# ALIGNMENT OF ELECTRON AND ION BEAM TRAJECTORIES IN NONMAGNETIZED ELECTRON COOLER * 

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#### Abstract

The cooling section (CS) of the low energy RHIC electron cooler (LEReC) consists of two 20 m long parts each containing several weak solenoids with trajectory correctors placed inside the solenoids and the BPMs located downstream of each solenoid. These weak solenoids are used to minimize divergence of electron beam due to the transverse space charge force of electron beam itself. To obtain the cooling it is required to keep the overall RMS electron angles in the cooling section below 100 urad. Possible mechanical misalignment, such as shift and inclination of the CS solenoids can cause an unacceptable misalignment of the e-beam trajectory with respect to the ideal trajectory set by ions. Therefore, it is critical to perform a beam based alignment of the CS solenoids. In this paper we suggest a procedure for such an alignment.


## INTRODUCTION

The LEReC accelerator [1, 2] consists of a 400 keV photo-gun followed by the SRF Booster, which accelerates the beam to $1.6-2.6 \mathrm{MeV}$, the transport beamline, the merger that brings the beam to the two cooling sections (in the Yellow and in the Blue RHIC rings) separated by the $180^{\circ}$ bending magnet and the extraction to the beam dump.
In the LEReC CS "cold" e-bunches co-propagate with "hot" ions reducing their emittance and energy spread.

Each LEReC CS (Fig. 1) contains 8 solenoids combined with trajectory correctors and the BPMs located downstream of each solenoid. The distance between solenoid centers is 3 m .


Figure 1: LEReC cooling sections.
Successful cooling of the ions requires RMS electron angles in the CS to be $<100$ urad. Figure 2 schematically demonstrates possible mechanical misalignment of the CS solenoids that can cause an unacceptable angle of the ebeam trajectory with respect to the ions.

We suggest [3] the following steps of the e-beam trajectory alignment.

1. Aligning the CS BPMs with respect to the ion beam.
2. Aligning the trajectory of low charge electron

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beam by zeroing the e-beam displacement in each BPM. For properly shielded cooling section this step automatically guarantees [4] that electron beam trajectory angles throughout most of the CS are below 100 urad.

3. Turning on and performing beam-based alignment of the CS solenoids.


Figure 2: A solenoid magnetic center is shifted by $d_{x}$ from and the solenoid is inclined by an angle $\alpha_{x}$ with respect to the optimal electron trajectory.

## CS BPM ALIGNMENT

The electron beam consists of macro-bunches repeated with 9 MHz frequency, which coincides with the frequency of stored ion bunches traveling through the cooling section. Each macro-bunch consists of 30 bunches repeated with the frequency of 704 MHz .
The following general algorithm for the alignment of the CS BPMs is suggested.

1. Send the e-beam with desired current through the CS and observe the intensity of the CS BPM signals with 9 MHz band pass filter.
2. Stop the electron beam and inject the ion beam of such current that the CS BPM signals have the same intensity that they did for the e-beam.
3. Set the desired ion trajectory through the cooling section using well-calibrated existing RHIC BPMs and centering the ion beam in the nearby RHIC quads.
4. Measure the positions of the ions in the CS BPMs with 9 MHz band pass filter. We will call these positions BPM9i0.
5. Turn off ions, turn on electrons and measure electron beam positions in the CS BPMs using 9 MHz filter. We call these positions BPM9e0.
6. Switch to 704 MHz filter, which will be routinely used to measure positions of electrons, and measure electron positions BPM704e0.

Apparently, the ion and electron beam trajectories coincide when BPM9e0=BPM9i0. Then, the optimal (absolute zero) electron beam position measured with 704 MHz filter is BPM704e $=$ BPM704e $0+($ BPM9i0-BPM9e0 $)$.

## CS SOLENOID ALIGNMENT

Each CS solenoidal module consists of the main solenoid and two weak correcting anti-solenoids (bucking coils), which reduce the longitudinal tails of the field. The field profile (see Fig. 3) of solenoids was modelled and measured for each module.


Figure 3: Measured (orange) and hard-edge approximation (pink) field of the CS solenoidal module.

