RF QUADRUPOLE STRUCTURES FOR TRANSVERSE LANDAU DAMPING IN CIRCULAR ACCELERATORS

M. Schenk*, X. Buffat, L. R. Carver, A. Grudiev, K. Li, E. Métral, K. Papke
CERN, CH-1211 Geneva, Switzerland
A. Maillard
ENS, 75005 Paris, France

Abstract

The beams required for the high luminosity upgrade of the Large Hadron Collider (HL-LHC) and other potential future circular colliders (FCC) call for efficient mechanisms to suppress transverse collective instabilities. In addition to octupole magnets installed for the purpose of Landau damping in the transverse planes, we propose to use radio frequency (rf) quadrupole structures to considerably enhance the aforementioned stabilising effect. By means of the PyHEADTAIL macroparticle tracking code as well as analytical studies, the stabilising mechanism introduced by an rf quadrupole is studied and explained. It is, furthermore, compared to the influence of the second order chromaticity on transverse beam stability.

INTRODUCTION

A potential advantage of an rf quadrupole over conventional instability mitigation measures such as magnetic octupoles arises from the fact that it produces the incoherent betatron tune spread $\Delta Q_{x,y}$ required for Landau damping more effectively [1,2]. Instead of generating a detuning with transverse action $\Delta Q_{x,y}(J_x, J_y)$ as is the case for magnetic octupoles, an rf quadrupole introduces a dependence of the transverse tunes on the longitudinal action $\Delta Q_{x,y}(J_z)$ [3]. Beams of hadron colliders like LHC, HL-LHC, or FCC have an order of magnitude larger spread in action in the longitudinal compared to the transverse planes [4,5]. As the beam energy increases, the detuning effect from magnetic octupoles becomes even less effective, due to (i) adiabatic damping, further decreasing the transverse emittance, and (ii) the higher beam rigidity. The longitudinal emittance on the other hand is blown up along the energy ramp and the handle of detuning with longitudinal amplitude is retained [6]. It must be pointed out, though, that the amount of tune spread alone does not give a full picture on the efficiency of Landau damping.

Several studies on the rf quadrupole were presented in the past. The original proposal of the device was based on theoretical considerations which had been formulated by J. Scott Berg and F. Ruggiero in 1997 [3,7]. The stabilising effect from an rf quadrupole was then further evaluated and thoroughly studied by means of macroparticle tracking simulations. It was demonstrated that the device can provide beam stabilisation, be it by itself [8], or in combination with magnetic octupoles [9]. These promising results have motivated further studies which are now ongoing in parallel to gain insight into the underlying mechanisms from an experimental point of view. The main path that is being followed in that respect is to study the effects of second order chromaticity Q'' on the beam dynamics. The rf quadrupole and Q'' are equivalent in a first approximation. Q'' is introduced for example in the LHC through changes of the machine optics, making experimental studies possible. It has already been demonstrated that Q'' can be accurately controlled in the LHC, and a first set of consistent experimental and simulation studies for single-bunch stabilisation at 6.5 TeV are also published at this conference [10]. The results demonstrate that the PyHEADTAIL code [11], used to study the rf quadrupole, accurately models the relevant beam dynamics and reproduces the experimental observations well.

As the basic working principle has been demonstrated in the past, this paper focuses on more detailed aspects of the interaction of the rf quadrupole with the beam. The equivalence between an rf quadrupole and Q'' is shown first. The existing theory is compared with tracking simulations, pointing out relevant approximations and the interplay between two beam dynamics effects. Finally, a two-family scheme for rf quadrupoles is proposed for improved efficiency.

BASIC FORMALISM

RF Quadrupole and Q''

A particle i that is subject to the transverse kicks from an rf quadrupole experiences a change of its betatron tunes

$$\Delta Q^i_{x,y} = \pm\beta_{x,y} \frac{b^{(2)}}{4\pi B_0\rho} \cos\left[\frac{\omega z_i}{\beta c} + \varphi_0\right]. \qquad (1)$$

$\beta_{x,y}$ are the transverse beta functions at the location of the kicks, $b^{(2)}$ is the rf quadrupolar integrated gradient in units of [T/m m], $B_0\rho$ denotes the magnetic rigidity of the beam, ω is the angular frequency of the rf quadrupole field, z_i is the longitudinal position of the particle measured with respect to the zero crossing of the main rf wave, β and c denote the relativistic beta and the speed of light respectively, and φ_0 is a constant phase. Slow head-tail instabilities develop over time scales of many synchrotron periods as the interplay between transverse and longitudinal particle motion is an important ingredient for their formation [2]. To ensure that the Landau damping mechanism works against this type of instabilities, the incoherent tune spread must not vanish over the given

