# PHASE SPACE FOLDING STUDIES FOR BEAM LOSS REDUCTION DURING RESONANT SLOW EXTRACTION AT THE CERN SPS

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*Abstract*

The requested number of protons slow-extracted from the CERN Super Proton Synchrotron (SPS) for Fixed Target (FT) physics is expected to continue increasing in the coming years, especially if the proposed SPS Beam Dump Facility is realised. Limits on the extracted intensity are already being considered to mitigate the dose to personnel during interventions required to maintain the extraction equipment, especially the electrostatic extraction septum. In addition to other on-going studies and technical developments, a reduction of the beam loss per extracted proton will play a crucial role in the future performance reach of the FT experimental programme at the SPS. In this paper a concept is investigated to reduce the fraction of beam impacting the extraction septum by folding the arm of the phase space separatrix. Beam dynamics simulations for the concept are presented and compared to the phase space acceptance of the extraction channel. The performance potential of the concept at SPS is evaluated and discussed alongside the necessary changes to the non-linear optical elements in the machine.

## INTRODUCTION

At the CERN SPS a standard sextupole-driven slow extraction at a 1/3-integer tune [1] takes place. During such an extraction some particles will hit the anode wires of the electrostatic extraction septum (ES), causing radio-activation. Without significant improvements, this is expected to limit the number of extracted protons to far less than the expected requests of potential experiments, e.g. SHiP, at the proposed SPS Beam Dump Facility [2].

It was shown in [3, 4] that higher order multipoles can be used to change the profile of the extracted beam in such a way that fewer particles will hit the ES wires, with decapoles being considered for 1/3-integer extraction in the first and octupoles and duodecapoles in the second reference. In this paper the possibility of adding decapoles to the SPS to reduce beam loss during extraction is investigated.

## HAMILTONIAN

The 2-dimensional phase space dynamics near a 1/3-integer resonance can be well approximated using the Kobayashi Hamiltonian[1] [5]

$$H = \frac{\epsilon}{2}\left(X^2 + P^2\right) + \frac{1}{4}K_2\left(X^3 - 3XP^2\right) + \frac{3}{16}K_4\left(X^2 + P^2\right)\left(X^3 - 3XP^2\right),$$

or equivalently

$$H = \frac{\epsilon}{2}A^2 + \frac{1}{4}K_2A^3\cos(3\theta) + \frac{3}{16}K_4A^5\cos(3\theta).$$

Here $\epsilon = 6\pi(Q - Q_{\text{res}})$ is a measure of the tune distance from the resonance, $X$ and $P$ are the normalised phase space coordinates, $(A, \phi)$ are the polar coordinates for $(X, P)$ and

$$K_2 = \frac{1}{2}\frac{L}{B\rho}\left[\frac{\partial^2 B_y}{\partial x^2}\right]_{x=y=0}\beta_x^{3/2}$$

$$K_4 = \frac{1}{24}\frac{L}{B\rho}\left[\frac{\partial^4 B_y}{\partial x^4}\right]_{x=y=0}\beta_x^{5/2}$$

are the normalised sextupole and decapole strengths, assuming a single thin lens of effective length $L$. For a machine with multiple sextupoles and decapoles the parameters of an equivalent virtual sextupole and decapole, which may replace the original multipoles, can be calculated in order to apply the Hamiltonian theory.

Some interesting properties of the dynamics become clear immediately upon examining this Hamiltonian. Most notably $H$ has a threefold rotational symmetry and when $K_2$ and $K_4$ differ in sign, the level set $H = -\frac{K_2}{K_4}\frac{2\epsilon}{3}$ contains the circle $X^2 + P^2 = -\frac{4}{3}\frac{K_2}{K_4}$. This property is helpful when searching for parameters that will cause the separatrix to bend in phase space inside the extraction aperture, since it defines a theoretical maximum amplitude particles originating near the centre of phase space can reach.

