UPDATE ON BMAD SIMULATIONS FROM TARGET TO STORAGE RING FOR THE NEW MUON $g-2$ EXPERIMENT AT FERMILAB*

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Abstract

The new muon g-2 experiment at Fermilab (E989) aims to measure the anomalous magnetic moment of the muon to an uncertainty of 140 ppb. The existing accelerator facility at Fermilab is being adapted to the requirements of the g-2 experiment and the baseline lattice design is now established. This paper presents the results of beam simulations and spin tracking carried out using the Bmad software package for the g-2 beam transport system, including a variant which bypasses the delivery ring as proposed for the beam commissioning.

INTRODUCTION

The anomalous magnetic moment is a fundamental constant which relates the magnetic moment and the spin. Precise experimental determination of the anomalous magnetic moment of the muon (muon anomaly) $a_\mu = (g-2)/2$ has the potential to reveal the effects of particle physics beyond the Standard Model.

The E821 experiment at Brookhaven determined the muon anomaly to a precision of 540 ppb in 2004. The new muon g-2 experiment at Fermilab (E989) [1] will measure the muon anomaly to an unprecedented precision of 140 ppb. The key improvement of the new muon g-2 measurement is that the Fermilab accelerator facility after significant modifications can provide a high intensity, pure muon beam to increase the statistics for the g-2 experiment.

A high intensity 8 GeV proton beam coming from the Recycler Ring impinges on a nickel alloy target and produces a beam of secondary particles which consists mainly of protons and positive pions. The target station is optimized for maximum production of positive pions since the g-2 experiment will utilize 3.1 GeV positive muons born from the pion decay. The secondary particles are focused by a lithium lens and then pass through a pulsed dipole magnet which selects positively-charged particles with momenta in the range ±10% from 3.1 GeV. The M2M3 beamline has been rebuilt to efficiently capture muons from pion decay (most of the pions decay in this beamline) and transport the mixed beam to the Delivery Ring (DR). Effectively, the rest of the pions decay before the third turn in the DR, while on the fourth turn the longitudinal separation between the protons and muons is sufficient for a fast kicker to remove cleanly the trailing proton beam without the loss of muons. Then, on the same turn, a beam of longitudinally-polarized muons with "magic" momentum $p_m = 3.094$ GeV/c is extracted into the newly-built M4M5 beamline that transfers it to the storage ring where the muons circulate in a highly-uniform magnetic field and decay into positrons and neutrinos. For muons with the magic momentum, the difference in angular frequency between the precession of muon spin and the muon momentum is given by a simple formula: $\omega_a = -\frac{e}{m} a_\mu B$. The muon anomaly a_μ can be determined by measuring two quantities: the storage ring magnetic field B using a number of NMR probes, and the angular frequency ω_a using a variety of detectors including trackers and calorimeters distributed inside the storage ring. The measured energies of the positrons are correlated with the directions of the muon spin vectors. After tuning the calorimeters to register only high-energy positrons, the number of positrons detected is modulated by the angular frequency ω_a as the spin precesses.

RESULTS OF PARTICLE TRACKING

Our ultimate goal is to construct a model that can be used for particle and spin tracking, including all of the necessary components and features to be able to investigate a range of effects contributing to the systematic error on the measurement of g-2.

In the g-2 team at the Cockcroft Institute, the main simulation code being used for the model is Bmad [2]. At each stage of development, comparisons have been made with simulation results from other codes and models (including, for example, G4Beamline [3]), to validate the results from the Bmad model.

At present, a model corresponding to the baseline engineering design has been developed and allows particle and spin tracking from the target, through the M2M3 transport lines, Delivery Ring, M4M5 transport lines and storage ring injection channel to the exit of the inflector, though further work is still needed to include an accurate representation of the full injection process through the inflector. The model also allows particle and spin tracking of the stored muon beam within the storage ring [4]. The simulation includes the dominant particle decay processes and detailed models of the magnet apertures that makes it possible to evaluate the population of muons (and other particle types) at different points along the beam transport lines, as well as the 6D phase space distribution and polarization.

The initial particle distributions used in the tracking simulations are based on the 6D distribution of secondary particles (generated in MARS [5]) 43 cm downstream of the

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lithium lens using 10^{11} protons incident on the target (POT). The evolution of particle populations along the beamlines including 4 turns in the DR was previously presented in [6]. In the updated study presented in this paper significant improvements have been made to a special module which was developed in F90 to include pion decay kinematics in the Bmad tracking.

Spin Polarization

Understanding the spin distribution of muons injected into the storage ring will be crucial for determining the systematic error on the g-2 measurement. Figure 1 shows the distribution of spin at the inflector entrance after four turns in the delivery ring found from Bmad spin tracking for the lattice without any alignment imperfections. The results of particle tracking performed using G4Beamline are shown as a black spots in Fig. 1. There is good agreement between the two different codes regarding the spin distributions of the muon beam. The longitudinal z and horizontal x spin polarizations are -0.63 with standard deviation 0.17, and -0.73 with standard deviation 0.14, respectively, while the vertical y polarization is zero on average with standard deviation 0.16. The spin polarization is directed 0.858 rad from the beam direction.

