

# PROGRESS TOWARDS A SINGLE-SHOT EMITTANCE MEASUREMENT TECHNIQUE AT AWAKE

J. Chappell<sup>†</sup>, D. Cooke, L. Deacon, S. Jolly, F. Keeble, M. Wing, UCL, London, United Kingdom

## Abstract

Externally injected electrons are captured and accelerated in the plasma wake of a self-modulated proton beam at the Advanced Wakefield Experiment (AWAKE) at CERN. The energy distribution of the accelerated electron beam is measured using a dipole spectrometer in combination with a scintillator screen, with two upstream quadrupoles providing energy-dependent focusing. Measuring the vertical beam size variation with horizontal position along the scintillator screen, and therefore energy, results in an effective quadrupole scan permitting single shot vertical geometric emittance measurements. Limitations of the method due to effects such as imperfect beam focusing and finite resolution are explored via simulations using the beam tracking code BDSIM.

## INTRODUCTION

Plasma wakefield acceleration (PWFA) is a promising technique for the acceleration of charged particles due to the large accelerating gradients that can be provided, three orders of magnitude large than those produced via traditional methods. At AWAKE, an 18 MeV electron beam is injected into the wakefield driven by a self-modulated SPS proton beam at an oblique angle [1]. Electrons are trapped within the wakefield structure and have shown to be accelerated to energies exceeding 2 GeV over a 10 m long plasma cell before being captured by a magnetic dipole spectrometer. Two quadrupoles provide energy-dependent focusing which, in combination with the dispersive dipole and typically large energy spread beams ( $\sigma_E/\mu_E \sim 0.1$ ) produced in the current AWAKE scheme, results in an effective single-shot quadrupole scan which is imaged on a scintillating screen that forms part of the spectrometer.

## SPECTROMETER DESIGN

The spectrometer beamline is shown in Fig. 1, with components beginning approximately 4.5 m downstream of the exit of the plasma cell. It consists of two quadrupoles to capture and focus the accelerated beam, and a magnetic dipole, used to separate the accelerated electrons from the proton beam [2]. After passing through the dipole magnet, the electrons are incident upon a 0.5 mm thick Gadolinium Oxysulfide ( $Gd_2O_2S : Tb$ ) scintillating screen at an angle of  $44.8 \pm 0.1^\circ$  to the beamline. Light emitted by the screen is transported over  $\sim 16$  m to the camera darkroom via a series of three highly reflective, optical grade mirrors where it is captured by a large diameter, 400 mm focal length lens and imaged onto an intensified CCD. The relationship between

an electron's energy and its incident position in the plane of the scintillator screen is derived using the beam tracking code BDSIM [3] using measured dipole field maps as input. A constant offset of 6% in the current supplied to, and hence field strength of, the quadrupoles was applied to enable approximate matching of their focal planes despite their spatial separation of 0.21 m. The optimal value of this offset is energy-dependent but the chosen value was found to be suitable for the energy range of interest (0.2 – 1.3 GeV).

## EMITTANCE MEASUREMENT

It is possible to estimate the emittance of the accelerated beam at the exit of the plasma by measuring the variation in the height of the beam on the scintillator screen. This is because both the quadrupole focusing and horizontal dispersion introduced by the dipole are energy dependent, resulting in an effective quadrupole scan in a single shot.

The evolution of the electron beam in the vertical plane from the plasma exit to the scintillating screen can be described using a linear transport matrix model:

$$\mathbf{R} = \begin{bmatrix} 1 & d(E) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \sqrt{k}l & \frac{\sin \sqrt{k}l}{\sqrt{k}} \\ -\sqrt{k} \sin \sqrt{k}l & \cos \sqrt{k}l \end{bmatrix} \begin{bmatrix} 1 & d_{qsep} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cosh \sqrt{k}l & \frac{\sinh \sqrt{k}l}{\sqrt{k}} \\ \sqrt{k} \sinh \sqrt{k}l & \cosh \sqrt{k}l \end{bmatrix} \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \quad (1)$$

where  $k(E) = (dB/dx) \cdot (3.3E)^{-1}$  is the normalised quadrupole strength,  $d(E)$  is the energy-dependent path

