# MATCHING SPACE-CHARGE DOMINATED ELECTRON BUNCHES INTO THE PLASMA ACCELERATOR AT SINBAD 

J. Zhu ${ }^{1,2}$, R. Assmann ${ }^{1}$, U. Dorda ${ }^{1}$, B. Marchetti ${ }^{1}$<br>${ }^{1}$ Deutsches Elektronen-Synchrotron, DESY, Hamburg, Germany<br>${ }^{2}$ Universität Hamburg, Hamburg, Germany

## Abstract

The SINBAD facility (Short and INnovative Bunches and Accelerators at DESY) is foreseen to provide sub-fs to tens of fs electron bunches for Laser Wake-Field Acceleration (LWFA) experiments. In order to avoid emittance growth in plasma cells with ultra-high accelerating gradients the injection and transport of electron bunches with beta functions of mm-size or even smaller are required. This kind of bunch is usually spacecharged dominated since the energy is low ( $\sim 100 \mathrm{MeV}$ ) while the peak current is high for allowing the electron bunches to be used for Free Electron-Laser (FEL) generation. We present the beamline design and explore the possible beam parameters at the SINBAD linac by start-to-end simulations.

## INTRODUCTION

The ARES linac hosted at SINBAD aims to produce sub-fs to several fs electron bunches with excellent bunch arrival-time jitter for the development of novel concept accelerators [1]. Matching of these electron bunches into the focusing field of an accelerating structure is of vital importance for preserving the beam quality. Assuming a constant focusing channel, the Twiss parameters for a matched emittance-dominated beam are given by

$$
\begin{equation*}
\beta_{\mathrm{x}}=\beta_{\mathrm{y}} \approx 1 / \sqrt{\mathrm{K}_{\mathrm{r}}}, \alpha_{\mathrm{x}}=\alpha_{\mathrm{y}} \approx 0 \tag{1}
\end{equation*}
$$

where $K_{r}$ is the focusing strength. For a plasma accelerator, $K_{r}$ is given by [2]

$$
\begin{equation*}
K_{r}(\xi)=\frac{2 k_{p} \sigma_{z, L}}{\gamma} \sqrt{\frac{\pi}{2}} \frac{a_{0}^{2}}{w^{2}(z)} \exp \left(-\frac{k_{p}^{2} \sigma_{z, L}^{2}}{2}\right) \sin \left(k_{p} \xi\right) \tag{2}
\end{equation*}
$$

in the linear regime and

$$
\begin{equation*}
K_{r}=k_{p}^{2} / 2 \gamma \tag{3}
\end{equation*}
$$

in the blow-out regime. Here $k_{p}$ is the plasma wave number, $\gamma$ is the Lorentz factor, $a_{0}$ is the laser strength parameter, $w(z)$ is the laser spot size, $\sigma_{z, L}$ is the rms laser pulse length and $\xi$ is the co-moving coordinate.

The matched beta function is typically extremely small ( $<1 \mathrm{~mm}$ ) for a $\sim 100 \mathrm{MeV}$ externally injected beam and a hard-edge plasma. However, the requirement on the matched beta function can be relaxed by using a tailored longitudinal plasma profile (up-ramp) [3], i.e. a proper density transition from the vacuum to the plasma channel with constant density. A mismatched beam will undergo
betatron oscillation inside the plasma. The varying betatron frequency along the bunch, which is caused by both the finite bunch length (only in the linear regime) and the energy chirp, will result in a considerable projected emittance growth [2].

Due to the strong space-charge effects of the compressed beam downstream of the chicane [7], a long transfer beamline between the chicane and the final focusing triplet should be avoided. This excludes the choice of using sextupole magnets to correct the chromatic aberration. As a result, a PMQ triplet is used to strongly focus the transverse beam size immediately downstream of the chicane bunch compressor, as shown in Figure 1. The Twiss parameters of the beam at the entrance of the PMQ triplet can be finely adjusted by six quadrupole magnets upstream of the chicane.


Figure 1: Cartoon of the main beamline from the entrance of the matching section to the focus.

