

CONTROL OF THE NONLINEAR DYNAMICS FOR MEDIUM ENERGY SYNCHROTRON LIGHT SOURCES

J. Bengtsson, A. Streun¹, B. Singh, H. Ghasem, R. Bartolini,
Diamond Light Source, Oxfordshire, U.K.
¹Paul Scherrer Institut, Villigen, Switzerland

Abstract

MAX-IV has introduced a paradigm shift in the design philosophy for the “Engineering-Science” in the quest for a diffraction limited Synchrotron Light Source. Similarly, SLS-2 has introduced a systematic method for controlling the Linear Optics beyond some 20 years of TME inspired paper designs; by introducing Reverse Bends to disentangle dispersion and focusing, which enables Longitudinal Gradient Bends to efficiently reduce the emittance. Correspondingly, here we outline a systematic approach for how to control the Nonlinear Dynamics for these Systems, by a method that was pioneered for the conceptual design of the Swiss Light Source in the mid-1990s; subsequently benchmarked and validated by the commissioning. Simply put, for predictable results.

INTRODUCTION

Control theory for nonlinear systems of ordinary differential equations remains an open ended research level problem; largely, because a reductionist approach is ineffective. In other words, an incremental approach for e.g. 1 Degree-of-Freedom considers what force to add to the R.H.S. of Hill’s Equation

$$x'' + \frac{K(s)}{1 + \delta}x = f(x) \quad (1)$$

to obtain the required performance for the System; after Linear Stability has been established by Linear Optics Design (aka Linear Control Theory for Engineers).

A Systems Approach, e.g. Optimal Control Theory, instead tries to figure out how to tailor both $K(s)$ and $f(x)$ for an Optimal Design, and it becomes an Art, aka Engineering-Science [1]. The Second-Order-Achromat concept dates back to spectrometer design [2] and was also applied for the design of the first dedicated Synchrotron Light Sources leading to Chasman-Green type lattices [3] comprising of: 2nd-Order-Achromats interspaced by Straight-Sections.

Over time various incremental improvements have been made. For the Linear Optics design, the Theoretical Minimum Emittance (TME) Cell [4, 5] became a guiding principle bordering on orthodoxy but exceptions do exist; i.e., ignoring TME and instead focusing on/pondering the entire parameter space up-front as an Engineering-Science Optimal Problem for a User Facility, e.g. MAX-IV [6, 7] and now SLS-2 [8–10]. As a side-effect, the Linear Optics design for the former is actually quite relaxed, i.e., can be pushed over time [11]. Among the incremental improvements, SOLEIL introduced a (high dispersion) Mid-Straight inside the Double-Bend-Achromat (D-BA), for e.g.

Three-Pole-Wigglers [12]; which was adapted for NSLS-II as well [27]. As a refinement, DIAMOND has introduced a Low Dispersion Mid-Straight, first by an incremental approach for the Facility [13, 14]; and now as a requirement for DIAMOND-II [15].

On the other hand, while Gröbner has introduced Lie Series [16] to Celestial Mechanics in the 1960s [17, 18], i.e., a Recursive (can be automated by Computer Algebra) mathematical formalism, it is only in the early 1990s that it has been applied for guiding particle accelerator design, e.g. ref. [19–21]. Similarly, the introduction of Frequency Map Analysis [22, 23] has become quite fashionable as well. In other words, while a very useful quantitative diagnostic tool, it does not replace 6D Phase Space Tracking for an Ensemble of Accelerators, i.e., statistics for the impact of Engineering Tolerances on the System. Besides, by definition, it is for 2.5 Degrees-of-Freedom (aka Adiabatic Approximation); i.e., without RF Cavity.

In addition, Genetic Algorithms from Computer Science have been used to solve the related Optimal Problems numerically, first for accelerator components [24, 25] and, eventually, for the ring as well [26]. For the latter case though, mainly by a brute force reductionist numerical approach; a case study/Lessons Learnt is provided below. However, e.g. APS-U has put it to good use: first by adopting the MAX-IV approach (Higher-Order-Achromat) and eventually ESRF-U (-I Transformer). The two basic strategies for controlling the nonlinear dynamics for Synchrotrons, originated in the 1970s, are outlined below.