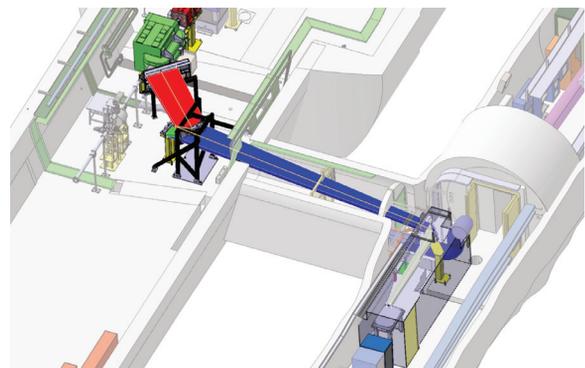


Figure 1: AWAKE spectrometer components highlighted within the experimental area. Two quadrupoles (red) focus the beam before entering the dipole (green). Light emitted by the scintillating screen follows the path shown in red, green, blue and gold to the CCD housed in the darkroom.

<sup>†</sup> james.chappell.17@ucl.ac.uk

length from the exit of the second quadrupole through the dipole to the screen,  $d_{qsep}$  is the separation between the quadrupoles, and  $d_1$  is the distance from the exit of the plasma cell to the first quadrupole. The functional form of  $d(E)$  is derived assuming a uniform field within the dipole. The 1D beam matrix can be described by the second order moments of the distribution in position, angle and their correlation:

$$\Sigma = \begin{bmatrix} \langle y^2 \rangle & \langle yy' \rangle \\ \langle yy' \rangle & \langle y'^2 \rangle \end{bmatrix} = \varepsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix} \quad (2)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are the Twiss parameters. Hence the transformation of the beam from the plasma exit to the screen is  $\Sigma_{scr} = \mathbf{R} \Sigma \mathbf{R}^T$ . The measured beam size at the screen,  $\sigma_y = \sqrt{\langle y^2 \rangle}$ , can be related to its size at the exit of the plasma cell via the relation

$$\sigma_{y,scr}^2 = R_{11}^2 \beta \varepsilon - 2R_{11}R_{12} \alpha \varepsilon + R_{12}^2 \gamma \varepsilon. \quad (3)$$

The matrix coefficients, e.g.  $R_{11}$ , are evaluated for the central energy of each pixel in the plane of the scintillator and fill a design matrix  $\mathbf{X}$ . A vector of  $n$  measurements of the square of the vertical beam size around the focus on the screen,  $\sigma = [\sigma_{y,1}^2, \sigma_{y,2}^2, \dots, \sigma_{y,n}^2]^T$ , is populated. The uncertainty on each measurement of the beam size squared, calculated via combining the uncertainty on the fit on the beam size and the energy uncertainty for each pixel,  $u(\sigma_{y,i}^2) = 2\sigma_{y,i}u(\sigma_{y,i})$ , is used to form a diagonal weighting matrix,  $\mathbf{W}$ . The beam parameters at the exit of the plasma cell,  $\beta = [\langle \hat{y}^2 \rangle, \langle \hat{y}\hat{y}' \rangle, \langle \hat{y}'^2 \rangle]^T$  can then be estimated by solving the weighted least-squares equation:

$$\mathbf{W} \sigma = \mathbf{W} \mathbf{X} \beta. \quad (4)$$

The geometric emittance is calculated using the relation,

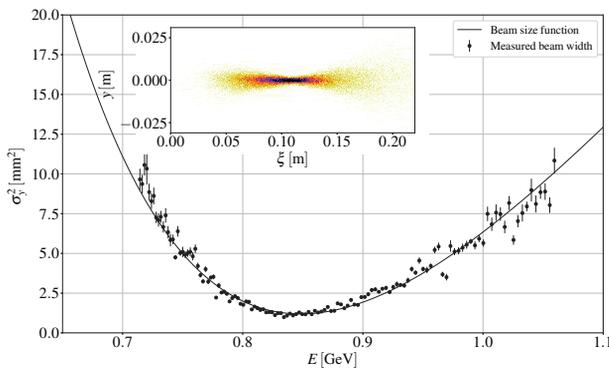


Figure 2: Example fit to the measured vertical beam size around the central 5.4 cm in the focus for a simulated beam of  $\varepsilon_y = 1$  mm-mrad. Horizontal bin width corresponds to the resolution of the camera, assuming perfect transmission. The measured emittance value of this beam is  $\varepsilon_y = 1.02 \pm 0.03$  mm-mrad. Inset: simulated spectrometer image showing the typical butterfly shape.

$$\varepsilon_y = \sqrt{\langle \hat{y}^2 \rangle \langle \hat{y}'^2 \rangle - \langle \hat{y}\hat{y}' \rangle^2}. \quad (5)$$

An example simulated beam image on the spectrometer screen is demonstrated in Fig. 2 with its associated fit, the beam size function, to the measurements around its focus.

