringe electrostatic field has been computed using conformal mapping and is including realistic electrodes as well as boundary conditions (i.e. vacuum chamber + diaphragm).

Trajectories are computed from the Hamilton equations up to order two and solved by perturbation with the same formula as the one used for spin.

We used the T-BMT equation and a perturbation method to solve the spin dynamics. The result is a rotation matrix, different for the inner part or the fringe fields of the deflector.

These matrices can be implemented in a tracking code to make faster simulations and to study precisely the dependencies of the spin dynamics in a deflector.

We are currently testing this model with the BMAD code on a simplified reference scenario with only electric fields.

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