MEASUREMENT OF TRANSVERSE MULTIPOLE MOMENTS OF THE PROTON BEAM IN THE J-PARC MR*

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Abstract

Transverse multipole moments (quadrupole and more) of the beam may give important information of the beam such as beam sizes, nonlinear resonances and so on. However higher moments are difficult to measure because signal-to-noise-ratio becomes smaller proportional to $(x/R)^n$, $(y/R)^n$, where x: horizontal position of the charged particle, y: vertical position of the charged particle, R: vacuum-duct-radius. In order to increase the SNR and to extend the multipole order, we developed and installed a 16 electrode beam monitor in the J-PARC MR. Principle, results from BEM calculation and wire mapping, and a beam test are presented.

INTRODUCTION

At the high intensity proton accelerator such as J-PARC, a strong demand for non-destructive beam monitors which can provide the quadrupole and higher moments of the beam as well as the dipole moments (horizontal- and vertical-center-of-mass position). We have been developing the 16-electrode monitor [1-3]. Making full use of multiple electrodes, the signal to noise ratio of quadrupole moment measurement is expected better than the four electrode case. In addition higher moments such as sextupole, octupole come in the scope of measurement.

PRINCIPLE

We consider the system consists of a infinitely-long cylindrical perfect-conductor pipe and a infinitely-long thin linear charged-beam in the pipe (Fig. 1). The coordinate of the beam is (r,ϕ) and the coordinate of the inner surface of the pipe is (R, θ). The induced charge at the inner surface is expressed as

$$\sigma(r,\phi,R,\theta)=\frac{\lambda(r,\phi)}{2\pi R}\frac{R^2-r^2}{R^2+r^2-2rR\cos(\theta-\phi)}. \tag{1}$$

Expanding this equation by r, we obtain:

$$\sigma(r,\phi,R,\theta)=\frac{\lambda(r,\phi)}{2\pi R}\left[1+2\sum_{n=1}^{\infty}\left(\frac{r}{R}\right)^n\cos\{n(\theta-\phi)\}\right],$$
$$=\frac{\lambda(r,\phi)}{2\pi R}\left[1+\text{ higher-order}\right]. \tag{2}$$

The higher order expression, up to 4th order is:

$$\begin{aligned}\text{higher-order}&=2\frac{x}{R}\cos\theta+2\frac{y}{R}\sin\theta\\ &+2\frac{x^2-y^2}{R^2}\cos2\theta+4\frac{xy}{R^2}\sin2\theta\\ &+2\frac{x^3-3xy^2}{R^3}\cos3\theta+2\frac{-3x^2y+y^3}{R^3}\sin3\theta\\ &+2\frac{x^4-6x^2y^2+y^4}{R^4}\cos4\theta+8\frac{x^3y-xy^3}{R^4}\sin4\theta.\end{aligned} \tag{3}$$

With the thin electrodes which have an opening angle of Δθ, the integrated charge of the n-th electrode is:

$$\begin{aligned}\sigma(r,\phi,R,\theta_n)\approx\frac{\lambda(r,\phi)\Delta\theta}{2\pi R}[&1+2\frac{x}{R}\cos\theta_n+2\frac{y}{R}\sin\theta_n\\ &+2\frac{x^2-y^2}{R^2}\cos2\theta_n+4\frac{xy}{R^2}\sin2\theta_n\\ &+2\frac{x^3-3xy^2}{R^3}\cos3\theta_n+2\frac{-3x^2y+y^3}{R^3}\sin3\theta_n\\ &+2\frac{x^4-6x^2y^2+y^4}{R^4}\cos4\theta_n+8\frac{x^3y-xy^3}{R^4}\sin4\theta_n\\ &\cdots].\end{aligned} \tag{4}$$

Rewriting this equation as a matrix form, we get:

$$\begin{bmatrix}\hat{V}_1\\ \vdots\\ \hat{V}_{16}\end{bmatrix}=\mathrm{M}\begin{bmatrix}1\\ x_w\\ y_w\\ x_w^2-y_w^2\\ 2x_wy_w\\ \vdots\end{bmatrix}, \tag{5}$$

where M is:

$$M=\begin{bmatrix}c_0 & a_1\cos\theta_1 & b_1\sin\theta_1 & a_2\cos2\theta_1 & b_2\sin2\theta_1 & \cdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots\\ c_0 & a_1\cos\theta_{16} & b_1\sin\theta_{16} & a_2\cos2\theta_{16} & b_2\sin2\theta_{16} & \cdots\end{bmatrix}.$$

Therefore measuring the electrode signals V_n's, we can derive the multipole moments with multiplying the V vector by the inverse matrix of M.

