# DETAILED ANALYSIS OF A LINEAR BEAM TRANSPORT LINE FROM A LASER-WAKEFIELD ACCELERATOR TO A TRANSVERSE-GRADIENT UNDULATOR 

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#### Abstract

A linear beam transport system, experimentally tested at a Laser Wakefield Accelerator setup at the JETI-laser facility in Jena, Germany, has been carefully analyzed in order to gain a deeper understanding of the experimental results and to develop experimental strategies for the future. This analysis encompassed a detailed characterization of the focusing magnets and an investigation of the effects of source parameters as well as magnet and alignment errors on the observables accessible in the experiment. A dedicated tracking tool was developed for these investigations. In this contribution we review the main results of these studies.


## INTRODUCTION

Electron bunches from a Laser-Wakefield Accelerator (LWFA) typically have an energy spread of a few percent and a divergence of up to few millirad. To produce monochromatic radiation in an undulator despite the large energy spread, the idea is to induce a transverse spectral dispersion within the bunches with a double dipole chicane and to lead the shaped bunch into a transverse gradient undulator (TGU). The TGU provides the necessary field gradient to let the particles of different energies emit monochromatic synchrotron light [1].

As described in [2], a small series of two different types of in-vacuum, high-gradient quadrupoles has been built to be used as part of a linear transport system for a LWFA [3]. Both types have the same aperture radius and yoke length but differ in the number of windings per coil. The smaller quadrupoles will be referred to as QK , the larger ones as QG.
A first experiment has been performed to test the linear beam transport system without the TGU $[3,4]$. There the beam transport system was set up with two sets of triplets consisting of QK-QG-QK and a dipole after each triplet [4]. The main challenges were the limited diagnostics, consisting of fluorescent screens and a magnetic spectrometer, and the necessity to perform an on-line beam based alignment with the limited information. Therefore we systematically investigated the influence of magnet positioning errors on the characteristics of the particle distributions at the available beam screens in order to establish a fast and simple online analysis and a magnet alignment procedure for future experiments.

[^0]03 Novel Particle Sources and Acceleration Techniques


Figure 1: Results of the circular magnetic field measurements of QG1. Shown are the first three multipole strengthes $\hat{F}_{n=1,2,3}$ and the according phases $\varphi_{n=1,2,3}$ along the nominal beam axis. The yokes reach from $\pm 40 \mathrm{~mm}$.

Furthermore, in order to gain a deeper understanding of the previous experiment we performed a complete and detailed analysis of the quadrupoles by measuring the magnetic field as well as by investigating the influences of possible field errors on the beam transport system.

## QUADRUPOLE-CHARACTERIZATION

Both types of quadrupoles have an aperture radius of $r_{d}=11 \mathrm{~mm}$ and a yoke width of 80 mm . The QK-types are designed to have a maximum field gradient of $30 \mathrm{~T} / \mathrm{m}$, the QG-types reach a maximum gradient of $39 \mathrm{~T} / \mathrm{m}$.

Magnetic field measurements have been performed for each quadrupole at the recently commissioned magnet laboratory at KIT [5] using hall probe scans. This section shows the results of the characterization of the first quadrupole of type QG as an example.

## Circular Magnetic Field Scans

Figure 1 shows the first three multipole strengths $\hat{F}_{n=1,2,3}$ and the according phases $\varphi_{n=1,2,3}$ of QG1 along the beam axis e.g. the scan axis, derived from the field harmonics. The field harmonics were obtained by measuring the radial field component along a circle within the aperture in the

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$(x, y)$-plane and analyzing the data using a Fourier transformation. Scans for discrete steps along the beam-axis were recorded and therefore we could obtain the distribution of the individual multipole moments along the beam axis $z$. This distribution can be used to simplify the modeling and calculation of realistic magnetic fields as shown in the following section.

The measured dipole moment results from feed-down harmonics and was used to determine the magnetic axis as offset from the scan axis [6].

It can be seen, that the phase of the quadrupole field shows an absolute offset as well as a linear slope along the beam axis. This behavior is likely a result of an imperfect stacking of the lamellae forming the yokes. In an ongoing improvement process of the quadrupoles, this issue will be dealt with by an improved frame for the yokes.

Higher order multipole terms, especially the sextupole strength, are present mainly in the edge area of the yokes.

## START-TO-END SIMULATION

## Methods

A numerical simulation framework has been developed in Python to investigate (1) the effect of the initial parameters of the bunches, (2) magnet alignment errors and (3) the influence of the measured field quality on the observables of the experiment. The investigations have been performed for single quadrupoles as well as for the complete beam transport system. The goal was to gain a deeper understanding of the data measured in the first experiment and also to deduct valid simulation models for the magnets that reproduce the results obtained with the real field data. The gathered knowledge provides a basis for an improved future experiment.