## VARIATION WITH BEAM PARAMETERS

The accuracy of the emittance measurement technique with varying emittances and energy spreads was tested in simulation. Emittances from 1 to 20 mm-mrad were used, with the beam size never exceeding 2 mm. This was chosen because plasma accelerated beams are expected to be transversely small due to the size of the accelerating structure ( $\sim 1.5$  mm for AWAKE baseline parameters). At present, typical plasma accelerated witness beams have large energy spreads ( $> 10\%$ ) due to the lack of optimal beam loading of the wakefield causing varying accelerating gradients for different parts of the witness beam. The results are summarised in Fig. 3.

To estimate the systematic uncertainty on the measurement process, the emittance of each beam was measured while varying the size of the fitting region around the focus of the beam. These measurements were then combined with a weighting according to their respective  $\chi^2$  values to calculate the measured emittance and associated uncertainty.

The width of the beam in a given column of the image is measured by fitting a Gaussian profile to the signal and extracting its width. As expected, the fit quality away from the focus improves with increasing signal, giving a more accurate measurement for beams with larger energy spread. As the divergence of the beam increases for larger emittance beams, the accuracy of the emittance measurement decreases. This is caused by the limited aperture of the beam pipe in combination with the distance to the first quadrupole where the beam is captured. At AWAKE, the beam pipe radius is 35 mm while the distance from the exit of the plasma cell to the first quadrupole is 4.5 m. Therefore, any particles with divergences exceeding 7.8 mrad will interact with the beam pipe and will not be captured by the first quadrupole.

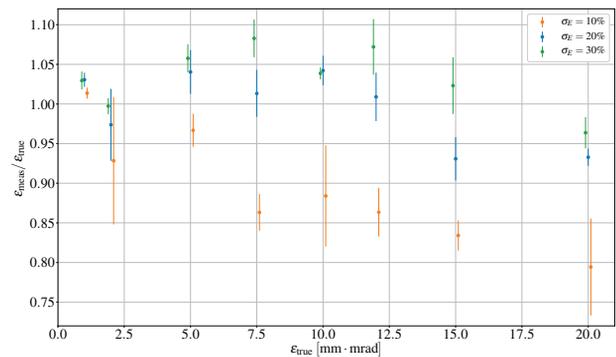


Figure 3: Ratio of the measured emittance to the true emittance for a range of emittances and energy spreads. Data points are horizontally shifted to avoid overlap.

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2019). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI

This leads to a systematic underestimation of the beam divergence, and hence emittance, for large emittances.

## RESOLUTION EFFECTS

The finite resolution of the optical system places a limit on the minimum beam size, and hence emittance, it would be possible to measure. The pixel size of the camera sensor corresponds to a physical size of 0.54 mm in the plane of the scintillator screen and this therefore represents the minimum possible resolution, assuming a perfect optical path. The entire spectrometer optical path, including mirrors, was designed such that the resolution of the system did not exceed 1.0 mm. However, a rectangular fire safety window had to be placed into the optical path. Due to its large aspect ratio, the glass was found to curve in the vertical plane when fixed in place, causing a decrease in the vertical resolution. Measurements showed a minimum resolvable size of  $3.78 \pm 0.11$  mm in the vertical plane with the window in place, which improved to  $0.92 \pm 0.02$  mm without the window. All measurements taken during the experimental running periods had the fire window in place, limiting the resolution in the vertical plane.

The effect of decreased resolution was simulated by convolving the scintillating screen image corresponding to ideal resolution with a two dimensional Gaussian kernel. The widths of this kernel in each plane were chosen such that an increase in the measured width of a single test pixel was equal to the equivalent decrease in resolution. Figure 4 shows the effect of the decreased vertical resolution on the measurement of the beam size at the focus and subsequent emittance measurement. For a test beam with an emittance of  $\epsilon_y = 1$  mm·mrad, the ideal resolution case measures the size of the beam at focus to be  $\sigma_{y,foc} = 1.12 \pm 0.05$  mm, close to the true value of 1 mm despite the limited number of pixels over which the measurement is made. In contrast, when the resolution is decreased to the experimentally determined value with the safety window in place, the measured beam size is  $\sigma_{y,foc} = 3.82 \pm 0.14$  mm. This leads to an overestimation of the emittance,  $\epsilon_y = 2.99 \pm 0.08$  mm·mrad.

