AN MDM SPIN TRANSPARENT QUADRUPOLE FOR STORAGE RING BASED EDM SEARCH

M. Bai, Y. Dutheil, Forschungszentrum, Jülich, Germany
D. Sagan, Cornell University, Ithaca, NY, USA

Abstract

A storage ring provides an attractive option for directly measuring the electric dipole moment (EDM) of charged particles. To reach a sensitivity of 10^{-29} e·cm, it is critical to mitigate the systematic errors from all sources. This daunting task is pushing the precision frontier of accelerator science and technology beyond its current state of the art. Here, we present a unique idea of a magnetic dipole moment (MDM) spin transparent quadrupole that can significantly reduce the systematic errors due to the transverse electric and magnetic fields that particle encounters.

INTRODUCTION

The quest for understanding the fate of antimatter in the universe – and to fully understand CP violation – has triggered a hunt to measure the electric dipole moment (EDM). So far, all measurements have shown the upper limit of the neutron EDM is on the order of 10^{-26} e·cm. However, no direct EDM measurement of charged light ions including protons, have yet been performed. The concept of a storage ring based EDM search has been proposed and investigated by the Brookhaven National Laboratory (BNL) as well as by the Forschungszentrum Juelich (FZJ) [1, 2].

In a storage ring, the spin motion is governed by the Thomas-BMT equation

$$\frac{d\vec{S}}{dt} = \vec{S} \times \frac{e}{m}\Big[\left(\frac{1}{\gamma} + G\right)\vec{B}_\perp + \left(\frac{1+G}{\gamma}\right)\vec{B}_\parallel + \left(G + \frac{1}{\gamma+1}\right)\frac{\vec{E}\times\vec{\beta}}{c} + \frac{\eta}{2c}\left(\vec{E} + c\vec{\beta}\times\vec{B}\right)\Big] \quad (1)$$

,where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields in the laboratory frame, G is the anomalous g-factor of the particle, $\vec{d} = \frac{e\eta}{2m}\vec{S}$ is the electric dipole moment, $\vec{\mu} = \frac{g}{2}\frac{e}{m}\vec{S}$ is the magnetic dipole moment, and $\vec{\beta}$ is the particle's velocity normalized by the speed of the light. In Eq. 1, $\vec{S}$ is the spin vector in the particle's frame, while $\vec{B}_\perp$ and $\vec{B}_\parallel$ are the magnetic field perpendicular and parallel to the particle's velocity, respectively.

Since the EDM is expected to be significantly smaller than the magnetic dipole moment (MDM), the spin motion is dominated by the MDM. The concept of measuring the EDM in a storage ring with the MDM part of the spin motion frozen along the velocity of the particle was developed and investigated by the BNL srEDM collaboration [3, 4]. The proposed storage ring for a deuteron EDM search uses static hybrid electro and magnetic deflectors to freeze the MDM part of the spin motion. The possible sources of systematic errors for such a high precision EDM storage ring was evaluated and the residual non-zero average vertical electric field was identified as the only source of first order systematic errors. Other error sources, such as fringe fields as well as the electric and magnetic fields in the RF cavity, introduce higher order systematic errors to the EDM measurement. The spin precession due to the average non-zero vertical electric field $\langle E_v\rangle$ is given by [3]

$$\omega_{E_v} = (G+1)\frac{e}{mc}\frac{\langle E_v\rangle}{\beta^2\gamma^2}. \quad (2)$$

For a partially spin frozen based EDM search in a storage ring, such as the proposed direct measurement of the deuteron EDM at the Cooler SYnchrotron (COSY) [2, 5], the additional vertical polarization buildup from the radial magnetic fields is the dominant source of systematic error. Hence, the closed orbit has to be corrected and controlled to a high precision [6]. This is currently the limiting factor for achieving the proposed measurement sensitivity. Hence, the MDM spin transparent quadrupole proposed below may also be beneficial by mitigating the systematic error from the radial magnetic fields due to the off-center trajectory in the magnetic quadrupole fields.