## SIMPLE MODEL

In order to simulate the dynamics however, we do not rely on the Hamiltonian, but rather use a simple tracking code based on thin kicks in normalised phase space. The map $R_\phi$ for a given phase advance of $\phi$ radians is simply a rotation

$$R_\phi : \begin{pmatrix} X \\ P \end{pmatrix} \mapsto \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} X \\ P \end{pmatrix},$$

and the thin lens multipole kick is described by

$$K : \begin{pmatrix} X \\ P \end{pmatrix} \mapsto \begin{pmatrix} X \\ P + K_2X^2 + K_4X^4 \end{pmatrix}.$$

Note that this model, like the Hamiltonian in the previous section, only allows phase advances of integer multiples of

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[1] We use a right-handed $(x, y, s)$ coordinate system with positive transverse directions being outward and upward, which gives subtle differences in the appearance of the Hamiltonian compared to a right-handed $(x, s, z)$ coordinate system with that same convention.


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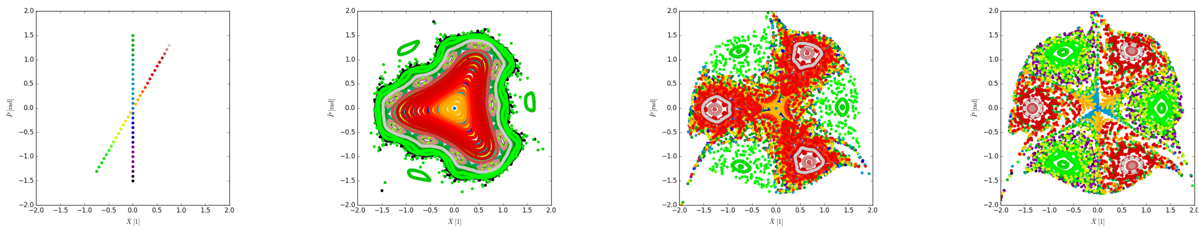
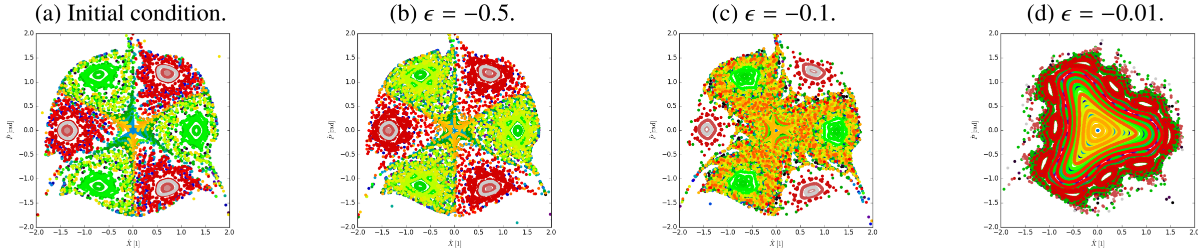


(a) Initial condition. (b) $\epsilon = -0.5$. (c) $\epsilon = -0.1$. (d) $\epsilon = -0.01$.



(e) $\epsilon = 0$. (f) $\epsilon = 0.01$. (g) $\epsilon = 0.1$. (h) $\epsilon = 0.5$.

Figure 1: Simulated normalised $(\hat{X}, \hat{P})$ phase space at the first multipole for $\kappa = -0.43$ for various values of $\epsilon$. Particles are initialised according to the first figure and tracked for 1000 turns, the coordinates of each particle at each turn is plotted in the color corresponding to that particle.

$2\pi$ between the sextupole and decapole. For future studies, the code can be easily generalised to apply two different kicks separated by a phase advance.

It is clear that the single kick described by the map $K$ will not have the threefold rotational symmetry in phase space that is present in the Hamiltonian description. In order to make the dynamics easier to understand, an approximate threefold symmetry was introduced by simulating three multipoles with a third of the original strength at a phase advance of $2\pi/3$ from each other. Furthermore, a coordinate transformation $(\hat{X}, \hat{P}) = K_2 \cdot (X, P)$ is used. These new coordinates are similar to the ones used in [6] for tracking. In this coordinate system the map describing phase advance is unchanged, but the map describing the thin-lens multipole kick reads

$$\hat{K} : \begin{pmatrix} \hat{X} \\ \hat{P} \end{pmatrix} \mapsto \begin{pmatrix} \hat{X} \\ \hat{P} + \hat{X}^2 + \kappa \hat{X}^4 \end{pmatrix},$$

where $\kappa = K_4/K_2^3$. The new coordinates show that the nature of the dynamics is fully determined by the ratio of multipole strengths $\kappa$, while variations in $K_2$ at fixed $\kappa$ only give a scaling factor for the conversion to physical coordinates. When we add fixed parameters for the extraction aperture however, this scaling in real space becomes important.