HIGHER ORDER ACHROMATS

By starting from/standing on Best Practices for Spectrometer Design and Chasman-Green lattices in the 1970s, i.e., Double-Bend-2nd-Order-Achromats (D-BAs) interspaced with Straight Sections, it is conceptually straightforward to generalize to:

- Multi-Bend-Achromats & Straight Sections; to reduce the horizontal emittance,
- and Higher-Order-Achromats; to improve control of the nonlinear dynamics

which we will refer to as M-BAs for short. The latter is typically done in hindsight, i.e., after a brute-force, numerical approach has been unable of providing a satisfactory solution when drilling down towards an Engineering Design; eventually, prompting or inspiring a revisit of:

- First Principles,
- a Systems Approach,
- and Hamiltonian Dynamics

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2018). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI.

often requiring a Re-Design of an ad hoc Conceptual Linear Optics Design; if not a re-baselining of the entire project [21, 27].

CONTROL OF NONLINEAR DYNAMICS

The Lattice Designer is dealing with a Periodic (essentially) Hamiltonian System:

$$H(s) = H_2(s) + \alpha V(s), \quad H(s + C) = H(s) \quad (2)$$

where C is the circumference, H_2 the quadratic Hamiltonian (Linear Optics) and αV the Nonlinear Terms; for which α is not small.

To control the nonlinear dynamics for synchrotrons, when chromatic sextupoles are introduced for linear chromatic control, there are two basic strategies; both based on symmetry:

1. -I Transformer: introduce sextupole pairs separated by $n \cdot \pi$ phase advance in both planes e.g. [28].
2. Higher-Order-Achromat [29]: introduce a Unit Cell, repeat it four or more times to generate a super period, and adjust the total phase advance to $n \cdot 2\pi$ in both planes.

The first approach is Standard Practice for Collider Design, e.g. ref. [28], and the second, either, by a systematic approach [30] or numerical optimization by a random search, e.g. ref. [31], is implicit for high periodicity/performance lattices. The latter is typically pursued by an ad hoc partitioning of the parameter space, e.g. first optimize $H_2(s)$, then $\alpha V(s)$, and trial-and-error approach towards some local optimum. In conclusion, an incremental approach.

Application to SLS

For the SLS conceptual design, both strategies were considered [30]. However, with a circumference limited to ~ 300 m the first approach appeared to be “academic” (interesting but not that useful). Besides, the strategy only works on-momentum, i.e., it systematically drives the Lie Generators: h_{20001} and h_{00201} ; which generate 2nd order chromaticity. Still, the method has been used for ESRF-U, a High-Energy Synchrotron Light course: essentially a hybridization of MAX-IV and SuperB (a discontinued e^+e^- collider [32]). Interestingly, the strategy has been adopted for not only APS-U (High Energy) but also ALS-U (Low Energy). Nota Bene: Touschek Life Time scales with γ^3 .

Application to NSLS-II

The 2nd strategy was applied for the re-baselining of the Lattice Design for the NSLS-II CDR [27, 33]. Also, guidelines for the Linear Optics and Engineering Tolerances, and (transparent) Integration of Insertion Devices (IDs), with Local Current Strip Correction for the APPLE-II devices, for the Phase-I and II IDs have been provided; based on systematic benchmarks of the System by numerical simulations [34–38]. However, as a “result” of a subsequent ad hoc design change of the sextupole scheme for the PDR [39], the requirement for the off-momentum dynamic aperture

eventually had to be re-baselined/downgraded from 3% to 2.5% for the PDR [40]. Hence, since the Touschek life time scales roughly with the cube, the expected Beam Life time at full current is expected to be significantly less for NSLS-II vs. MAX-IV [6, 7].