MDM SPIN TRANSPARENT QUADRUPOLE

Similar to the condition for freezing spin in a dipole, in order to make the MDM part of the spin motion to be transparent in a quadrupole, both electric and magnetic quadrupole fields are needed. For a linear electric quadrupole, the electric field can be described as

$$E_x - iE_y = (b_{e1} + ia_{e1})\frac{x+iy}{r_0} \quad (3)$$

with r_0 being the radius of the quadrupole, b_{e1} and a_{e1} being the normal and skew component, respectively. x and y are the horizontal and vertical distances from the center of the quadrupole. The magnetic field of a quadrupole can be described as

$$B_y + iB_x = (b_1 + ia_1)(x+iy) \quad (4)$$

with b_1 and a_1 being the magnetic normal component and skew component of the quadrupole, respectively.

Two conditions have to be met to achieve spin transparency. First of all, the electric and magnetic fields have to be perpendicular to each other everywhere in the quadrupole. If there are no electric nor magnetic skew fields, that is, $a_{e1} = a_1 = 0$, it is easily shown that the electric and magnetic fields stay perpendicular everywhere if

$$\vec{B}\cdot\vec{E} = 0. \quad (5)$$

Similar to the approach for spin frozen deflector [7], the strength of the electric and magnetic fields for the MDM spin transparent quadrupole have to follow the given relationship in Eq. 6,

$$B_x = -\left[1 - \frac{1}{(\gamma+1)(1+G\gamma)}\right]\frac{\beta E_y}{c}$$
$$B_y = \left[1 - \frac{1}{(\gamma+1)(1+G\gamma)}\right]\frac{\beta E_x}{c}. \tag{6}$$

Thus,

$$b_1 = \left[1 - \frac{1}{(\gamma+1)(1+G\gamma)}\right]\frac{\beta b_{e1}}{r_0 c}. \tag{7}$$

The equivalent field gradient of a combined EB quadrupole is $k_1 = b_1 + \frac{b_{e1}}{r_0\beta c}$. Thus, the equivalent field gradient of an MDM spin transparent quadrupole is

$$k_1 = \left[\left[1 - \frac{1}{(\gamma+1)(1+G\gamma)}\right]\frac{\beta}{c} + \frac{1}{\beta c}\right]\frac{b_{e1}}{r_0}. \tag{8}$$

For a 1.0 GeV/c deuteron in a combined quadrupole with a field gradient of $k_1 = 1.0\ m^{-1}$, the corresponding electric field gradient is 6.4 MV/m at a radius of 50 mm and the magnetic field gradient is 0.2 T/m.

BENEFITS OF MDM SPIN TRANSPARENT QUADRUPOLE

As discussed in the original BNL deuteron EDM storage ring proposal, where the lattice consists of spin frozen electric and magnetic deflectors, the residual average non-zero vertical electric field $\langle E_v \rangle$ is the primary source of first order contribution to the systematic error. In the BNL proposal [3], the additional spin precession due to the average vertical electric field is given by Eq. 2.

For deuterons with a momentum of 1 GeV/c, $G = -0.14$ and $\gamma = 1.14$, the additional spin precession due to non-zero vertical electric field is about $2.0\frac{e}{mc}\langle E_v \rangle$.

