**INTRODUCTION**

Beam-beam effects are of great importance for all future machines which aim at running with high energies and luminosities. It is well known that the ring betatron tunes should be carefully chosen to avoid beam-beam instabilities, which may result in transverse beam size blowup thus limiting the performance of the machine [1].

Beam-beam simulations with beamstrahlung have been performed for the FCC-ee machine at the initial beam energy for \(t\bar{t}\) running of 175 GeV (the final beam energy in the \(t\bar{t}\) operation mode is 182.5 GeV [2]), using the parameters of Table 1. Beam-beam simulations are carried out by a weak-strong beam-beam model (BBWS) [3] implemented in SAD [4]. Earlier tracking simulations were performed in a coupled lattice by introducing vertical random misalignments for all the ring sextupoles so as to achieve the design coupling value of 0.2%. In these simulations, a vertical emittance blowup was observed, both with and also without the beam-beam element. Such blowup in the absence of beam collisions, is predicted to be due to chromatic effects in electron storage rings [5]. In addition, the blowup was observed to depend on the random-number generator seed used to misalign the sextupoles [6, 7]. In this paper, we report and discuss results from beam-beam tracking in a more realistic \(t\bar{t}\) lattice after optics correction and emittance tuning, and the effect of a vertical beam-beam offset on the emittance blowup.

**TRACKING IN THE \(t\bar{t}\) LATTICE**

**Emittance Tuning**

The low-emittance tuning for FCC-ee lattices has been studied thoroughly in the last few years [8–10]. So far, these studies confirm the possibility of achieving an adequately low vertical emittance for the \(t\bar{t}\) lattice in the presence of different quantitative and qualitative machine errors and after performing a series of iterative optics corrections [10]. The machine errors follow a Gaussian distribution, where quadrupoles (including the final-focus quadrupoles) and sextupoles are misaligned in both horizontal and vertical planes (RMS=100 \(\mu \text{m}\)), and quadrupoles are tilted (RMS=100 \(\mu \text{rad}\)). The optics corrections applied consist of orbit, \(\beta\) beating, dispersions, coupling, tune, and chromaticity corrections.

So far only a single random generator seed has been considered from among the successful seeds used for the optics corrections and emittance tuning, in our attempt to study the characteristics and physical origin of the beam blowup. Simulations with a larger number of seeds (also including various errors/corrections combinations) are planned for the future.

**Tracking Without Beam-beam Element**

The low-emittance tuning for the chosen seed results in an invariant vertical equilibrium emittance of 0.08 pm (about 34 times lower than the design value of \(\epsilon_y = 2.7 \text{ pm}\)), as computed by SAD. The vertical dispersion and coupling parameters (R1, R2, R3 and R4) [7] around the ring are presented in Figs. 1 and 2 respectively.
$10^4$ macroparticles representing the beam are generated at one of the two Interaction Points (IPs) and then tracked through the corrected lattice for 500 turns, which corresponds to about 10 transverse radiation-damping periods at a beam energy of 175 GeV. The projected vertical emittance of the beam at the IP increases by about a factor of four to 0.3 pm, as is illustrated in Fig. 3.

**Tracking With Beam-beam Collisions**

We next perform a weak-strong beam-beam simulation. To this end we insert two strong beam-beam elements at the two IPs. Tracking in the presence of the beam-beam element dramatically blows up the vertical emittance to 25 pm; (see Fig. 4).

To understand the mechanism causing this huge blowup, and its dependence on residual optics errors, we investigated a number of points.

**Waist scan** A shift in the waist position will lead to larger beam-beam tune shift and beam-beam induced tune modulation. To investigate this effect, a waist scan was performed by shifting the strong beam at both IPs longitudinally in the range from $-4$ mm to $+4$ mm (several times the IP beta function). Figure 5 shows that shifting the waist of the strong beam enhances the blowup of the weak beam, and that the initial waist location was already the optimum.

**IP Offset Scan** For the corrected lattice we observe a residual position offset at the IP of $-0.5$ µm and $-1.5$ µm in the horizontal and vertical planes, respectively. While the horizontal offset is negligible compared with the horizontal...
beam size at the IP ($\sigma^*_{x} = 37 \, \mu m$), the vertical offset is enormous compared with the vertical beam size at the IP ($\sigma^*_{y} = 66 \, nm$). It turns out that this vertical offset is the primary cause of the emittance blowup observed with beam-beam elements.

To correct for this offset, we again track the weak beam in the presence of the beam-beam element, but this time on every turn we calculate the vertical offset of the weak beam at the IP just prior to the beam-beam elements. Figure 6 reveals that the mean values of the vertical offsets are pretty similar at both IPs, in the order of $\pm 1 \, \mu m$. The correction for such offsets is done by displacing the strong beam at both IPs by the corresponding mean values from Fig. 6. This correction of vertical offset results in a much smaller vertical emittance blowup of the weak beam, which shrinks from 25 pm to $\approx 0.8 \, pm$ (Fig. 7). Furthermore, matching of the horizontal and vertical angles at the IP was performed in the same way, with no significant improvement of the blowup.

**Achieving Design Emittance Including Beam-beam Blowup**  
In view of such a beam-beam induced emittance blowup, we may need to define an upper bound on the high-est vertical emittance that should be achieved by emittance tuning without beam-beam collisions. As a first step towards setting such a limit, we studied the beam-beam blowup for different values of invariant vertical emittance. The vertical emittance of the ring was varied by changing the strength of skew quadrupole elements. It has been observed that an invariant emittance of 0.4 pm is enough to achieve the design emittance of 2.7 pm in the presence of beam-beam blowup (Fig. 8). This value of 0.4 pm is so far only indicative and not yet definitive, since it needs to be confirmed by further simulations with other optics random seeds.

**CONCLUSIONS**

Beam-beam simulations have been performed in the FCC-ee for a lattice with errors after optics correction and emittance tuning. Residual beam-beam offsets at the collision point can cause a large vertical emittance blowup. If these offsets are corrected, the beam-beam induced emittance blowup is much reduced, but still a factor of a few. Consequently, the beam-beam blowup will impose limitations on low emittance tuning which need to be studied carefully and in greater detail in the future.

**ACKNOWLEDGEMENTS**

The authors thank S. Aumon, M. Benedikt, T.K. Charles, K. Ohmi, D. Shatilov, T. Tydecks, D. Zhou and the entire FCC-ee Collaboration Team for encouraging the research, useful discussions, and suggestions.
REFERENCES