Application to SLS-2

The Linear Optics Design for SLS-2 has avoided the TME Cell limit by introducing Reverse and Longitudinal Gradient Bends [8]: to transcend the current state-of-the-arts (MAX-IV). Besides, to control the nonlinear dynamics, the Ratchet Wall will be tweaked to enable the implementation of a 12 vs 3-fold Seven-BA. Control of the nonlinear dynamics for the preliminary Conceptual Design was based on numerical optimizations with MOGA (Multi Objective Genetic Algorithm) [31]. However, some detailed benchmarking revealed that the multipole scheme and obtained solution are not Robust. It goes to show that care is required to obtain practical results from advanced numerical algorithms & methods.

In fact, given the magnitude of the Tune Footprint see Figs. 1-2, the impact of Mechanical Misalignments and Magnetic Multipole Errors on the Frequency Maps for a “Realistic Lattice” is not surprising. To resolve the issue, a Higher-Order-Achromat was implemented, see Figs. 3-4 and ref. [41] for the details.

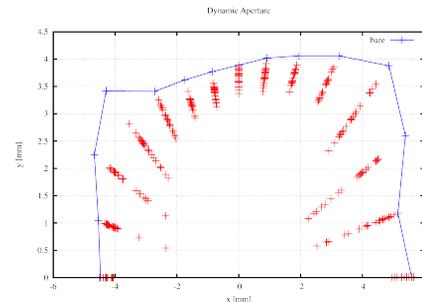


Figure 1: Dynamic Aperture with RF-Cavity for MOGA Optimized SLS-2 Lattice (with Eng. Tol.): Not Robust.

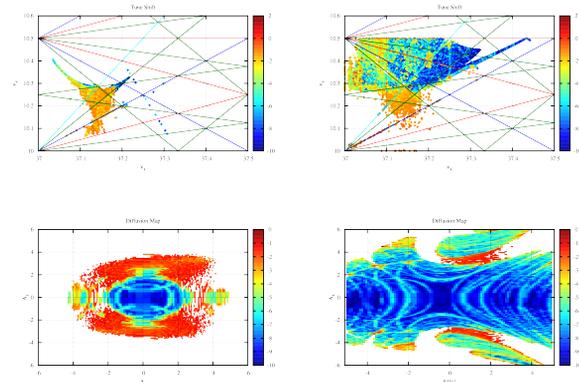


Figure 2: On and Off-Momentum Diffusion Maps for MOGA Optimized SLS-2 Lattice (with Eng. Tol.): Not Robust.

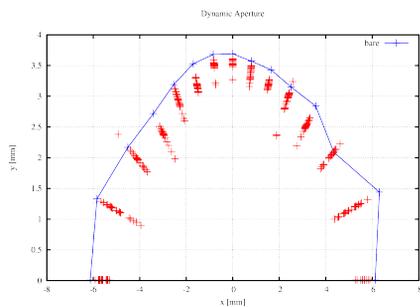


Figure 3: Dynamic Aperture with RF-Cavity for Robust Design of SLS-2 Lattice (with Eng. Tol.).

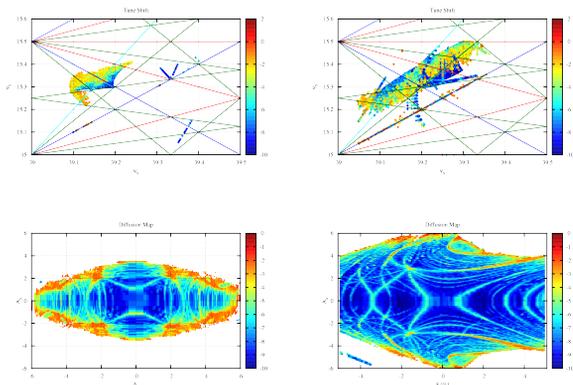


Figure 4: On and Off-Momentum Frequency Maps for Robust Design of SLS-2 Lattice (with Engineering Tolerances).

Application to DIAMOND-II

The prototype lattice for DIAMOND-II has been inspired by ESRF-U: a Hybrid-6B with a Low Dispersion Mid-Straight Cell (with transverse and longitudinal gradients) and the -I Transformer strategy outlined above [15]. However, given its limitations, while it might be adequate for High-Energy Facilities, the Linear Optics Design has been re-designed into a Higher-Order-Achromat, see ref. [42]. Interestingly, similar exploratory work has been pursued for ALS-U as well [43].

