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LOSSLESS CROSSING OF 1/2 RESONANCE STOPBAND BY SYNCHROTRON OSCILLATIONS *

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Abstract

Modern high-performance circular accelerators require sophisticated corrections of nonlinear lattices. The beam betatron tune footprint may cross many resonances, reducing dynamic aperture and causing particle loss.

However, if particles cross a resonance reasonably fast, the beam deterioration may be minimized. This paper describes the experiments with the beam passing through a half-integer resonance stopband via tune modulation by exciting synchrotron oscillations. This is the first time that beam dynamics have been kept under precise control while the beam crosses a half-integer resonance. Our results convincingly demonstrate that particles can cross the half-integer resonance without being lost if the passage is reasonably fast and the resonance stopband is sufficiently narrow.

INTRODUCTION

It has become standard practice to constrain the particle's tune footprint while designing the storage ring lattice so that the particle tunes fit between harmful resonances, which limit ring dynamic aperture (DA) [1]. This approach, known as "tune confinement", puts tight limits on the magnitude of the tune shifts with amplitude and with momentum. The latter requires labor-intensive optimization of the off-momentum DA and the corresponding tune footprint for the large momentum deviations to achieve a reasonable lifetime.

As nonlinearities of the modern ring lattices are much enhanced as compared with the previous generation of synchrotrons, it is becoming more and more difficult to keep the off-momentum tune footprint inside the range surrounded by the resonance lines [2-4]. One of the major resonances is the half-integer resonance and it is always treated as an unstable working point that may cause beam loss. The half-integer resonance poses concerns in many circular accelerators, such as modern synchrotron light sources [2, 3], heavy ion medical accelerators [5] and non-scaling fixed-field alternating-gradient (FFAG) accelerators [6].

Intuitively, if the particle crosses the stopband quickly, one may expect that the betatron oscillation amplitude will not increase substantially thereby keeping the particle within the machine acceptance. At the same time, the tight tolerances with which modern lattice elements can be designed and produced afford much narrower resonance stopbands when compared with machines built decades ago.

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Recently modern synchrotrons advanced to Multi-Bend Achromat lattices featuring small dispersion and low beta functions, and high nonlinearity of the particle motion due to stronger sextupoles. In certain cases [2, 3], the tune spread for on-energy beam was successfully minimized, but the off-momentum tunes swing across the major resonances, as shown in Fig. 1. However, the tracking result did not show particle losses in contrast to the experiments [5, 7] on resonance crossing where the beam losses were observed.

In this paper, we investigated the beam dynamics during crossing of a major resonance in NSLS-II, both by design and by experiment, to achieve the storage ring conditions where the beam crosses the ½ resonance without particle loss [8].

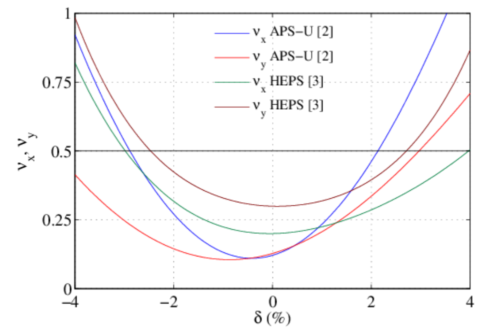

Figure 1: Fractional tune shift with momentum deviation as presented in [2, 3].

DYNAMICS OF CROSSING A STATIC RESONANCE STOPBAND

We consider a storage ring model with large chromatic tune shift and a particle with momentum deviation $\delta = \frac{\Delta p}{p}$ up to the second order writing the particle's tune shift as:

$$\nu(\delta) = \nu_0 + \xi_1\delta + \xi_2\delta^2 + O(\delta^3), \qquad (1)$$

where ξ_1 and ξ_2 are linear and 2nd order chromaticities. In the following, we constrain our analysis to the 2-dimensional case of y and δ. For our experiments we kept ξ_{1y}=+1 and tuned the 2nd order chromaticity to ξ_{2y}=+300 (the same value as in [2, 3]) by changing ring sextupoles while maintaining small tune shifts with amplitude.