To simulate motion of the e-beam centroid in magnetic field we use an approach suggested in [5]. Introducing $\xi=x+i y, \theta=\theta_{x}+i \theta_{y}$ and $B_{\perp}=B_{x}+i B_{y}$ we can write equation of motion in a paraxial approximation as:

$$
\left\{\begin{array}{c}
\xi^{\prime}=\theta  \tag{1}\\
\theta^{\prime}=\frac{i}{B \rho}\left(B_{\perp}-B_{z} \theta\right)
\end{array}\right.
$$

Here a prime defines differentiation with respect to longitudinal coordinate $z$.
In absence of additional transverse fields $\left(B_{\perp}\right)$ the off axis field is due to the non-uniformity of $B_{z}$ only and from Maxwell equations:

$$
\begin{equation*}
B_{\perp}=-\frac{\xi}{2} \frac{d B_{Z}}{d z} \tag{2}
\end{equation*}
$$

The stable numerical solution of (1) can be obtained with an implicit method:

$$
\left\{\begin{array}{c}
\xi^{n+1}=\xi^{n}+\frac{\Delta z}{2}\left(\theta^{n}+\theta^{n+1}\right)  \tag{3}\\
\theta^{n+1}=\frac{\theta^{n}+i \frac{\Delta z}{2 B \rho}\left(B_{\perp}{ }^{n}+B_{\perp}{ }^{n+1}-B_{Z}{ }^{n} \theta^{n}\right)}{1+i \frac{\Delta z}{2 B \rho} B_{Z}{ }^{n+1}}
\end{array}\right.
$$

Since the highest peak field of solenoids is $\sim 200 \mathrm{G}$, the limit on acceptable mechanical displacements of the CS solenoid calculated for such field will be a reasonable estimate of the tolerance of solenoid alignment.
Simulating beam motion through measured field according to (3) we find that the CS solenoid inclination with respect to the design beam trajectory must be measured and corrected with accuracy much better than 0.5 mrad and the displacement of the CS solenoid magnetic center with respect to the design beam trajectory must be measured and corrected with accuracy much better than 0.5 mm .

The hard-edge solenoid (see Fig. 3) field (B) and length $(L)$ shall satisfy the relations:

$$
\begin{equation*}
B=\frac{\int B(z)^{2} d z}{\int B(z) d z}, L=\frac{\left(\int B(z) d z\right)^{2}}{\int B(z)^{2} d z} \tag{4}
\end{equation*}
$$

To check the obtained approximation we compare results of simulations (3) to the beam trajectory found from a standard solenoid transfer matrix:

$$
M_{\text {sol }}=\left(\begin{array}{cccc}
\frac{1+c}{2} & \frac{s}{k} & \frac{s}{2} & \frac{1-c}{k}  \tag{5}\\
-\frac{k s}{4} & \frac{1+c}{2} & -k \frac{1-c}{4} & \frac{s}{2} \\
-\frac{s}{2} & -\frac{1-c}{k} & \frac{1+c}{2} & \frac{s}{k} \\
k \frac{1-c}{4} & -\frac{s}{2} & -\frac{k s}{4} & \frac{1+c}{2}
\end{array}\right)
$$

Here $s \equiv \sin k L, \quad c \equiv \cos k L, \quad k=\frac{B}{B \rho}$
We find a perfect agreement between the beam trajectory in the hard-edge model and in the real distribution of the solenoid field (Fig 4).


Figure 4: An example of the BPM readings simulated (red crosses) according to (3) and found in the hard-edge approximation (blue line). The BPM is located at 3 m from solenoid center. Beam trajectory at the entrance to the solenoid field region has an input angle $\theta_{x}=-10 \mathrm{mrad}$. $\mathrm{I}_{\text {sol }}=1$ corresponds to 205.7 G peak field of the solenoid.

Hence, we will use the hard-edge approximation to find the fitting function for the beam trajectory through the misaligned solenoid.

If the solenoid magnetic center is shifted from the ideal e-beam trajectory by $d_{x}$ and $d_{y}$, then beam trajectory at the exit of the solenoid is given by:

$$
\left(\begin{array}{c}
x_{\text {shift }}  \tag{6}\\
\theta x_{\text {shift }} \\
y_{\text {shift }} \\
\theta y_{\text {shift }}
\end{array}\right)=M_{\text {sol }}\left(\begin{array}{c}
d_{x} \\
0 \\
d_{y} \\
0
\end{array}\right)-\left(\begin{array}{c}
d_{x} \\
0 \\
d_{y} \\
0
\end{array}\right)
$$

The effect of the solenoid inclined by angles $\alpha_{x}$ and $\alpha_{y}$ on the beam trajectory is given by:

$$
\left(\begin{array}{c}
x_{\text {incl }}  \tag{7}\\
\theta x_{\text {incl }} \\
y_{\text {incl }} \\
\theta y_{\text {incl }}
\end{array}\right)=M_{\text {sol }}\left(\begin{array}{c}
-L \alpha_{x} / 2 \\
\alpha_{x} \\
-L \alpha_{y} / 2 \\
\alpha_{y}
\end{array}\right)-\left(\begin{array}{c}
L \alpha_{x} / 2 \\
\alpha_{x} \\
L \alpha_{y} / 2 \\
\alpha_{y}
\end{array}\right)
$$

Cooling section solenoids are combined with transverse correctors. According to the procedure outlined in Introduction these correctors will be set to compensate the effect of the residual earth field. In the model suggested in [4] these correctors will be providing 0.17 mrad bending angle (for 1.6 MeV beam). Due to the $\mathrm{x}-\mathrm{y}$ rotation in solenoidal field there will be an additional effect of trans-
verse correctors on beam trajectory for the turned on solenoid. This effect is small but not negligible.