Naturally, the boundary conditions for the given setup simplify the calculations. Space charge effects cause an increase of the geometrical divergence only within the first centimeters of the drift after the LWF accelerator and can afterwards be neglected [3]. There are no external electrical fields present. The framework therefore solves the equation of motion with a Runge-Kutta scheme of 4th order and adaptive step-size control according to Fehlberg [7] allowing the input of arbitrary magnetic fields. This is of particular interest to model the measured fields as well as to implement all degrees of freedom. Solving the equation of motion directly with the Runge-Kutta scheme lets the numerical error not be dependent of the strongly varying initial parameters but instead only on the step size. The higher calculation times compared to solving the reference orbit equation of motion with perturbation terms and matrix formalisms are bearable due to the limited number of components and the single-pass nature of the setup.

## Models

The simulation framework handles three different types of magnet models. The simplest one is a rectangularly shaped model with an analytically calculated field of arbitrary shape, which may be used e.g. to model ideal multipole magnets


Figure 2: Simulated beam profiles for a bunch with gaussian distributed momentum and divergence for the three different quadrupole models and the corrected multipole model, each focusing in the $y$-plane.


Figure 3: Integrated beam profiles of the focal and non focal plane for the three quadrupole models and the corrected multipole model shown in Fig. 2
with user-defined multipole moments and phases. The second model reads a 3D field map and numerically interpolates the field components by using the python-scipy package [8]. The third type can read a map of measured multipole values and phases along the beam axis (as seen in Fig. 1) and create an adequate 3D model thereof via the Fourier-series, which is much faster than interpolating the whole field map. All magnet models can be arbitrarily positioned within the cartesian space and rotated with arbitrary rotation matrices specified by the user.

## Alignment Error-Studies

A crucial part of the beamline setup is the alignment of the magnets with respect to the beam axis. Within the ex-


Figure 4: Comparison of the statistical moments of the particle distribution on the simulated beam screen for different misalignments between the analytical and multipole magnet models.
periment it is possible to measure the particle distribution at beam monitors. A study on the influences of misalignments of the magnets on the statistical moments of this distribution was performed. The particle distribution in the focal plane for a single bunch with central momentum of $p_{0}=50 \mathrm{MeV} / \mathrm{c}$, $\Delta p / p=10 \%$ and a divergence of $u^{\prime}=3.8 \mathrm{mrad}$ when passing a quadrupole is shown in Fig. 2 and Fig. 3. The quadrupole focuses in $y$-direction.

The statistical moments of the particle intensities for the different field models created with measured field data differ from the analytic case. The found deviations of the magnetic axis according to the found dipole moment in Fig. 1 was therefore corrected by a shift in $y$ as well as a slight rotation about the $x$-axis and the same particle distribution was tracked again. As can be seen in Fig. 3 the particle distribution then deviates only little from the analytic case. The found focusing error could be mostly compensated by only using geometrical degrees of freedom and hence could be lead back to an alignment error rather than to the presence of the measured higher multipole moments.

With regard to the future experiments it is of interest to apply such corrections after evaluating the observed particle distribution on the beam screens. Therefore rotational as well as translational misalignments were simulated for analytic and multipole models for a broader range of shifts and rotations to distinguish them. Fig. 4 shows the statistical moments of the particle distribution as seen on a beam screen in the focusing plane for a shift in $y$-direction as well as a rotation about the $x$-axis. This shows, that the dependency of the statistical moments differs between the rotational and translational errors as well in strength as in the behavior along $z$. In the experimental situation this should allow a beam based adjustment of the magnets for the different degrees of freedom independently. Performing these calculations according to the measured particle distribution will then guide the alignment process.

## Simulation of the Complete Setup

To simulate the whole setup as realized in the first experiment, the same optic as in [4] for a central momentum of


Figure 5: Left: Measured intensity on the fluorescent screen at the end of the whole beam transport line as measured in $[3,4]$
Right: Intensity on the same screen as predicted by the simulation framework.
$p_{0}=60 \mathrm{MeV} / \mathrm{c}$ was chosen and a parameter study was performed varying central momentum $p_{0}$, momentum spread $\sigma_{p}$ and divergence $\sigma_{p^{\prime}}$.

Figure 5 shows the measured beam screen intensity in the first experiment next to the newly simulated results as predicted by the framework and found in the parameter study. Especially the splitting of the intensity in the $x$-direction could be reproduced and its origin could be explained by the large initial divergence.

## SUMMARY AND CONCLUSION

The quadrupoles of the beam transport system have been analyzed by detailed measurements of their magnetic field. Simulation models from the measured field data have been created and validated. For detailed tracking simulations a customized framework has been developed to include the measured field data as well as alignment error studies.

The simulations showed the possibility to distinguish the different degrees of freedom in a beam based alignment procedure, by evaluating the particle distribution as measured on fluorescent screens.

Furthermore previously gathered experimental data could be assigned to the according bunch parameters by a parameter study of the possible initial bunch parameters.

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