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ON WAKEFIELD IN DIELECTRIC WAVEGUIDE WITH SHALLOW CORRUGATION OF METALLIC WALL *

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Abstract

We study the radiation of a bunch moving along the axis of a circular corrugated waveguide filled with dielectric. It is assumed that Cherenkov effect takes place in the filling medium. We consider the "long-wave radiation" with wavelengths much larger than the corrugation period. The exact boundary conditions on the complicated periodic surface are replaced with the equivalent boundary conditions which should be fulfilled on the smooth surface. Analytical and numerical results for the mode frequencies and amplitudes are presented.

INTRODUCTION

One of conventional methods of microwave radiation generation is excitation of electromagnetic waves by a charged particle bunch moving in a periodic metallic waveguide. As a rule, researchers consider the range of wavelengths which are comparable to or less than the period of the structure (Smith-Purcell radiation). However, it is interesting as well principally different situation when the wavelengths under consideration significantly exceed the period of the structure [1–4]. In these cases the periodic conductive structure can be approximately described with help of so-called averaged boundary conditions for a grid waveguide [1] or equivalent boundary conditions (EBC) for a corrugated waveguide [2–5] (EBC are known also as Vainstein-Sivov conditions). These conditions should be fulfilled on the smooth surface instead the real waveguide wall.

The problems with corrugated waveguide were analyzed earlier, for example, in [2, 3] where authors examined the electromagnetic field of a charge moving along the axis of an empty round waveguide with a finely corrugated wall. The similar problem was also researched in [4], where we investigated some important aspects which were not noted earlier. In particular, in [4] the dependence of the radiation properties on the charge velocity has been analyzed. Underline that the paper [4] includes the comparison of theoretical results and results of COMSOL Multiphysics simulations as well. We have demonstrated that the EBC are applicable even for situation when the excited wavelength is more than the structure period in 10 times only.

Here we analyze an analogous problem for the waveguide with corrugated wall and dielectric filling under condition that Cherenkov effect takes place. Due to this fact the radiation differs significantly from the one in the vacuum structure. At the same time, the radiation has essential distinctions from the one in the dielectric waveguide with smooth wall [6–8].

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EQUIVALENT BOUNDARY CONDITIONS

We consider a circular waveguide having a wall with rectangular corrugation (Fig. 1). The waveguide is filled with a nondispersive isotropic dielectric having permittivity ε, permeability μ, and refractive index $n = \sqrt{\varepsilon\mu}$. The period of corrugation d and the depth d_3 are assumed to be much less than the waveguide radius a and the wavelengths under consideration λ:

$$d \ll a, \quad d_3 \ll a, \quad d \ll \lambda, \quad d_3 \ll \lambda. \tag{1}$$

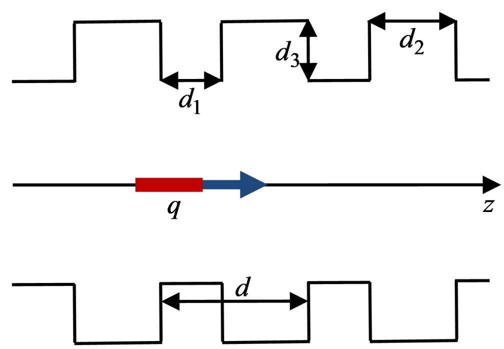

Figure 1: Longitudinal section of the waveguide.

The equivalent boundary conditions for Fourier-transforms of the electric and magnetic fields have the following view [5]:

$$E_{\omega z}\big|_{r=a} = \eta^m H_{\omega\varphi}, \quad E_{\omega\varphi}\big|_{r=a} = \eta^e H_{\omega z}, \tag{2}$$

where η^m and η^e are the "impedances" which are imaginary for perfectly conductive structures (we use cylindrical coordinates r, φ, z). In the case of the structure shown in Fig. 1, we have [5]

$$\eta^m = \frac{i\omega n}{c}\left(\frac{d_2 d_3}{d} - \frac{\delta\alpha_z^2}{1-\alpha_\varphi^2}\right), \ \eta^e = -\frac{i\omega n}{c}\delta\left(1-\alpha_\varphi^2\right), \tag{3}$$