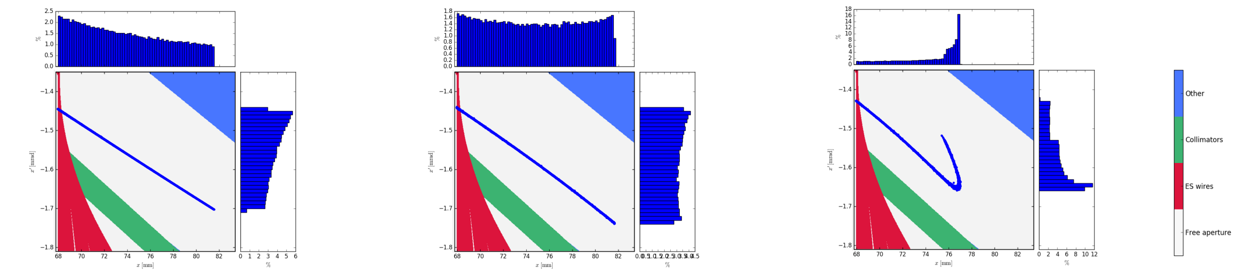
Each turn in the simulations consists of the kicks by the multipoles with the appropriate rotations between them, followed by a rotation according to the tune of the particle. An example of the simulated normalised phase space at the first multipole for various tune distances can be found in Fig. 1. For each value of $\epsilon$ an approximate threefold rotational symmetry can be seen, alongside an apparant circular boundary at roughly $\hat{X}^2 + \hat{P}^2 = \sqrt{-4/(3\kappa)}$, as expected. This circular boundary is broken at angles where the amplitude growth due to the decapole is particularly large, and significant deviations from the symmetrical expectation can be seen particularly at larger amplitudes. This demonstrates that the tracking routine will simulate more complicated dynamics than the one described by the Kobayashi Hamiltonian.

If we additionally define $(\phi_s + 2\pi)$ as the phase advance from the first multipole to the start of the ES, the tracking routine used to simulate extraction can be summarised as follows.

- Initialise particles $i \in \{1, \ldots, N_{\text{part}}\}$ with initial coordinates $(\hat{X}_0, \hat{P}_0)_i$, at the multipole.
- For every turn $j \in \{1, \ldots, N_{\text{turns}}\}$ do
  - $\begin{pmatrix} \hat{X}_j \\ \hat{P}_j \end{pmatrix}_i = \left(R_{2\pi/3} \circ \hat{K}\right)^3 \begin{pmatrix} \hat{X}_{j-1} \\ \hat{P}_{j-1} \end{pmatrix}_i$ (multipole kicks)
  - $\begin{pmatrix} \hat{X}_s \\ \hat{P}_s \end{pmatrix}_i = R_{\phi_s} \begin{pmatrix} \hat{X}_j \\ \hat{P}_j \end{pmatrix}_i$, if $\hat{X}_s$ is outside of the circulating beam aperture of the ES particle $i$ is lost/extracted in turn $j$ with these coordinates.
  - $\begin{pmatrix} \hat{X}_j \\ \hat{P}_j \end{pmatrix}_i = R_{(4\pi+\epsilon)/3} \begin{pmatrix} \hat{X}_j \\ \hat{P}_j \end{pmatrix}_i$ (phase advance due to the tune)

Here $\epsilon$ should be set for every particle according to the difference between the machine tune and the resonant tune. For the simulations presented in Fig. 2 dispersion and momentum spread were not taken into consideration and the tune was kept fixed at $\epsilon = 0$, though future studies are planned to include these variables. The nominal SPS extraction was reproduced in Fig. 2(a) by correctly setting $\phi_s$ and $K_2$, at $K_4 = 0$. The value of $\phi_s$ was then kept fixed for the other simulations.

In Fig. 2(b) the results extracted beam for $K_2 = 365\,\text{m}^{-1/2}$ and $\kappa = -0.43$ is shown. We see a clear reduction of the density at the ES wires, at 68 mm. This combination of sextupole and decapole strengths leads to a much more uniform spatial density of the extracted beam. The beam density at




(a) Nominal extraction, $K_2 = 185\,\mathrm{m}^{-1/2}$, $\kappa = 0$.

(b) More uniform beam, $K_2 = 365\,\mathrm{m}^{-1/2}$, $\kappa = -0.43$.