## FINAL FOCUSING

By taking into account the beam emittance and the chromatic aberration, the beta function $\beta_{x_{f}}$ at the focal point is given by

$$
\begin{equation*}
\beta_{x_{f}}=f^{2} / \beta_{x_{i}}+\sigma_{\delta}^{2} \beta_{x_{i}} \tag{4}
\end{equation*}
$$

where $\beta_{x_{i}}$ is the incoming beta function, $\delta=\Delta p / p$ denotes the fractional momentum spread with $p_{0}$ being the reference momentum and $\Delta p=p-p_{0}$. According to equation (4), the minimum achievable beta function is $2 f \sigma_{\delta}$ at $\beta_{x_{i}}=f / \sigma_{\delta}$. Since a considerable energy spread must be introduced to compress the bunch, a shorter focal length is desired. The gradient of the permanent magnet quadrupole (PMQ) can reach as high as $600 \mathrm{~T} / \mathrm{m}$ [4], and the focal length of a PMQ triplet can be as short as several cm for $\mathrm{a} \sim 100 \mathrm{MeV}$ beam.

A PMQ triplet with a symmetric incoming beam at its waist is considered, as illustrated in Figure 2. The transfer matrix of this triplet is given by

$$
\mathrm{M}_{\mathrm{FF}}=\left[\begin{array}{cc}
0 & 2 f  \tag{5}\\
-1 /(2 f) & \mp 1
\end{array}\right]
$$

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This solution maximizes the convergence angle at the focus for a given maximum obtainable focusing strength of a single PMQ [5].


Figure 2: Illustration of the triplet optics for the symmetric focus.

## CHROMATIC EFFECT

The influence of the chromatic effect was first evaluated by beam dynamics simulations from the entrance of the PMQ triplet to the focus using ELEGANT [6]. An incoming cylindrical symmetric Gaussian beam $(\alpha=0)$ with transverse emittance of $0.2 \mu \mathrm{~m}$ was assumed. The PMQ triplet used in these simulations composed of three 2-cm-long PMQs with gradients of $250 \mathrm{~T} / \mathrm{m}, 500$ $\mathrm{T} / \mathrm{m}$ and $500 \mathrm{~T} / \mathrm{m}$ respectively. The beta functions of the incoming beam and the locations of the PMQs were adjusted to focus beams with different energies to $\beta_{x}=\beta_{y} \approx 1 \mathrm{~mm}$ and $\alpha_{x}=\alpha_{y} \approx 0$ at the fixed focal spot. The results are shown in Figure 3 and Figure 4. Here the mismatch factor shown in Figure 4 is defined as

$$
\begin{equation*}
M_{S}=B_{m a g}+\sqrt{B_{m a g}^{2}-1} \tag{6}
\end{equation*}
$$

where $B_{\text {mag }}$ is given by

$$
\begin{equation*}
B_{m a g}=\left(\beta \gamma_{m}+\beta_{m} \gamma-2 \alpha \alpha_{m}\right) / 2 \tag{7}
\end{equation*}
$$

with $\alpha_{m}, \beta_{m}$ and $\gamma_{m}$ the matched Twiss parameters.


Figure 3: Transverse emittance growths at the focus for different beam momentums and rms momentum spreads.


Figure 4: Same as Figure 3 but the distributions of the mismatch factors in both planes are shown.
The PMQ triplet composed of three 2-cm-long PMQs with gradients of $250 \mathrm{~T} / \mathrm{m}, 500 \mathrm{~T} / \mathrm{m}$ and $500 \mathrm{~T} / \mathrm{m}$
respectively. The incoming beta function ranges from 6 m to 22 m .

Although the incoming beam is cylindrical symmetric, the influences of the chromatic effect in the horizontal and vertical planes are not equal. At the first PMQ, the beam will be focused in one plane and defocused in the other. In the plane where the beam is defocused, the emittance growth due to the chromatic effect will be stronger because of the larger beta function. Therefore, one should be careful about the sign of the PMQ gradient if the emittance and mismatch factor in one plane is more important than in the other plane.

The beam energy affects the chromatic effect in several competitive ways. Although the fractional energy spread of the beam decreases as the energy increases, the beta function of the incoming beam increases as the beam energy increases given the PMQ gradients. Moreover, the focal length and the distances between PMQs also increase as the beam energy increases, which affect the beta functions at the second and third PMQ. The simulation results show that the beam energy does not have a significant impact on the emittance and the mismatch factor. However, it must be pointed out that the matched beta function increases as the beam energy increases according to equation (1-3). In other words, the matching condition is more relaxed for a beam with higher energy. In practice, nevertheless, one must also consider the constraint of beam optics at the entrance of the PMQ triplet.