where d_2 is the width of groove, and α_φ, α_z are directing cosines of the incident wave with respect to $\vec{e}_\varphi$ and $\vec{e}_z$ accordingly: $\alpha_\varphi = k_{0\varphi}/k_0$, $\alpha_z = k_{0z}/k_0$ ($\vec{k}_0$ is the wave vector of the incident wave, $k_0 = \omega n/c$). The parameter δ is determined with use of certain system of transcendent equations [4, 5]. In the case of the diaphragm system ($d_1 \to 0$), the formula for δ is known [5]:

$$\delta = d_3 - \frac{d}{\pi}\ln\left[\cosh\left(\frac{\pi d_3}{d}\right)\right]. \tag{4}$$

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WAKEFIELD

We consider the field of the charged particle bunch moving along the waveguide axis (z-axis) with the velocity $\vec{V} = c\beta\vec{e}_z$. It is assumed that the bunch thickness is negligible, and the charge density is $\rho = q\delta(x)\delta(y)f(z - Vt)$ where $f(z - Vt)$ is the longitudinal distribution of the charge. In this case $\alpha_\varphi = 0$, $\alpha_z = 1/(n\beta)$, only symmetrical TM-field is excited, and we can use only the 1st condition (2).

It is assumed that $n\beta > 1$ i.e. Cherenkov radiation is generated. Omitting the transformations, we give only the main results for longitudinal component of the electric field. The longitudinal component of the total field has the following form:

$$E_z = -\frac{q\left(n^2\beta^2 - 1\right)}{2c^2\beta^2\varepsilon}\int\limits_{-\infty}^{\infty}\omega\tilde{f}(\omega)\times \\ \times\left[H_0^{(1)}(sr) + RJ_0(sr)\right]\exp\left(\frac{i\omega\zeta}{V}\right)d\omega, \quad (5)$$

where $\zeta = z - Vt$, $s = \frac{\omega}{V}\sqrt{n^2\beta^2 - 1}$,

$$R = -\frac{H_0^{(1)}(sa) - g\cdot sa\cdot H_1^{(1)}(sa)}{J_0(sa) - g\cdot sa\cdot J_1(sa)}, \quad (6)$$

$$g = \frac{\varepsilon\eta}{\alpha^2},\ \ \eta = \frac{n}{a}\left(\frac{d_2 d_3}{d} - \frac{\delta}{n^2\beta^2}\right),\ \ \alpha = \frac{\sqrt{n^2\beta^2 - 1}}{\beta}. \quad (7)$$

Here $\tilde{f}(\omega)$ is the normalised Fourier transform of the longitudinal distribution of the charge. For example, for Gaussian bunch

$$f(\varsigma) = \frac{\exp\left(\frac{-\zeta^2}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma},\ \ \tilde{f}(\omega) = \exp\left(-\frac{\omega^2\sigma^2}{2\beta^2c^2}\right). \quad (8)$$

The mode frequencies ω_m are determined by the dispersion equation

$$J_0(sa) = gsaJ_1(sa). \quad (9)$$

Calculation of the contribution of these poles in (5) gives the following expression for the wave field (so called "wakefield") which exists behind the bunch:

$$E_z^W = \sum_{m=1}^{\infty} E_{0mz}\cos\left(\Omega_m\frac{\zeta}{a\beta}\right)\Theta(-\zeta), \quad (10)$$

$$E_{0mz} = \tilde{f}(\omega_m)W_m J_0\left(\alpha\,\Omega_m\frac{r}{a}\right), \quad (11)$$

$$W_m = -\frac{2\pi q\alpha^2\Omega_m\left[\alpha N_0(\alpha\Omega_m) - \varepsilon\eta\Omega_m N_1(\alpha\Omega_m)\right]}{\varepsilon a^2 J_1(\alpha\Omega_m)\left[\alpha^2 + (\varepsilon\eta\Omega_m)^2\right]}, \quad (12)$$

where $\Omega_m = a\omega_m/c$, $J_k(\xi)$ and $N_k(\xi)$ are Bessel and Neumann functions accordingly, $\Theta(-\zeta)$ is Heaviside step function. Naturally, we have the infinite series of propagating waveguide modes as in the smooth waveguide with dielectric filling.