In these studies we assume that longitudinal distribution of the correctors' field repeats the field distribution of the solenoidal module.

Solving (1) in a hard-edge approximation we obtain the following formula for effect of transverse correctors in solenoidal field on the beam trajectory at solenoid exit:

$$
\left(\begin{array}{c}
x_{c o r r}  \tag{8}\\
\theta x_{c o r r} \\
y_{c o r r} \\
\theta y_{c o r r}
\end{array}\right)=\left(\begin{array}{l}
\frac{B_{y}(c-1)+B_{x}(k L-s)}{B k} \\
\frac{B_{x}(1-c)-B_{y}(s+k L)}{2 B} \\
\frac{B_{x}(1-c)+B_{y}(k L-s)}{B k} \\
\frac{B_{y}(1-c)+B_{x}(s+k L)}{2 B}
\end{array}\right)-\left(\begin{array}{c}
-\frac{B_{y} L^{2}}{2 B \rho} \\
-\frac{B_{y} L}{B \rho} \\
\frac{B_{x} L^{2}}{2 B \rho} \\
\frac{B_{x} L}{B \rho}
\end{array}\right)
$$

It is worth mentioning that in the model suggested in [4] the beam will be entering the CS solenoid with a small angle of about 20 urad. The effect from this angle on beam-based alignment procedure is negligibly small. In the following considerations we include these small angles into the numerical model but we do not account for the effect from these angles in the fitting functions.
To find the inclination and the shift of each CS solenoid we will measure dependence of the readings of the downstream BPM (located at distance $z$ from the solenoid exit) on the solenoid current and fit obtained dependence with the following fitting functions:

$$
\left\{\begin{array}{c}
x_{B P M}=x_{\text {shift }}+x_{\text {incl }}+x_{\text {corr }}+  \tag{9}\\
z \cdot\left(\theta x_{\text {shift }}+\theta x_{\text {incl }}+\theta x_{\text {corr }}\right) \\
y_{B P M}=y_{\text {shift }}+y_{\text {incl }}+y_{\text {corr }}+ \\
z \cdot\left(\theta y_{\text {shift }}+\theta y_{\text {incl }}+\theta y_{\text {corr }}\right)
\end{array}\right.
$$

The fit will be obtained by minimizing:

$$
\begin{equation*}
\chi^{2}=\frac{1}{N-4} \sum_{n=1}^{N}\left\{\left[\frac{x_{n}-x_{B P M}}{\sigma_{x}}\right]^{2}+\left[\frac{y_{n}-y_{B P M}}{\sigma_{y}}\right]^{2}\right\} \tag{10}
\end{equation*}
$$

Here $x_{n}$ and $y_{n}$ are BPM readings corresponding to solenoid field $B_{n}$, and $\sigma_{x}$ and $\sigma_{y}$ is RMS spread of BPM readings.

We modelled the outlined beam -based alignment procedure. We used (3) to simulate $x_{n}$ and $y_{n}$, assumed 10 um for both $\sigma_{x}$ and $\sigma_{y}$, and chose $\mathrm{N}=21$.

As a result, even if we use just one BPM (at 3.2 m from solenoid center in our model) for the fitting, the solenoid shift can be found with $\sim 90$ um accuracy and solenoid inclination can be found with an accuracy of about 70 urad.

We can improve this result by using two BPMs for fitting - one at 3.2 m downstream of solenoid center and one at 6.2 m downstream of the solenoid. In that case we obtain the solenoid shift with 40 um accuracy and solenoid inclination with 30 urad accuracy. Figure 5 shows an example of the fitting performed with two BPMs.

## CONCLUSION

In this paper we considered the steps required to align the electron beam trajectory through the LEReC cooling section. We devised a detailed procedure for the
beam-based alignment of the cooling section solenoids. We modelled the alignment procedure and showed that with two BPM fitting the solenoid shift can be measured with 40 um accuracy and the solenoid inclination can be measured with 30 urad accuracy. These accuracies are well within the tolerances of the cooling section solenoid alignment.


Figure 5: Fitting (green curve) BPM readings (blue crosses) with two BPMs. First BPM (two top plots) is located at 3.2 m downstream of the solenoid center and second BPM (two bottom plots) is located at 6.2 m downstream of the solenoid. In this example $d_{x}=5 \mathrm{~mm}, d_{y}=$ $-2 \mathrm{~mm}, \alpha_{x}=1.5 \mathrm{mrad}, \alpha_{y}=-3 \mathrm{mrad} . \mathrm{I}_{\mathrm{sol}}=1$ corresponds to 205.7 G peak field of the solenoid.

## REFERENCES

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