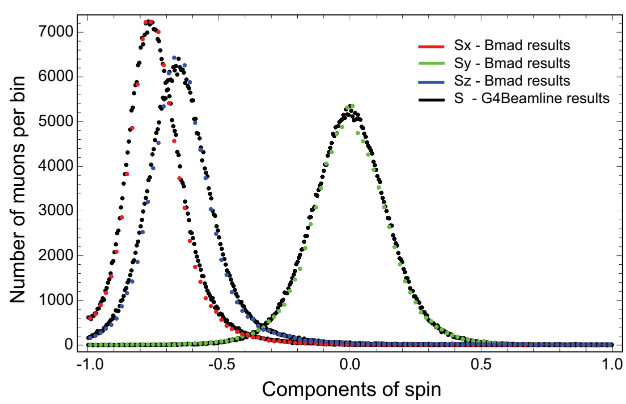

Figure 1: Spin distributions at the end of M4M5 beamline.

Tolerance to Alignment Errors

Recent work on the section of the model from the pion production target to the storage ring has focused on investigating the impact of magnet alignment errors on the population, phase space distribution and polarization of the muon beam at the end of the M4M5 beam line (beginning of the inflector into the storage ring).

Beam simulations have been performed with different seeds of horizontal and vertical alignment errors on all dipoles and quadrupoles, with rms 250 μm, and then with added tilt errors with rms 5 mrad. No correction or tuning algorithms (e.g. for beam trajectory or dispersion) have been implemented, so the results in Table 1 indicate the sensitivity of various muon beam quantities to the errors, rather than representing a realistic estimate of what may be expected in machine operation. The main impact is on the population of the muon beam: with 250 μm rms alignment errors and 5 mrad tilt errors, the muon population at the end of the M4M5 beam line is reduced by roughly 46% (averaged over 20 seeds of alignment errors). There is relatively little impact on the beam emittance ϵ_x and ϵ_y, energy spread or polarization. The beam essentially fills the acceptance of the beam line from the target to the storage ring, which is limited by the physical apertures: alignment and tilt errors have little impact on the acceptance, but the orbit distortion resulting from the errors leads to a larger number of particles falling outside the acceptance.

Table 1: Parameters of muon beam at the end of the M4M5 beamline as a results of particle tracking performed with 20 different seeds of horizontal and vertical alignment errors on all dipoles and quadrupoles, with rms 250 μm, and with added tilt errors of rms 5 mrad.

	rms 250 μm	rms 250 μm rms 5 mrad
μ^+ per POT $\times 10^{-7}$	4.8 ± rms 0.7	4.2 ± rms 0.8
ϵ_x, mm.mrad	10.1 ± rms 0.5	10.1 ± rms 0.7
ϵ_y, mm.mrad	9.3 ± rms 1.0	9.5 ± rms 1.2
rms $\Delta p/p$	1.17%	1.1%

Beam Commissioning Scenario

The beam commissioning plan for the g-2 experiment has been established. It will include a phase of the beam delivery to the storage ring straight through the injection/extraction section of the DR, i.e. without the beam circulating around the DR (the injection and extraction kickers share the same straight section). Figure 2 shows the evolution of the population of secondary particles from the target to the storage ring found from tracking simulations performed for the commissioning scenario. Table 2 summarizes and compares the beam parameters of all the particles at the inflector entrance and the particles within the momentum acceptance of the storage ring, 0.5%. Figure 3 shows the momentum distribution of muons in which 83% of muons are centered on the magic momentum within one standard deviation of 2.1% (the main peak) while the rest of the muons (low-momentum tail) have a momentum below 2.9 GeV/c. The momentum of a newly-born muon can differ from the momentum of the parent pion by a factor ranging from 0.57 to 1.0 with uniform probability. A significant loss of muons with momentum outside ±2% from 3.094 GeV/c occurs in the achromatic bend of the M2M3 beamline (around s = 160 m) and in the change in elevation of the M4M5 beamline (around s = 330 m) due to horizontal and vertical dispersion, respectively. Pions continue to decay throughout the last 100 m of the M4M5 beamline. This, first, contributes to the number of low-momentum muons and second, increases the transverse emittance of muon beam.