If MDM spin transparent (MDMST) quadrupoles are added to the lattice, the non-zero vertical electric field will force the particles to experience additional Lorentz forces from all the quadrupoles to stay stable. In this case the average of electric and magnetic forces due to the quadrupoles will balance the residual electric field.

$$\left\langle \beta c B_{qx} + E_{qy} \right\rangle + \langle E_v \rangle = 0 \tag{9}$$

where B_{qx} and E_{qy} are the radial magnetic field and vertical electric field a particle sees in a MDMST quadrupole, and the resulting spin precession $\omega_{\mathrm{MDMST}} = 0$. So, the spin precession due to non-zero average vertical electric field is given by Eq. 10

$$\omega_{E_v} = \frac{e}{mc}\left(G + \frac{1}{\gamma+1}\right)\langle E_v \rangle \beta. \tag{10}$$

For deuterons at momentum of 1 GeV/c, $G = -0.14$ and $\gamma = 1.14$, this additional spin precession due to non-zero vertical electric field is about $0.2\frac{e}{mc}\langle E_v \rangle$, about an order of magnitude smaller than the case with pure magnetic quadrupoles.

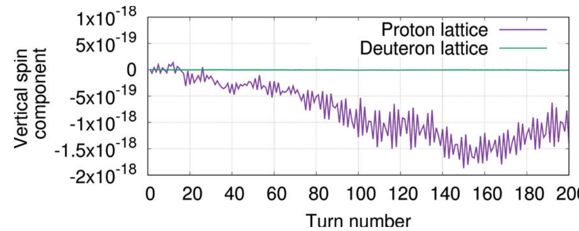

Figure 1: Average vertical spin components for 8 particles uniformly distributed on a vertical invariant of 10^{-7} mm mrad, as a function of the turn number.

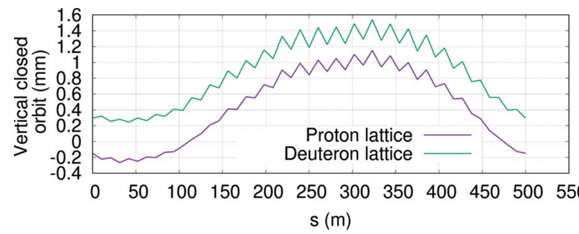

Figure 2: Vertical closed orbit along the ring for the proton and deuteron lattices.

Contribution to the EDM Signal

Using combined electric and magnetic quadrupoles, a non-zero closed orbit results to a net contribution to the EDM part of the spin precession of

$$\frac{d\vec{S}}{dt} = \frac{e}{\gamma m}\frac{\eta}{2c}\vec{S}\times[E_{bx} + c\vec{\beta}\times B_{by} + \frac{b_{e1}}{r_0}(x\hat{x} - y\hat{y}) + c\vec{\beta}\times b_1(x\hat{x} + y\hat{y})] \tag{11}$$

Where E_{bx} and B_{by} are the radial electric and vertical magnetic fields of the deflector, and $c\vec{\beta}$ is the particle's velocity. b_1 and b_{e1} are the magnetic and electric field gradients of the quadrupole, and x and y are the radial and vertical closed orbit distortions in the quadrupole. Unlike a pure electric or magnetic deflector, this perturbation on the EDM part of the spin motion depends on the beam position as shown in Eq. 11

Again, as long as beam motion is stable, the average net Lorentz force from the quadrupoles has to be zero on the closed orbit, That is,

$$\left\langle \frac{b_{e1}}{r_0}(x\hat{x} - y\hat{y}) + c\vec{\beta}\times b_1(x\hat{x} + y\hat{y}) \right\rangle = 0. \tag{12}$$

Hence, the average EDM buildup from the MDMST quadrupoles has to be also averaged to zero, and the net EDM buildup rate is given by

$$\left\langle \frac{d\vec{S}}{dt} \right\rangle = \frac{e}{\gamma m}\frac{\eta}{2c}\vec{S}\times[E_{bx} + c\vec{\beta}\times B_{by}] \tag{13}$$

Preliminary Simulation Results

To study the benefits of using MDMST quadrupoles, two lattices were employed. One was the lattice of Y. Semertzidis et al [8]. This is a proposed design for an EDM search using a pure electric proton storage ring. The other

ISBN 978-3-95450-182-3

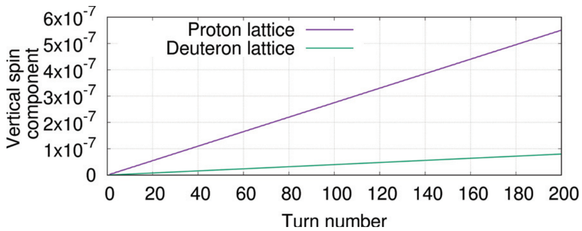

Figure 3: Average vertical spin components for 8 particles uniformly distributed on a vertical invariant of 0.1 mm mrad, as a function of the turn number, in a lattice with vertical closed orbit.