Next we assume that the particle energy oscillates with the maximum deviation δ0 and this synchrotron oscillation, for simplicity, is taken as $\delta(n) = \delta_0 \sin(2\pi\nu_s n)$, where ν_s is the synchrotron tune and n is the number of turns around the ring. An illustration of the problem under consideration is shown in Fig. 2. As can be seen, the betatron tune of a longitudinally oscillating particle crosses the half integer resonance νR=p/2, which has a stopband width that depends on quadrupole errors. The resonance is

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characterized by a stopband with the width J_p, which is heuristically defined as the boundary of the tune range where the peak beta-beat $\Delta\beta/\beta = \frac{\beta-\beta_0}{\beta_0}$ reaches 100% [9]. Here p is an integer number, β_0 is the reference beta function calculated from the unperturbed lattice model, and β is the measured beta function obtained from beam oscillations excited by a pulsed kicker and measured by beam position monitors (BPMs) distributed around the ring [10].

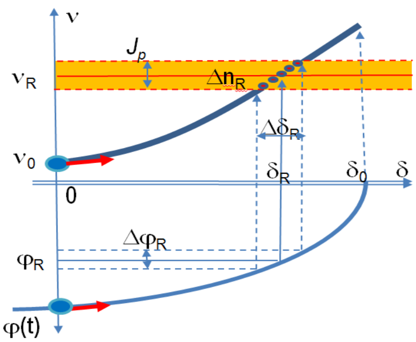

Figure 2: Particle's synchrotron oscillations with maximum amplitude δ_0 crossing resonance at νR=p/2 with the stopband width of J_p.

We define $\delta_R = \delta(\nu_R)$ as the value of the energy deviation where the particle's tune crosses the resonance ν_R. The boundaries of energy deviation that correspond to the resonance stopband J_p are (neglecting the contribution from the linear chromaticity ξ_1 and assuming that $\frac{p}{2} - \nu_0 > 0$ and ξ_2 is positive):

$$\delta_R \pm \frac{\Delta\delta_R}{2} = \sqrt{\frac{(p/2 - \nu_0) \pm J_p/2}{\xi_2}}$$

For calculating the number of turns the particle takes to cross the stopband we get:

$$\Delta n_R = \begin{cases} \left(\mathrm{asin}\left(\left(\delta_R + \frac{\Delta\delta_R}{2}\right)\delta_0^{-1}\right) - \mathrm{asin}\left(\left(\delta_R - \frac{\Delta\delta_R}{2}\right)\delta_0^{-1}\right)\right)/(2\pi\nu_s), & if\ \delta_0 > \delta_R + \frac{\Delta\delta_R}{2} \\ \mathrm{acos}\left(\left(\delta_R - \frac{\Delta\delta_R}{2}\right)\delta_0^{-1}\right)/(\pi\nu_s), & if\ \delta_R - \frac{\Delta\delta_R}{2} < \delta_0 \le \delta_R + \frac{\Delta\delta_R}{2} \end{cases} \quad (2)$$

Due to the radiation damping the amplitude of energy oscillations δ will decay below δ_R after the time interval $\Delta T = N_{cross} T_s/2$, where T_s is the synchrotron period, $N_{cross} = -2\frac{\tau_s}{T_s} ln\left(\frac{\delta_R}{\delta_0}\right)$ corresponds to the number of crossings of the resonance stopband when $\delta_0 > \delta_R$ and τ_s is the damping time. This expression is an approximate result since we are not taking quantum excitation into account.