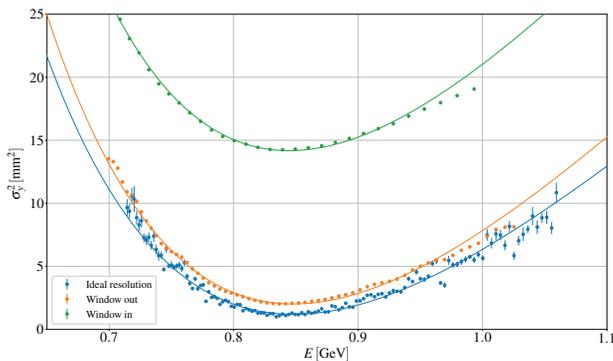


Figure 4: Comparison between simulated measured beam sizes on the scintillator screen for an ideal optical resolution (blue), the measured resolution with the safety window removed (orange), and in place (green).

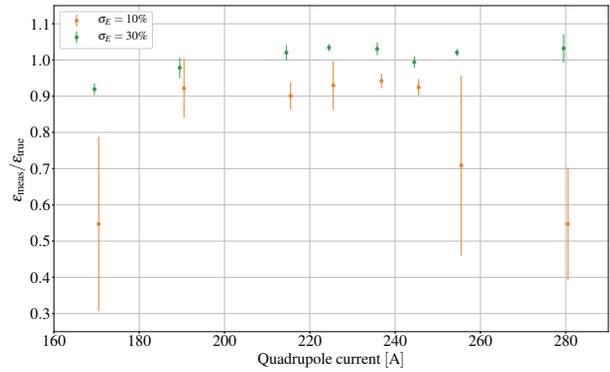


Figure 5: Ratio of measured emittance to true emittance for a beam with  $\epsilon_y = 5$  mm·mrad under varying quadrupole focusing conditions.

Conversely, with the resolution mimicking when the safety window is removed, the beam size at focus was measured to be  $\sigma_{y,foc} = 1.50 \pm 0.12$  mm with an emittance of  $\epsilon_y = 1.10 \pm 0.07$  mm·mrad.

## FOCAL ENERGY FLUCTUATIONS

An inherent difficulty with measuring the emittance of a plasma accelerated witness bunch is the shot-to-shot energy fluctuations that can occur due to, for example, plasma density fluctuations. As the witness mean energy fluctuates, the quadrupoles will focus a limited energy range that may not correspond to the bulk of the energy distribution of the accelerated beam. The effect of this on the measurement of emittance was simulated by varying the quadrupole current while keeping the beam energy constant. The results are shown in Fig. 5.

The measured emittance differs greatly from the true emittance when the quadrupole current is much lower or higher than that corresponding to a focal energy of the mean energy of the beam ( $I_{foc} = 236.6$  A,  $\mu_E = 0.8$  GeV) due to the loss of the butterfly shape. The emittance measurement is more stable for larger energy spread beams due to the increased charge density in the wings that still allows accurate measurement of the beam size despite reduced focusing. In addition to this, the non-linearity in the relationship between energy and horizontal position on the screen means that there is a higher charge density for higher energies. Therefore, the emittance measurement remains more stable for lower quadrupole currents as there is still sufficient charge within the wings at higher energies for accurate fitting of the beam size.

This also demonstrates that in a quadrupole current range of  $\sim 40$  A around the focal current, the measurement is stable and accurate to within 5% for beams with a large energy spread. In the AWAKE spectrometer setup this current range corresponds to a focal energy range of 710 – 840 MeV and hence shows the applicability of this method even with a beam with shot-to-shot mean energy fluctuations on the order of 5%.

## REFERENCES

- [1] The AWAKE Collaboration, "Acceleration of electrons in the plasma wakefield of a proton bunch", *Nature* 561, 363–367, 2018.
- [2] F. Keeble *et al.*, "A magnetic spectrometer to measure electron bunches accelerated at AWAKE", arXiv:1902.05752, 2019.
- [3] L. Nevay *et al.*, "BDSIM: An Accelerator Tracking Code with Particle-Matter Interactions", arXiv:1808.10745, 2018.