Figure 4 shows the longitudinal and transverse spin distributions at the inflector entrance. The vertical and horizontal

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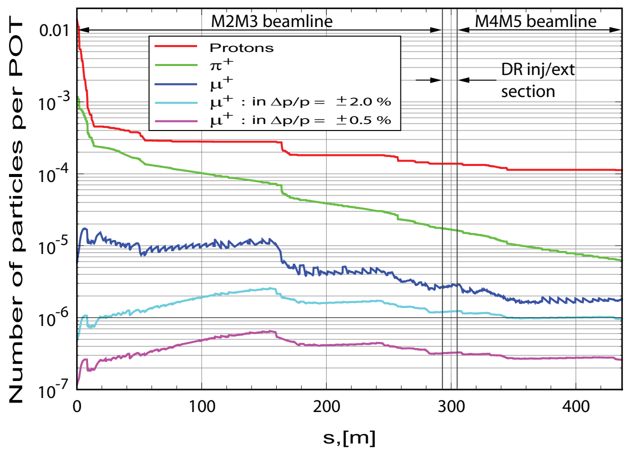

Figure 2: Number of particles per POT along the M2M3 beamline, injection/extraction straight section of DR and M4M5 beamline, showing protons (red line), pions (green line), muons (blue line). Muons with momentum in the range of ±2% and ±0.5% with respect to 3.094 GeV/c are shown by light blue and magenta lines, respectively.

Table 2: Beam Parameters at the End of M4M5 Beamline

	within $\pm\Delta p/p_m$	Muons	Pions	Protons
Number of particles per POT $\times 10^{-7}$	100 %	18	62	1100
	2 %	9.5	38	680
	0.5 %	2.6	10	180
Emittance ϵ_x rms, [mm·rad]	100 %	63	9.3	9.5
	0.5 %	16.6	8.2	8.3
Emittance ϵ_y rms, [mm·rad]	100 %	72	9.4	9.6
	0.5 %	17.8	8.7	8.9
Average Polarization, [%]	100 %	−80	n/a	n/a
	0.5 %	−96	n/a	n/a

spin polarization is zero on average with one standard deviation of 0.3 while the longitudinal polarization is 80%, being affected by the presence of a small number of low-momentum muons. Taking into account the momentum acceptance of the storage ring, the longitudinal polarization for muons with momentum ±0.5% from 3.094 GeV/c is 96%.

The beam population is dominated by protons with a ratio of 60 protons per muon in total, and 70 protons per muon within the momentum acceptance of the storage ring. The bunch length of both proton and muon beams is 40 m. The velocity difference between protons and muons leads to a longitudinal separation of 19.5 m (65 ns) between the centres of mass of the proton and muon beams at the end of the M4M5 beamline. Consequently, the back half of the muon beam is overlapped by protons.

The results of Bmad tracking simulations performed for the beam commissioning scenario are in good agreement with the results [7] obtained using the G4Beamline code.

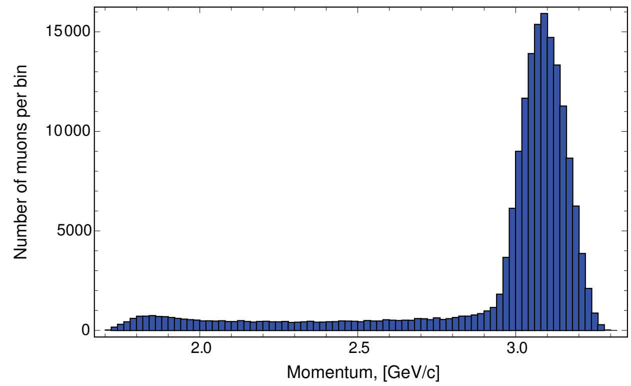

Figure 3: Momentum distribution of muons at the end of M4M5 beamline.

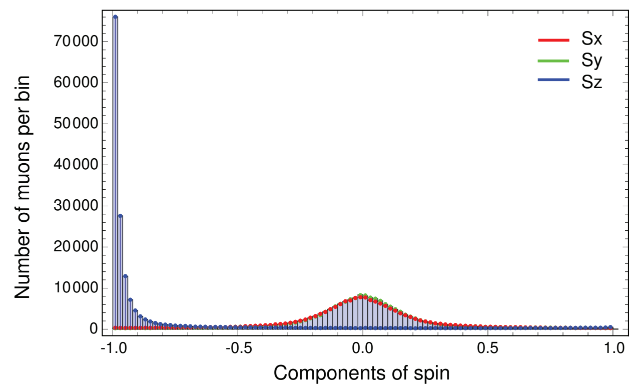

Figure 4: Spin distributions at the end of M4M5 beamline.

CONCLUSIONS

A basic model of the beam delivery from the target to the storage ring has been developed in Bmad for the nominal and commissioning scenarios. Beam simulations based on the tracking of particles with decay processes have been carried out to characterize the beam dynamics and spin polarization. The results of the spin tracking show that the polarization of muons within the momentum acceptance of the storage ring is not less than 96%, as required, for both the nominal and the commissioning scenarios. Lattice alignment errors with rms 250 μm have relatively little impact on the phase space and spin polarization but affect significantly the population of muons. A number of muons 2.6×10^{-7} per POT with momentum within ±0.5% from 3.094 GeV/c can be expected in the commissioning scenario for the lattice without any imperfections. There is good agreement between the results from Bmad and from an independent model in G4Beamline, regarding the spin distributions, population of particles along the transport lines and the beam parameters at the inflector entrance.

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