Abstract

The MAX IV 3 GeV storage ring lattice contains several strong sextupoles. In order to achieve nominal lattice performance it is important to be able to characterise and correct the higher order magnets and optics of the lattice. This has been done through the analysis of the Off-Energy Response Matrix (OEORM). Its approximate linearity in sextupole strength has been utilised to identify sextupole errors, as well as symmetrise the 2nd order optics. The symmetrisation was able to correct chromaticity, and increase horizontal acceptance by 50 %, compared to magnet settings based solely on rotating coil measurements. An approximate decrease of 10 % in vertical acceptance was detected. This work was inspired by similar investigations at ESRF [1].

THE CHROMATIC FUNCTIONS

The first derivative of the beta functions with respect to the relative momentum deviation, referred to as the chromatic functions in this paper, are given by [2]:

\[
\frac{d\beta}{d\delta}(\varsigma) = \frac{\beta(s)}{2 \sin 2\pi \nu_0} \times \\
\int_s^{s+L} \beta(\zeta)(k(\zeta) - m(\zeta)\eta(\zeta)) \times \\
\cos\left[2\nu_0(\varphi(s) - \varphi(\zeta) + 2\pi)\right] \, d\zeta
\]

where \(\varphi\) is the phase advance and \(\nu_0\) is the unperturbed tune.

Note that the chromatic functions are linear in sextupole strength \(m\). In addition, the sensitivity of the chromatic function to a change of sextupole strength in a certain magnet is dependent on the product of the beta function and the horizontal dispersion at the location of the magnet.

THE MAX IV 3 GEV LATTICE

The MAX IV 3 GeV storage ring has a 20-fold symmetry. Each of the 20 achromats contain five sextupole families, three focusing and two defocusing. All sextupoles are chromatic, meaning that they will contribute to the chromatic functions according to Eq. (1). The SDend, SFi, SFo, and SFm families consist of two magnets connected in series, while the SD family consists of ten. In achromat 8, one of the SFi circuits is split up for BPM offset experiments, resulting in a total of 101 sextupole circuits. The location of the sextupoles in each achromat can be seen in Fig. 1.

METHOD AND SIMULATIONS

Off-Energy Orbit Response

Since \(\beta\) and \(\eta\) in Eq. (1) are functions of \(\delta\) the approximate linearity of the Off-Energy Orbit Response (OEOR) of the MAX IV 3 GeV ring was investigated in simulation. In Fig. 2 it can be clearly seen that the OEOR is approximately linear up to a relative change of > 50 % of the nominal sextupole strength.

The linearity of the OEOR also depends on the magnitude of the change in momentum. As the change in momentum increases, the linearity of the OEOR with sextupole strength deteriorates. Based on this alone, it is preferable to choose a as small as possible change in momentum. However, when including BPM noise, a very small change in momentum has a poor signal-to-noise. For every sextupole magnet, there exists a change in momentum for which the residual of a linear fit is minimised, as seen in Fig. 3. Empirically, a 100 Hz change in accelerating momentum was chosen for both simulation and measurements of the OEOR.

2nd Order Optics Symmetrisation

From the OEOR of every dipole corrector an Off-Energy Orbit Response Matrix (OEORM) can be constructed. It was measured by taking the difference between two orbit response matrices measured at accelerating frequencies ±50 Hz from nominal.

An array of normally distributed sextupole strength errors with a STD of 4 % of the nominal values were introduced to the sextupole circuits of the model. Using a Gauss-Newton fitting procedure the OEORM of the nominal model could be...
fitted to the OEORM measured on the model with errors. A noise of 0.4 μm STD, corresponding to the BPM noise level at 3 mA and 10 Hz data stream, was added to the OEORM to simulate BPM noise. The parameters to fit were each of the 101 sextupole circuits (see Fig. 4), chosen because these are independently controlled by individual power supplies in the real machine.

Errors introduced by sextupole circuit are fully within the space spanned by the OEORM Jacobian (the 101 sextupole circuits), thus a good identification of the errors is to be expected. In the case of the real machine sextupole strength errors may also be errors by magnet as opposed to by circuit. Such errors were introduced to the model, in addition to alignment errors to all quadrupoles and sextupoles. Unlike the errors by circuit, these errors are clearly not spanned by the Jacobian when fitting by sextupole circuit. The introduced errors and the values found from the fit can be seen in Fig. 5.