CONTROLLING THE RESONANCE STOPBAND WIDTH

Quadrupole imperfections of the linear lattice lead to a betatron tune shift as well as forming a finite bandwidth of resonances on the tune diagram. The tune shift and half-integer stopband width are determined correspondingly by the 0th and pth harmonics of quadrupole perturbations around the machine:

$$\Delta\nu_t = \frac{1}{4\pi}\sum_q \beta_q (\Delta k_1 L)_q$$
$$J_p = \frac{1}{2\pi}\left|\sum_q \beta_q (\Delta k_1 L)_q e^{-ip\phi_q}\right| \quad (3)$$

where p is close to 2ν, q runs over the lattice quadrupoles, β and $\phi = \frac{1}{\nu_0}\int_0^s \frac{ds}{\beta}$ are betatron amplitude and phase and $\Delta k_1 L = \Delta B' L/(B\rho)$ is the perturbed quadrupole focusing strength.

The way to control the resonance stopband width J_p is to act on the pth harmonic of $(\Delta k_1 L)_q$ while maintaining the 0th harmonic caused by the same $(\Delta k_1 L)_q$ equal to zero. Methods of minimizing the stopband width were presented in [11].

In the experiments we characterized the resonance stopband using the two ways of tune scans described above, resulting in the measured J_p of 0.016 with accuracy of about ±0.0025. We assume the beta-beat distribution along the ring as the sum of harmonic functions and calculate the r.m.s. beta-beat driven by random Gaussian distributed errors in the ring quadrupole settings as:

$$\langle\frac{\Delta\beta}{\beta}\rangle_{max} \approx \frac{\sqrt{M_q}}{2\sin(2\pi\nu)}\langle\beta\Delta k_1 L\rangle$$

where Mq is the total number of quadrupoles. Using the measured value of beta-beat (3%) we calculate the r.m.s perturbations in Mq=300 NSLS-II quadrupoles as $\langle\Delta k_1/k_1\rangle$=0.13%. From these perturbations we estimate the ½ resonance stopband width J_p via (3) as 0.015, which closely corresponds to our measurements.

To control the stopband width, defined in (3), we selected several quadrupoles separated by $n\cdot\pi + \pi/2$ in betatron phase advance $\varphi = \int_0^s \frac{ds}{\beta}$ and changed their strength by $(\Delta k_1 L)_q$ yielding the maximum change in the stopband width of $\frac{1}{2\pi}\sum_{q=1}^{N_q}\beta_q(\Delta k_1 L)_q$.

EXPERIMENTAL RESULTS

We carried out our experimental studies at the NSLS-II storage ring. The NSLS-II is a high-brightness synchrotron light source based upon a 3-GeV storage ring with a 30-cell double-bend-achromat lattice complimented by damping wigglers in order to reduce the emittance below 1 nm·rad [12]. In the following table we present the beam parameters of the NSLS-II storage ring relevant to our experiments.

Table 1: NSLS-II Storage Ring Beam Parameters

Vertical betatron tune	16.26…16.55
Revolution period, µsec	2.64
Synchrotron tune	0.00625
Damping time (x/y/z), msec	55.3/55.3/27.7
Vertical emittance, pm·rad	30
Energy spread, %	0.05

In the experiment, we stored a beam current of a few milliamperes, switched to the lattice with high ξ2 and then moved the betatron tune to a near half-integer reso-

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nance (ν0~16.47) by controlling non-dispersive quadrupoles. In order to modulate the off-energy tune in this experiment, we developed a method of rapid excitation of coherent beam energy oscillations ("RF jump" or "RF pinger", [13]). Turn-by-turn (TBT) beam transverse positions and beam relative intensity were measured with BPMs. Beam TBT energy oscillation was retrieved from the horizontal data of the BPMs located in the dispersion region.

The beta-beat along the ring at different tunes was retrieved from BPM TBT data to measure the stopband width. The beta beat for the nominal lattice was corrected to ~3% with stopband width at 0.016. We called these experimental conditions the "Small stopband" scenario.