(c) Folded beam, $K_2 = 510\,\mathrm{m}^{-1/2}$, $\kappa = -0.24$.

Figure 2: Simulated extracted beams at the ES entrance for various parameters (a-c), where the ES anode wires are located at 68 mm. The phase space acceptance at the ES entrance is superimposed.

the ES wires is roughly 25% lower while the density inside the extraction aperture is higher.

Figure 2(c) shows the extracted beam for $K_2 = 510\,\mathrm{m}^{-1/2}$ and $\kappa = -0.24$, which is folded in phase space. Apart from changing the multipole settings this simulation also uses a reduced extraction bump, which brings the centre of the circulating beam roughly 7.7 mm further from the ES wires, in order to influence the phase space step[2] of particles in the last three turns. For these settings there is roughly 50% loss reduction compared to the nominal case, even though the extracted beam is smaller. One could use a configuration like this to reduce losses while also being able to reduce the size of the extraction gap. This would be interesting since it would allow for a lower voltage while maintaining the current deflection angle, which would lead to less frequent sparking, and thus slow down aging effects.

The acceptance at the ES entrance is shown as well in Fig. 2. It is clear that even the folded beam could be transported through the extraction channel without any changes. The further transport of the beam through the transferline will need investigating.

## HARDWARE SPECIFICATION

The SPS lattice features ten extraction sextupoles, of which four are used for the nominal extraction. These sextupoles can be used up to a maximum gradient of $227\,\mathrm{T/m^2}$ and have a length of roughly 740 mm. There are currently no decapoles installed or foreseen, but to get an estimate of the integrated decapole strength and the associated space needed in the lattice, we will assuming a radius of 50 mm as the smallest acceptable aperture and use 1.8 T as the maximum pole tip field, which gives a maximal gradient of $2.88 \times 10^5\,\mathrm{T/m^4}$. We also assume that all multipoles can be placed at locations that have both the right phase advance and $\beta_x \approx 100\,\mathrm{m}$ in order to be able to convert normalised to physical strengths. Lastly, a beam energy of 400 GeV, or equivalently a beam rigidity of 1334 T m is assumed, as for the current extraction.

The simulation settings used for the configuration depicted in Fig. 2(b) are $K_2 = 365\,\mathrm{m}^{-1/2}$ and $\kappa = -0.43$. This means we need an integrated sextupole gradient of 973.8 T/m, which implies the use of a minimum of 6 extraction sextupoles. Additionaly an integrated decapole gradient of $6.7 \times 10^6\,\mathrm{T/m^3}$ would be needed, which with the parameters assumed above would mean a minimum of 23.24 m of decapoles to be manufactured and installed.

The simulation corresponding to Fig. 2(c) used stronger sextupole and decapole strengths, setting $K_2 = 510\,\mathrm{m}^{-1/2}$ and $\kappa = -0.24$. This would require using a minimum of 9 of the extraction sextupoles, and installing at least 35.39 m of decapoles, using the same assumptions.

Note that the estimates we make here are rather sensitive to some of the parameters, most notably the aperture of the decapoles, the required $K_2$ for a good loss reduction and the horizontal beta function at the multipoles.

## CONCLUSION & OUTLOOK

The multipole strengths presented here seem too high to be realistically implemented in the SPS, however it was shown that it should theoretically be possible to reduce slow extraction losses in the SPS by a factor ~2 with this technique without changing the extraction channel. A more exhaustive investigation of parameter space is needed to determine whether a similar loss reduction would be possible with lower multipole strengths, perhaps by also adjusting the phase advance between the sextupole and decapole family. Future studies will furthermore take momentum spread and dispersion into account and assess the expected blow-up in the emittance of the extracted beam.

The studies will be expanded to include different types of multipoles. Octupoles are a particularly interesting choice, since several are available in the SPS lattice, so that tests with beam could be realised for this case.

An implementation in MAD-X using an adapted version of the standard SPS model is being carried out in order to investigate more realistic placements of the multipoles along with more detailed dynamics. This will also allow simulating more realistic losses including a full-length ES model with scattering, rather than the approximations employed in this paper. If a realistic configuration with significant loss reduction is found, the sensitivity to errors should be investigated as well.

[2] We choose not to use the term ‘spiral step size’ which commonly denotes the amplitude jump in the last 3 turns, since it could be confusing for this folded phase space geometry.

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