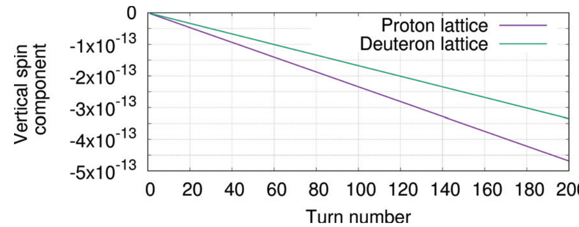

Figure 4: Average vertical spin components for 8 particles uniformly distributed on a vertical invariant of 0.1 mm mrad, as a function of the turn number, with an EDM of $\eta = 10^{-15}$.

lattice is based upon the first lattice but uses deuterons with MDMST quadrupoles and full spin frozen EB deflectors. Both lattices are 500 m long and feature transverse betatron tunes of 2.24 and 0.32 in the horizontal and vertical planes for the deuteron lattice. The proton lattice tunes are 2.42 and 0.44. All simulations here are done using the Bmad toolkit [9].

The first study had no misalignments. That is, the closed orbit is through the center of all quadrupoles. In that case the spin is perfectly frozen and remains along the longitudinal direction. No vertical buildup of the spin in the vertical direction occur since those simulations are done with no EDM. Figure 1 shows that in the absence of EDM and without vertical closed orbit the vertical spin buildup remains around zero, for both lattices and with vertical emittance.

Figure 2 shows vertical closed orbits generated by randomly misaligning quadrupoles. These two closed orbits are slightly different due to the small difference in the focusing of both lattices but otherwise generated by the same misalignments.

Figures 3 and 4 shows the average vertical component of the spin, as a function of orbital turn number, for 8 particles uniformly distributed with a vertical invariant $y y'$ of $0.1\,\pi$ mm-mrad. As a main source of systemic error it is important to look at the effect of vertical closed orbit on the vertical spin buildup. Figure 3 compares the effect on the vertical spin buildup of a vertical closed orbit for the proton and deuteron lattices, without EDM. It is clear here that the contribution of the vertical closed orbit on the vertical spin buildup is much lower in the case of the deuteron lattice.

A value of $\eta = 10^{-15}$ was used for the simulations showed Figure 4. For a deuteron, $\eta = 10^{-15}$ corresponds to an EDM on the order of 10^{-29} e·cm. Figure 4 shows that the vertical spin buildup due to the EDM and without of closed orbit. The vertical spin build up due to the EDM is similar in the deuteron and proton lattices but also much lower in amplitude than the one due to the vertical closed orbit.

CONCLUSION

A novel idea for a electrostatic and magnetic quadrupole transparent for the magnetic dipole moment was introduced. This element was introduced in a spin frozen deuteron lattice and tracking were carried using the Bmad library and the PTC tracker[10]. It aims at mitigating the effect of systematic errors on the EDM measurement and in particular the effect of non zero vertical closed orbit.

However, simulations of spin dynamics in electric fields are particularly challenging. Even more so in our case as we aim to track particle spin with an accuracy of the order a 64 bits floating point machine precision. The results showed here are preliminary and not yet exempt of numerical issues in particular associated to round off errors. Furthermore experience with those codes of spin dynamics in the presence of complex electric fields is limited. This justify the need for extensive benchmarking and validation of the simulated spin dynamics. Both of which are still ongoing and a major effort.

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