An attempt to introduce Reverse & Longitudinal Gradient Bends to the DIAMOND-II prototype lattice last year turned out to be nontrivial; due to the three kinds of Straight Sections: Long, Low-Dispersion Mid-Straight, and Standard. So, for a short-cut, we instead adopted the SLS-2 Unit and Matching Cells and scaled and matched them to fit in the DIAMOND tunnel; leading to a 6-Reverse-Bend-BA (6RB-BA) [44]. In particular, with Achromatic Mid-Straights and reduced horizontal emittance; for on-axis injection. Clearly, for off-axis injection e.g. a 4RB-BA could be considered.

The Chasman-Green structure for e.g. the 6RB-BA is outlined in Figs. 7 and 8 in ref. [42]. The global structure of the tunnel has 24 Achromats & Straights, which is reduced to a 6×4 -fold structure by Long vs. Standard Straights; by tweaking the Matching Sections for the Unit Cells. Simply

put: 6 Superperiods each one with 4 Cells. A generalized 8-Cell Higher-Order-Achromat is implemented by introducing 2+1 Chromatic Sextupole Families for the Unit Cells, an extra family is needed due to the Low-Dispersion Mid-Straight, and the Cell Tune is chosen to e.g. [23/8, 19/16]; to cancel all the Resonance Driving Terms to 2nd order in the sextupole strength over two Superperiods. A proof-of-concept for the Bare Lattice (without engineering tolerances; snapshot of work-in-progress) is shown in Figs. 5-6.

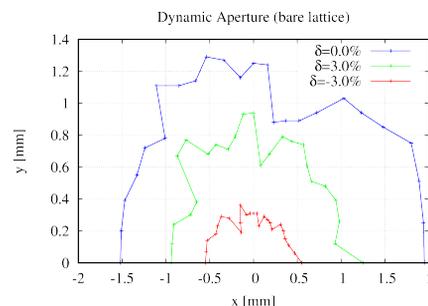


Figure 5: On and Off-Momentum Dynamic Aperture with RF-Cavity for 6RB-BA (Bare Lattice).

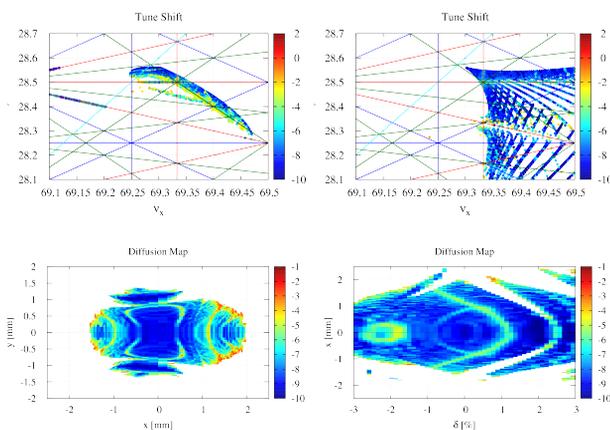


Figure 6: On and Off-Momentum Frequency Maps for 6RB-BA (Bare Lattice).

CONCLUSIONS

While Higher-Order-Achromats and Chasman-Green style lattices have been pursued for dedicated Synchrotron Light Sources since the 1970s, they still have a strong following. Besides, at the time of the NSLS-II Conceptual Design, the general perception in the field was that no further major improvements could be made for these Instruments. However, then came MAX-IV, a Paradigm Shift, by a bold Engineering-Science approach and, yet, with Predictable Results. Hence, finally there is light at the end of the tunnel: the quest for a Diffraction Limited Facility is now within the horizon; e.g. $\epsilon_x \sim \lambda/4\pi = 8 \text{ pm} \cdot \text{rad} @ 1 \text{ \AA} = 12.4 \text{ keV}$.