* michael.schenk@cern.ch

time scale. This is true for an rf quadrupole operating with a phase of $\varphi_0 = 0$ or $\varphi_0 = \pi$. In the following we assume $\varphi_0 = 0$. Equation 1 can be Taylor-expanded in z_i and the higher-order terms $O(z_i^4)$ are negligible if the wavelength of the rf wave is much larger than the bunch length σ_z, i.e. $\omega\sigma_z/\beta c \ll 1$. The zeroth-order term represents a constant detuning and is also neglected as it does not contribute to the incoherent tune spread. The average detuning is

$$\left\langle \Delta Q^i_{x,y} \right\rangle \approx \mp\beta_{x,y} \frac{b^{(2)}}{8\pi B_0 \rho} \left(\frac{\omega}{\beta c}\right)^2 \frac{\eta R}{Q_s} J^i_z = a^{x,y}_z J^i_z, \quad (2)$$

where J^i_z is the longitudinal action, η is the slip factor, R the machine radius, and Q_s denotes the synchrotron tune. All the constants have been absorbed in $a^{x,y}_z$.

For comparison, given a non-zero second order chromaticity $Q''_{x,y}$, a particle i with a relative momentum deviation $\delta_i = \Delta p_i/p$ experiences a betatron detuning of $\Delta Q^i_{x,y} = Q'_{x,y}\delta_i + Q''_{x,y}\delta_i^2/2$. The average detuning reads

$$\left\langle \Delta Q^i_{x,y} \right\rangle = \frac{Q''_{x,y}}{2} \frac{Q_s}{\eta R} J^i_z = \hat{a}^{x,y}_z J^i_z. \quad (3)$$

Hence, Q'' is equivalent to the rf quadrupole under the assumption that $\omega\sigma_z/\beta c \ll 1$. Q''_x and Q''_y can often be controlled independently (see e.g. [10]). This can be achieved also for the rf quadrupole by installing two different families as explained below. The advantage of the rf quadrupole over Q'' is that it serves as an independent knob and is hence not constrained by machine optics requirements.

Beam Dynamics

An approximate stability diagram theory for betatron detuning with longitudinal amplitude is derived from the dispersion relation in [7]. An example of the stability diagrams for a Gaussian beam is shown in Fig. 1 for an rf quadrupole with $\varphi_0 = 0$ for an azimuthal mode zero head-tail instability. The asymmetry of the stability curves has important consequences for the stabilising efficiency in the two transverse planes. It is a result of the one-sided detuning (see Eq. (2)) as detailed further below. The dispersion relation in [7] has been derived under various approximations. In particular, the explicit dependence on the impedance was

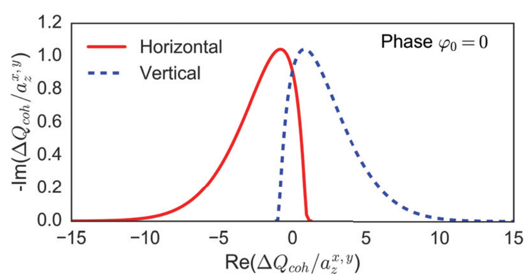

Figure 1: Stability diagrams for an rf quadrupole (second order) with $\varphi_0 = 0$ for the two transverse planes. It was determined by solving the dispersion relation for detuning with longitudinal amplitude numerically [7].

cancelled out by assuming that its content is limited to a frequency range much smaller than that of the bunch spectrum. The dispersion relation hence does not necessarily reflect all the aspects of the beam dynamics. It has been observed in tracking simulations that the rf quadrupole can directly impact on the interaction of the beam with the impedance, similarly to a first order chromaticity. This can lead to a change of the (unperturbed) unstable mode through a change of the effective impedance. What happens is that either the complex coherent tune shift of the instability is modified, with for instance the effect of an increase or decrease in the growth rate, or another head-tail mode is excited on a different synchrotron side band. A similar effect has also been observed for the experiments and the simulations with Q'' in the LHC [10].

To summarise, the beam dynamics in presence of an rf quadrupole is dominated by an interplay between two effects: (i) Landau damping from the incoherent tune spread, described by the approximate dispersion relation and stability diagram theory, and (ii) a change of the effective impedance, not related to Landau damping, but to how the beam interacts with the impedance. The latter is missing in the description by [7]. The interplay of the two effects is rather complex and depends mostly on the beam parameters and the details of the machine impedance. It can be favourable, or unfavourable for the stabilising efficiency of the rf quadrupole. Both mechanisms have been identified analytically by means of Vlasov theory [2] and were benchmarked against tracking simulations. The derivation and detailed discussion of these results, however, are out of the scope of this article and will be presented separately. At present, beam stabilisation with an rf quadrupole is most accurately addressed by means of particle tracking simulations.