It is interesting to compare the field in the corrugated dielectric waveguide with the field $\vec{E}^{(0)}$ generated in the ordinary smooth waveguide with the same filling [6,7] (the values related to a smooth waveguide are designated with the superscript (0)). Calculations show that $g > 0$ for the structure under consideration. As one can see from (7), if $n\beta$ is not close to 1 then $g \ll 1$. In this case one can obtain

$$\Omega_m \approx \Omega_m^{(0)}(1 - g), \quad (13)$$

where $\Omega_m^{(0)} \approx \mu_m^{(0)}/\alpha$ are the dimensionless mode frequencies in the smooth waveguide ($\mu_m^{(0)}$ are zeros of the Bessel function $J_0(x)$). The relation of the z-component of the mode amplitudes on the waveguide axis is

$$E_{0mz}/E_{0mz}^{(0)}\Big|_{r=0} \approx (1 - 2g)\frac{\tilde{f}(c\Omega_m/a)}{\tilde{f}(c\Omega_m^{(0)}/a)}. \quad (14)$$

Thus, in the case $g \ll 1$, the corrugation decreases the mode frequencies. The corrugation influence on the mode amplitude can be different. On the one hand, the factor $(1 - 2g)$ decreases U_m, but the second factor in (14) increases this relation because of decreasing the frequency.

For $g \gg 1$ one can obtain the following asymptotic for frequencies:

$$\Omega_1 \approx \sqrt{2/(g\alpha^2)}, \\ \Omega_m \approx \frac{\mu_{m-1}^{(1)}}{\alpha} + \frac{1}{g\alpha\mu_{m-1}^{(1)}} \quad (m = 2, 3, 4...). \quad (15)$$

Here $\mu_m^{(1)}$ are the zeros of function $J_1(x)$. In this case the mode amplitudes can be larger than amplitudes of the modes in the smooth waveguide even for the point charge. However, the case $g \gg 1$ can be realized for $n\beta \approx 1$ only.

Figures 2, 3 demonstrate dependencies of the mode frequencies and amplitudes on the bunch velocity and the dielectric permittivity. Calculations show that the diaphragm system ($d_1 \to 0$) affects the wave field in the strongest way, therefore we present results for this structure. One can see that the corrugation decreases the mode frequencies. Influence of corrugation on the amplitudes is more complex. If the bunch is the point charge ($\sigma = 0$) and $n\beta$ is not close to 1 then all amplitudes are less than in the smooth waveguide. But for the real bunch with $\sigma \neq 0$ some modes can be larger than in the smooth waveguide, as one can see from Fig. 3.

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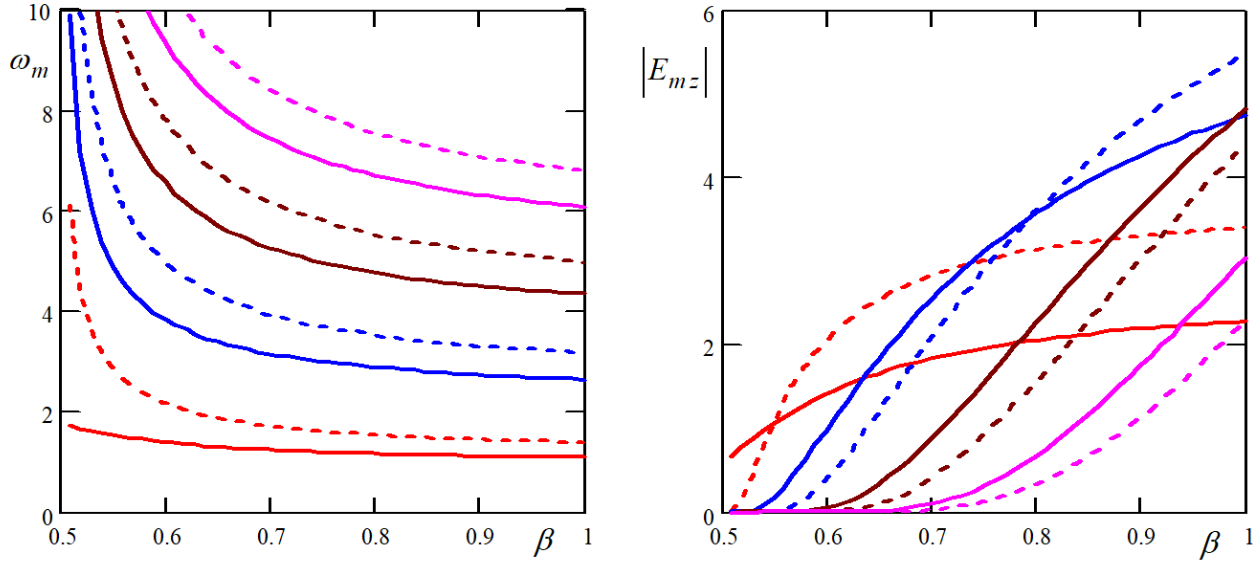