REFERENCES

- [1] S. Corneliussen, "From engineering science to big science", NASA SP-4119, Section 4, 1998, <https://history.nasa.gov/SP-4219/Contents.html>
- [2] K. Brown, "A first- and second-order matrix theory for the design of beam transport systems and charged particle spectrometers" SLAC-75, Section 5-6, (1982), <https://cds.cern.ch/record/283218>
- [3] R. Chasman, G. Green, and E. Rowe, "Preliminary design of a dedicated synchrotron radiation facility", in Proc. PAC'75, pp. 1765-7.
- [4] M. Sommer, "Optimization of the emittance of electrons (positrons) storage rings", LAL/RT/83-15 (1983), http://www.iaea.org/inis/collection/NCLCollectionStore/_Public/15/049/15049431.pdf?r=1
- [5] L. Teng, "Minimum emittance lattice for synchrotron radiation storage ring", FNAL/TM-1269 (1984), http://www.aps.anl.gov/Science/Publications/lnotes/content/files/aps_1417575.pdf
- [6] MAX IV Detailed Design Report (2010), <https://www.maxiv.lu.se/accelerators-beamlines/accelerators/accelerator-documentation/max-iv-ddr/>
- [7] S. Leemann *et al.*, "Beam dynamics and expected performance of Sweden's new storage-ring light source: MAX IV", Phys. Rev. ST-AB **12**, 120701 (2009), <https://doi.org/10.1103/PhysRevSTAB.12.120701>
- [8] A. Streun, "The anti-bend cell for ultralow emittance storage ring lattices", NIM A737, pp. 148-154 (2014), <https://doi.org/10.1016/j.nima.2013.11.064>
- [9] A. Streun and A. Wrulich, "Compact low emittance light sources based on longitudinal gradient bending magnets", NIM A770, pp. 98-112 (2015), <http://dx.doi.org/10.1016/j.nima.2014.10.002>
- [10] A. Streun (Ed.), "SLS-2 conceptual design report", Paul Scherrer Institut, Villigen, Switzerland, Rep. 17-03, Dec 2017., http://www.lib4ri.ch/archive/nebis/PSI-Berichte_000478272/PSI-Bericht_17-03.pdf
- [11] S. Leemann and M. Eriksson, "MAX IV emittance reduction and brightness improvement", in Proc. IPAC'14, paper TUPRI026.
- [12] A. Nadji *et al.* "A modified lattice for SOLEIL with a larger number of straight sections", 25th Symposium on Intermediate Energy Light Sources Shanghai, China, Sep 24-26, 2001.
- [13] R.P. Walker *et al.*, "The double-double bend achromat (DDBA) lattice modification for the Diamond storage ring", in Proc. IPAC'14, paper MOPRO103.
- [14] I. Martin *et al.*, "Electron beam commissioning of the DDBA modification to the DIAMOND storage ring", in Proc. IPAC'17, paper WEPAB095.
- [15] A. Alekou *et al.*, "Study of a double triple bend achromat (DTBA) lattice for a 3 GeV light source", in Proc. IPAC'16, paper WEPOW044.
- [16] S. Lie "Theorie der Transformationsgruppen, 1. Abschnitt", (Teubner, Leipzig 1888), Engl. Transl. J. Merkel arXiv:1003.3202 (2010), <https://arxiv.org/abs/1003.3202>
- [17] F. Cap, W. Gröbner, and P. Lesky, "The astronomical n-body problem with time-dependent forces", Acta Physica Austriaca **15**, pp. 213-216 (1962).
- [18] F. Cap, W. Gröbner, and J. Weil, "Soft landing on the moon with fuel minimization", Scientific Report 16 (1967).
- [19] F. Neri, "Lie algebras and canonical integration", Unpublished manuscript, Maryland (1987).
- [20] R. Ruth and E. Forest, "Fourth-order symplectic integration", Physica D **43**, pp. 105-117 (1990), [https://doi.org/10.1016/0167-2789\(90\)90019-L](https://doi.org/10.1016/0167-2789(90)90019-L)