TRACKING SIMULATIONS

The rf quadrupole is modelled in PyHEADTAIL, a highly modularised 6D macroparticle tracking code designed to study collective instabilities in circular colliders [11]. Various tracking studies concerning the stabilising effect from an rf quadrupole have been presented in the past. Stabilisation was demonstrated based on a head-tail mode observed experimentally in the LHC [8]. Like magnetic octupoles, the rf quadrupole suppresses the instability by introducing a similar amount of tune spread. Second, the combined effect from octupoles and an rf quadrupole was presented for the HL-LHC showing that a significant reduction of the stabilising octupolar strength can be achieved with the support from an rf quadrupole [9]. Finally, the PyHEADTAIL simulation model has been successfully benchmarked against experimental studies with Q'' in the LHC [10].

Illustrations of the Beam Dynamics

The studies are based on the impedance model of the Future Circular proton-proton Collider (FCC-hh) at injection energy (3.3 TeV) with nominal bunch parameters [12]. The model includes the beam screen treated as a cylindrical cop-

ISBN 978-3-95450-182-3

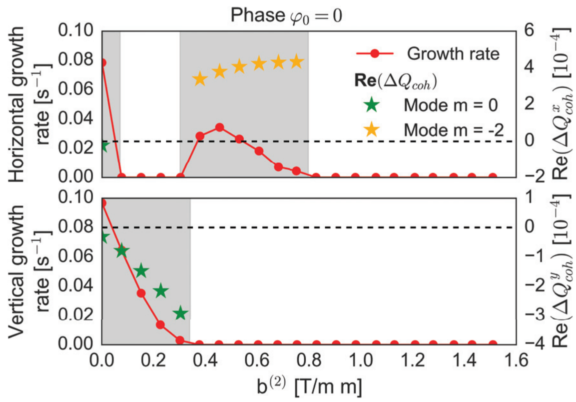

Figure 2: FCC-hh simulations for $\varphi_0 = 0$. Horizontal (top) and vertical (bottom) growth rates (circles, left axis) and $Re\,(\Delta Q_{coh})$ (stars, right axis) are shown vs. $b^{(2)}$.

per pipe, and the contribution from the collimators. The first order chromaticity is set to $Q'_{x,y} = 5$. A 50-turn ideal transverse feedback is part of the model as well as an 800 MHz rf quadrupole at a location with $\beta_{x,y} = 200$ m (conservative). No Landau octupoles are used in the set-up. Figure 2 summarises the results of a scan in rf quadrupole strength $b^{(2)}$ for $\varphi_0 = 0$. It shows the horizontal (top) and vertical (bottom) growth rates of the unstable mode(s) in red, as well as the corresponding real parts of the complex coherent tune shifts (stars). For an rf quadrupole operating with $\varphi_0 = \pi$, the dependencies observed in the horizontal and the vertical planes are exchanged. This is expected given the quadrupolar nature of the device. At $b^{(2)} = 0$ T/m m, the most unstable mode is a weak head-tail azimuthal mode $m = 0$. The complex coherent tune shifts $\Delta Q^{x,y}_{coh}$ are almost identical in the two planes. In particular, the real components $Re(\Delta Q^{x,y}_{coh})$ are both negative (green stars). Stability diagram theory (Fig. 1) predicts that in this case the rf quadrupole operating with $\varphi_0 = 0$ should demonstrate a better stabilising efficiency in the horizontal compared to the vertical plane. This is reflected in Fig. 2 at low $b^{(2)}$. The horizontal plane is stabilised immediately, while the vertical one requires a larger strength. As explained earlier, there is a second important beam dynamics effect. The rf quadrupole changes the effective impedance at sufficiently high strengths, and the horizontal plane becomes unstable again (second grey box), only that now the excited mode is an $m = -2$ (orange stars). This mode is not suppressed by Landau damping at that stage, because it has (i) a larger real coherent tune shift, and (ii) $Re(\Delta Q^x_{coh})$ has now a different sign, meaning that the mode has ‘moved out’ of the stable area. The reason why this mode was stable at intermediate strengths is that its corresponding imaginary coherent tune shift was small enough to fall within the stable area. By further increasing $b^{(2)}$, however, this mode can be suppressed as well. Within the scanned region, no other mode is observed. In this case, the required stabilising strength is $b^{(2)} \approx 0.8$ T/m m where it has to be emphasised that despite the conservative choice of $\beta_{x,y}$, this can be achieved with less than ten rf quadrupole cavities [3].

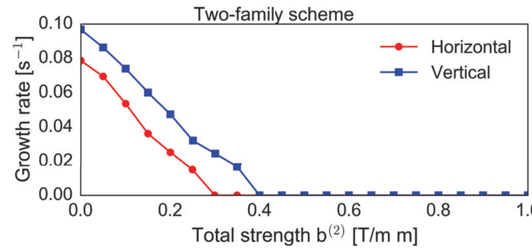

Figure 3: FCC-hh simulations for a two-family rf quadrupole scheme. Horizontal (red) and vertical (blue) growth rates are shown as a function of the *total* quadrupolar strength.