With the same RF jump and transverse kicker settings we designed another experimental scenario in which the quadrupole strength was adjusted to expand the stopband width from 0.016 to 0.038, so that the beam tune stays within the stopband much longer during the RF jump. We called these experimental conditions the "Large stopband" scenario.

The measurement results are shown in Fig. 3 including traces of the vertical tune, vertical beam position and beam normalized intensity. Beam energy oscillation amplitude is about +/-1.4% (peak to peak), as retrieved from the horizontal beam position measured by BPMs. The tune modulation is calculated using Eq. (1) and presented in the upper plots. Different colors correspond to the different values of the initial tune ν_0. When the tune approaches the resonance, motion in the vertical plane exhibits the behavior typical for parametric resonance, i.e. modulation at the detuning frequency Δν, which is in the range between 20 and 120 turns for our experimental conditions.

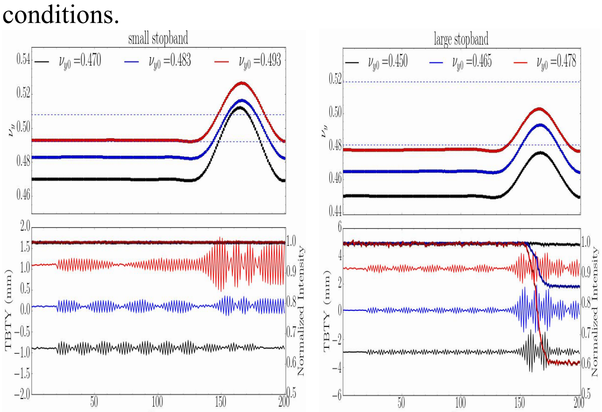

Figure 3: "Small stopband" and "Large stopband" case Turn-by-turn beam parameters (upper plot: calculated y-tunes with dashed lines indicating borders of the stopband, lower plot: BPM measured TBT y-position on left axis and normalized beam intensity on right axis) for the three separate experiments with different initial tunes.

In the left plot of Fig. 3, it includes the data with different initial vertical tunes: 0.470 (black), 0.483 (blue), 0.493 (red). With these initial conditions the beam takes approximately 11, 12 and 17 turns to cross the resonance. The difference in the oscillation amplitude after the first crossing is visible but in every case there is no beam loss. In the "Large stopband" scenario, as shown in lower plot of Fig. 3, there is no beam loss while the tune is outside of the resonance stopband, but when the beam is moving through the resonance for about 40 turns, particle loss occurs. The losses then repeat during subsequent synchrotron oscillations.

With Eq. (2) we estimate that the maximum number of turns the beam can spend within the "Small stopband" is about 25 and for the "Large stopband" it is about 49. We estimate the betatron amplitude growth [8] under our experimental conditions as a factor of 3.5 for the "Small stopband" and a factor of 350 for the "Large stopband". This large amplification factor for the "Large stopband" leads to significant beam loss as demonstrated by our experiments. Since about half of the beam intensity is lost after the first crossing of the stopband, the plots of vertical TBT data in this case are not representative of the actual beam betatron motion.

We note that we were able to study vertical TBT data corresponding to the first crossing of the resonance during the first half-period of the energy oscillation. Clear exponential-like growth of betatron motion is visible only during the first crossing. During the calculating few synchrotron oscillations the BPM TBT signal blurs due to the decoherence and filamentation of the beam as the particles are repetitively passing through the stopband.

CONCLUSIONS

In summary, we carried out a study focused on beam dynamics in a storage ring featuring a large chromatic tune footprint that can span across major resonances. We have shown that it is possible, both by design and by experiment, to achieve the storage ring conditions where the beam crosses the ½ resonance without particle loss. This can be accomplished if the stopband is narrow due to small residual field errors in the ring magnets and is further controlled by accurate cancellation of the harmful harmonic of the field errors around the ring. The combination of the small stopband width with a large magnitude of nonlinear chromaticity leads to the rapid crossing of the resonance, which does not cause loss of the particles as demonstrated by our experiments.

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