Figure 2: The mode frequencies (left; in unities c/a) and amplitudes of z-components on the waveguide axis (right; in unities q/a^2) depending on the bunch velocity. Solid curves refer to the corrugated waveguide, and dotted curves refer to the smooth waveguide. Red, blue, brown, and magenta curves refer to the 1st , 2nd, 3rd, and 4th modes, respectively. Parameters: $d = d_3 = d_2 = 0.1a$, $d_1 = 0$ (diaphragms), $\delta \approx 0.022a$, $\varepsilon = 4$, $\mu = 1$, $\sigma = 0.3a$.

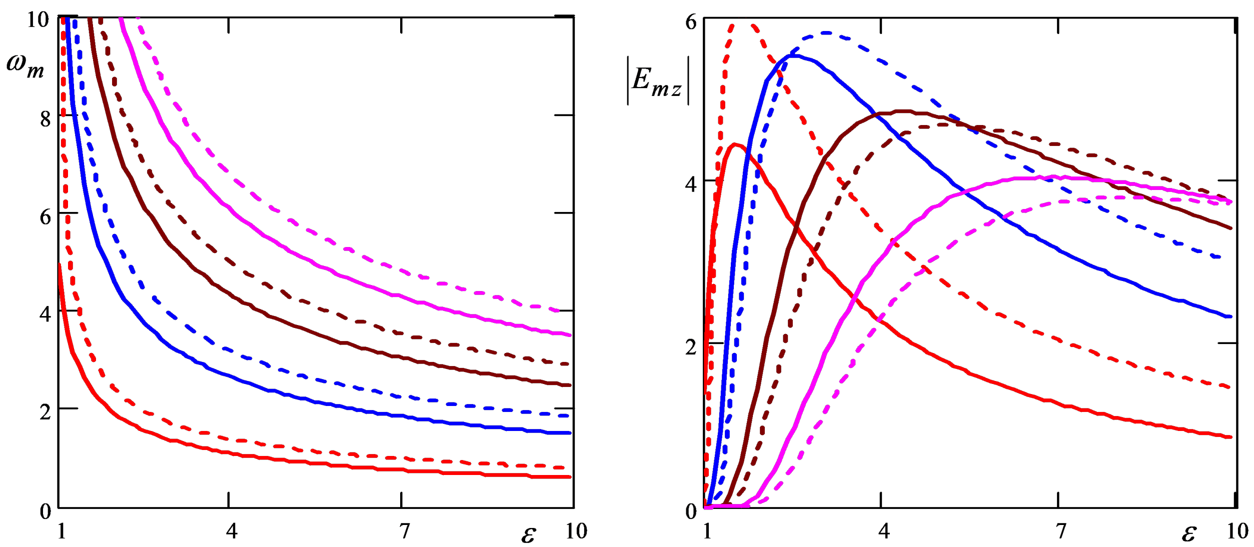

Figure 3: The mode frequencies (left; in unities c/a) and amplitudes of z-components on the waveguide axis (right; in unities q/a^2) depending on the permittivity in the case $\beta = 1$. Other parameters are the same as in Fig.2.

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