The sextupole circuit errors found when fitting to a model with sextupole errors by magnet and alignment errors do not correspond to the mean field in the circuit. Despite this, when comparing the chromatic functions of the model with errors, and the model fitted to the OEORM they show very good agreement (see Fig. 6).

MEASUREMENTS
In order to test the procedure, a set of 5 circuit errors were introduced to the MAX IV 3 GeV storage ring during a machine shift. These consisted of reductions of the corresponding power supplies by 20%, 15%, 10%, 5%, and 2% from the nominal sextupole field values as given by calibration curves based on rotating coil measurements. By measuring an OEORM before and after introducing the errors, and comparing the sextupole circuit strengths extracted from each of the two measurements, it was possible to identify the magnitude and source of most of the errors (see Fig. 7). The BPM readings from these measurements were linearised using a polynomial map calculated using the procedure in [4].

From the extracted sextupole settings the sextupole strengths of the machine could be corrected. The change in chromaticity from the introduced errors, and the change from the corresponding corrections can be seen in Table 1. After three iterations the chromaticity of the machine had reached the goal chromaticity (+1 / +1).

The three successive correction iterations based on measured OEORM resulted in the change in chromatic function seen in Fig. 8. The reduction in chromatic function beating resulted in a RMS reduction from 4.2002 m / 4.0971 m to 1.6417 m / 1.0534 m. The change corresponds to the relative changes in sextupole strengths seen in Fig. 9.

Lifetime-scraper measurements were done on both a lattice with nominal sextupole settings calculated from rotating coil measurements, and a lattice with sextupole settings from
Table 1: Change in chromaticity before and after introducing sextupole circuit errors, and after correcting the 2nd order optics via the OEORM for a number of iterations. The goal chromaticity was +1 / +1. The initial chromaticity was the resulting chromaticity after a linear optics correction using LOCO, with no subsequent chromaticity correction.

<table>
<thead>
<tr>
<th>Circuit Number</th>
<th>Initial</th>
<th>Post Error Introduction</th>
<th>1st Iteration</th>
<th>3rd Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>A - Acceptance</td>
<td>5.59 ±0.15</td>
<td>5.44 ±0.19</td>
<td>5.59 ±0.15</td>
<td>5.44 ±0.19</td>
</tr>
<tr>
<td>B - Acceptance</td>
<td>11.11 ±0.08</td>
<td>10.17 ±0.17</td>
<td>11.11 ±0.08</td>
<td>10.17 ±0.17</td>
</tr>
</tbody>
</table>

Table 2: Acceptance from lifetime-scaper measurements on machine with nominal sextupole settings and settings found from the OEORM fit. The required values are determined by injection requirements [3].

<table>
<thead>
<tr>
<th>Acceptance</th>
<th>Nominal</th>
<th>OEORM Fit</th>
<th>Physical</th>
<th>Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_x [mm mrad]</td>
<td>5.59 ±0.15</td>
<td>8.14 ±0.19</td>
<td>11.11</td>
<td>5.44</td>
</tr>
<tr>
<td>A_y [mm mrad]</td>
<td>3.48 ±0.19</td>
<td>3.03 ±0.17</td>
<td>8.00</td>
<td>2.00</td>
</tr>
</tbody>
</table>

**SUMMARY**

The linearity of the OEOR in sextupole field strengths has been utilized to identify and correct sextupole errors introduced to the MAX IV 3 GeV storage ring. Additionally, the method is able to symmetrise the 2nd order optics of the ring. By iteratively correcting the sextupole settings, it was possible to reduce the beating of the chromatic functions by a factor 2.6 in the horizontal plane, and 3.9 in the vertical plane, and increase the horizontal acceptance by 46%, while the vertical was reduced by 13%.

More exhaustive studies of choice of parameters and ways to increase the accuracy of the measurement are planned, with the goal of making 2nd order optics symmetrisation part of regular operation at MAX IV.

**REFERENCES**