Two-Family RF Quadrupole Scheme

The stabilising behaviour in the two transverse planes can be symmetrised by installing two independent rf quadrupole families operating with opposite signs, placed at two different locations in the machine: One of them at high β_x, low β_y to improve stability in the horizontal plane, and one at low β_x and high β_y for stability in the vertical plane. The difference between the local beta functions must be large to avoid that the two families counteract each other significantly. Here, the high and low beta function values are 200 m and 50 m, respectively, which is again very conservative. The results for the same FCC-hh configuration as before are summarised in Fig. 3, showing the horizontal (red) and vertical (blue) growth rates as a function of the *total* quadrupolar strength (both families are powered with identical strengths). There are two main advantages. First, the strong asymmetry between the two planes is removed and the required stabilising strength is now equalised for both planes. Second, the quadrupolar strength required for stability in both planes is significantly lower overall.

SUMMARY AND CONCLUSIONS

As a complementary article to the past proof-of-principle studies for an rf quadrupole for beam stabilisation, the beam dynamics was discussed in more detail based on tracking simulations. Two effects are mainly involved: (i) Landau damping, described by existing stability diagram theory, and (ii) a change of the effective impedance. Both effects have been observed by experiments and simulations for Q'' in the LHC, and for the rf quadrupole for an FCC-hh study case. Finally, a two-family rf quadrupole scheme was proposed and benchmarked to overcome the asymmetric stabilising efficiency in the transverse planes.

When considering an rf quadrupole for beam stabilisation instead of magnetic octupoles, it must be taken into account that the interplay of the device with the beam is much more involved. At intermediate strengths, other modes may be excited. Nevertheless, in all the studies done so far the device could ultimately stabilise the beams at reasonably high quadrupolar field strengths. A theory describing the mechanisms is under validation, but tracking codes are still the best tool to accurately evaluate the capabilities of an rf quadrupole.

REFERENCES

[1] L. Landau, "On the vibration of the electronic plasma", *J. Phys. USSR*, vol. 10, pp. 25–34, 1946.

[2] A. W. Chao, *Physics of collective beam instabilities in high energy accelerators*, Wiley Series in Beam Physics and Accelerator Technology, Wiley, 1993.

[3] A. Grudiev, "Radio frequency quadrupole for Landau damping in accelerators", *Phys. Rev. ST Accel. Beams*, vol. 17, p. 011001, Jan. 2014.

[4] O. Brüning *et al.*, "LHC Design Report", CERN Yellow Reports: Monographs, CERN, Geneva, Switzerland, CERN-2004-003-V-1, Jun. 2004.

[5] G. Apollinari, I. Béjar Alonso, O. Brüning, M. Lamont, and L. Rossi, "High-Luminosity Large Hadron Collider (HL-LHC): Preliminary Design Report", CERN Yellow Reports: Monographs, CERN, Geneva, Switzerland, CERN-2015-005, Dec. 2015.

[6] P. Baudrenghien and T. Mastoridis, "Longitudinal emittance blowup in the Large Hadron Collider", *Nucl. Instr. Meth. A*, vol. 726, pp. 181–190, 2013.

[7] J. S. Berg and F. Ruggiero, "Stability diagrams for Landau damping", *in Proc. 17th Particle Accelerator Conf. (PAC'97)*, vol. 2, Vancouver, B.C., Canada, May 1997, pp. 1712–1714.

[8] A. Grudiev, K. Li, and M. Schenk, "Radio frequency quadrupole for Landau damping in accelerators: analytical and numerical studies", *in Proc. 54th ICFA Advanced Beam Dynamics Workshop on High-Intensity and High-Brightness Hadron Beams (HB2014)*, East-Lansing, MI, USA, Nov. 2014, paper WEO4AB01, pp. 315–319.

[9] M. Schenk, A. Grudiev, K. Li, and K. Papke, "Use of rf quadrupole structures to enhance stability in accelerator rings", *in Proc. 54th ICFA Advanced Beam Dynamics Workshop on High-Intensity and High-Brightness Hadron Beams (HB2016)*, Malmö, Sweden, Jul. 2016, paper THPM7X01, pp. 505–510.

[10] M. Schenk *et al.*, "Practical stabilisation of transverse collective instabilities with second order chromaticity in the LHC", presented at the 8th Int. Particle Accelerator Conf. (IPAC'17), Copenhagen, Denmark, May 2017, paper THPVA026, this conference.

[11] E. Métral *et al.*, "Beam instabilities in hadron synchrotrons", *IEEE Transactions on Nuclear Science*, vol. 63, no. 2, pp. 1001–1050, Apr. 2016.

[12] The FCC-hh, `http://tlep.web.cern.ch/content/fcc-hh`