- [21] J. Bengtsson and J. Irwin, "Analytical calculations of smear and tune shift", SSC-232 (1990), <http://lss.fnal.gov/archive/other/ssc/ssc-232.pdf>
- [22] J. Laskar, "The chaotic motion of the solar system: a numerical estimate of the size of the chaotic zones", Icarus **88**, pp. 266-291 (1990), <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.464.3632&rep=rep1&type=pdf>
- [23] L. Nadolski, "Application of frequency map analysis to the study of the SOLEIL and ESRF beam dynamics", in Proc. EPAC'00, paper TUP2A09.
- [24] I. Bazarov *et al.*, "Multivariate optimization of a high brightness DC gun photoinjector", PR ST-AB **8**, 034202 (2005), <https://doi.org/10.1103/PhysRevSTAB.8.034202>
- [25] F. Briquez *et al.*, "Status of the SOLEIL insertion devices", in Proc. EPAC'06, paper THPLS118.
- [26] V. Sajaev *et al.*, "Multi-objective optimization of a lattice for potential upgrade of the Advanced Photon Source", in Proc. PAC'11, paper THP125.
- [27] NSLS-II CDR, 2006, <https://www.bnl.gov/ns1s2/project/CDR/>
- [28] "Design concept for a 100 GeV e^+e^- storage ring (LEP)", CERN 77-14 (1977), <http://dx.doi.org/10.5170/CERN-1977-014>
- [29] D. Kaltchev *et al.*, "Lattice studies for a high-brightness light source", in Proc. PAC'95, paper FAB14.
- [30] J. Bengtsson, "The sextupole scheme for the Swiss Light Source (SLS): an analytic approach", SLS 9/97 (1997), <http://slsbd.web.psi.ch/pub/slsnotes/sls0997.pdf>
- [31] M. Ehrlichman, "Genetic algorithm for chromaticity correction in diffraction limited storage rings", PR ST-AB **19**, 044001 (2016), <https://doi.org/10.1103/PhysRevAccelBeams.19.044001>
- [32] "SuperB: A high-luminosity asymmetric e^+e^- super flavour factory", SLAC-R-856 (2007), <http://slac.stanford.edu/pubs/slacreports/reports16/slac-r-856.pdf>
- [33] J. Bengtsson, "NLSL-II: control of dynamic aperture", BNL-81770-2008-IR, <https://www.bnl.gov/isd/documents/43740.pdf>
- [34] J. Bengtsson and I. Pinayev, "NSLS-II: control of vertical beam size", NSLS-II Tech Note 7 (2007).
- [35] J. Bengtsson, "NSLS-II: robust dynamic aperture", NSLS-II Tech Note 8 (2006).
- [36] B. Nash and J. Bengtsson, "Effects of magnetic multipole errors on dynamic aperture for NSLS-II", NSLS-II Tech Note 22 (2007).

- [37] J. Bengtsson, “NLSL-II: control of dynamic aperture with insertion devices”, NLSL-II Tech Note 58.
- [38] J. Bengtsson, “NLSL-II: magnet tolerances”, NLSL-II Tech Note 78 (2010).
- [39] J. Bengtsson, “NLSL-II: analysis of an asymmetric chromatic scheme”, NLSL-II Tech Note 90 (2009).
- [40] NLSL-II PDR, 2007, <https://www.bnl.gov/nsls2/project/PDR/default.asp>
- [41] J. Bengtsson, A. Streun “Robust design strategy for SLS-2”, SLS2-BJ84-001 (2017), <http://ados.web.psi.ch/SLS2/Notes/SLS2-BJ84-001.pdf>
- [42] B. Singh *et al.*, “Lattice options for DIAMOND-II”, in Proc. IPAC’18, paper THPMF009.
- [43] S. Leemann *et al.*, “A novel 7BA lattice for a 196-m circumference diffraction-limited soft X-ray storage ring”, in Proc. IPAC’18, paper THPMF077.
- [44] A 1-week visit to PSI by one of the authors, to ramp up on the (state-of-the) art for linear optics design, is acknowledged and highly appreciated.