The chromatic effect increases monotonically as the energy spread increases given the beam energy. For an energy spread of about $0.25 \mathrm{MeV}(0.25 \%$ for a 100 MeV beam), the emittance growth and beam mismatch in the horizontal plane are both negligible. However, the emittance growth in the vertical plane is about $20 \%$ and the mismatch factor is about 1.2.

As a comparison, the simulation results using shorter (1-cm-long) PMQs with the same gradients are shown in Figure 5 and Figure 6. It is apparent that the influences of the energy spread become much more significant due to the much larger incoming beta functions.


Figure 5: Same as Figure 3 but the length of the PMQ is reduced to 1 cm . The incoming beta function ranges from 18 m to 77 m .

As mentioned previously, the matched beta function can be increased by using a tailored longitudinal plasma profile. Considering that the matched beta functions increase to $\beta_{x}=\beta_{y} \approx 10 \mathrm{~mm}$, there are two options to adapt the focusing system in the previous case. One is to
reduce the incoming beta functions, and the other is to use a weaker PMQ triplet.


Figure 6: Same as Figure 5 but the distributions of the mismatch factors in both planes are shown.

Considering a $100-\mathrm{MeV}$ incident electron bunch, the incoming beta functions need to be smaller than 2 m in order to use the same PMQ triplet. However, this choice will significantly increase the space-charge effects before the PMQ triplet, particularly during the bunch compression in the chicane.

For the other option, a weaker PMQ triplet ( $100 \mathrm{~T} / \mathrm{m}$, $200 \mathrm{~T} / \mathrm{m}, 200 \mathrm{~T} / \mathrm{m}$ ) is considered and the simulation results are shown in Figure 7 and Figure 8. Although the chromatic effects are suppressed significantly and the incoming beta functions are comparable to the previous case, the total length of the triplet largely increases from $\sim 0.18 \mathrm{~m}$ to $\sim 0.48 \mathrm{~m}$. For a space-charge dominated electron bunch, the degradation of the beam quality due to the space-charge effects in this extra distance must be taken into account.


Figure 7: Same as Figure 5 but the gradients of the PMQ triplet reduce to $100 \mathrm{~T} / \mathrm{m}, 200 \mathrm{~T} / \mathrm{m}$ and $200 \mathrm{~T} / \mathrm{m}$. The incoming beta function ranges from 10 m to 46 m .


Figure 8: Same as Figure 7 but the distributions of the mismatch factors in both planes are shown.

## SPACE-CHARGE EFFECT

Figure 9 shows a start-to-end simulation result using ASTRA (linac) [9] and IMPACT-T (after linac) [10]. The distance between the exit of the chicane and the focal point is about 1.5 m in consideration of the spaces for the BAC, BPM, screen, collimator, vacuum valve, etc. The electron bunch is compressed by a hybrid-compression
scheme. The slit collimator located between the second and third dipole magnet of the chicane is used to slice the central part of the beam, so that the high-order terms in the longitudinal profile and the energy spread can be controlled [8]. Due to the space-charge effects, the final beta functions are 4.5 mm and 0.9 mm in the horizontal and vertical planes, respectively, which differ from the optics design. However, the mismatch can be further improved by optimization.


Figure 9: Start-to-end simulation result. The first row: linac exit; the second row: chicane exit and the third row: plasma entrance. The final bunch charge is about 17 pC with $\varepsilon_{x} \approx 0.33 \mu \mathrm{~m}$ and $\varepsilon_{y} \approx 0.52 \mu \mathrm{~m}$.

## CONCLUSION

In this paper, we have discussed various considerations in the design of the final focusing beamline at the ARES linac. The space-charge effects limit the space and choice of the final focusing system. In the start-to-end simulation, a $100-\mathrm{MeV}, 17-\mathrm{pC}, 12-\mathrm{fs}$ ( $\sim 1.5 \mathrm{kA}$ peak current) electron bunch with mm scale beta functions has been achieved.

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