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CONSIDERATION OF THE PRACTICAL GEOMETRY

Practical shape is shown in Fig. 2. In this case the matrix M should be replaced by the one which reflects the geometry change. The modified M (M') is:

$$M' = \begin{bmatrix} c_1 & a_{1,1} & b_{1,1} & a_{1,2} & b_{1,2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\ c_{16} & a_{16,1} & b_{16,1} & a_{16,2} & b_{16,2} & \cdots \end{bmatrix}.$$

These elements of M' are determined by the Boundary Element Method (BEM) with transmission-line analysis or the wire mapping.

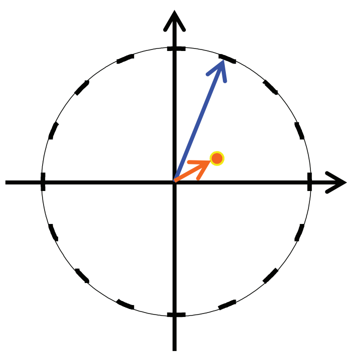

Figure 1: Idealized geometry of a beam and a beam duct.

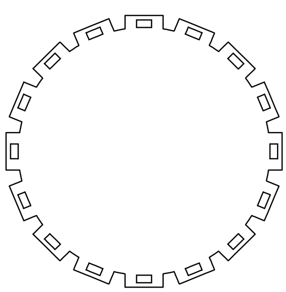

Figure 2: Real geometry of electrodes and a beam duct.

RESPONSE EVALUATION

BEM Simulation

The matrix M' was evaluated with the method described in [4, 5]. The x- and y-positions reconstructed with this M' are well agree with the original wire positions (Fig. 3 right plot). Using only the four electrodes at $\theta = 0$, $\pi/2$, π and $3\pi/2$, the reconstructed wire position becomes deformed (Fig. 3 left plot).

Wire Mapping

Using the results of wire mapping [2, 3], reconstructed positions are well agree with the BEM results. The left plot of Fig. 4 is reconstructed using only the four electrodes as the above. The right plot of Fig. 4 is reconstructed using the full matrix M'.

Examining the matrix M, the n-th column forms sinusoidal function. Even in the practical geometry this characteristic is kept. The elements corresponding to 1st and 2nd moments are plotted in Fig. 5. The function shapes as $\cos\theta$, $\sin\theta$, $\cos 2\theta$ and $\sin 2\theta$ are clear. This implies that the process of multiplying by M' equals to extracting the certain Fourier component from the charge distribution at the inner surface of the pipe.

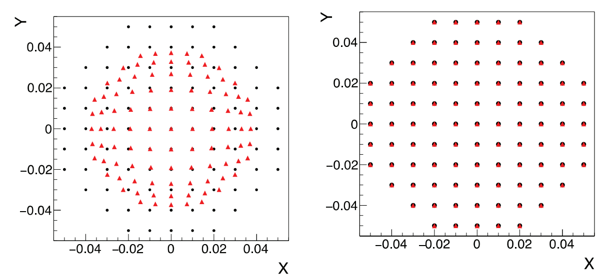

Figure 3: Reconstructed wire positions with M' reduced from the BEM simulation.

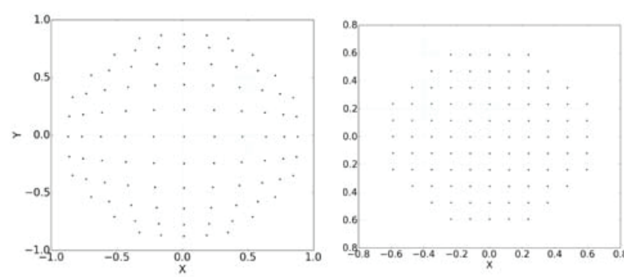

Figure 4: Reconstructed wire positions with M' reduced from the wire mapping.

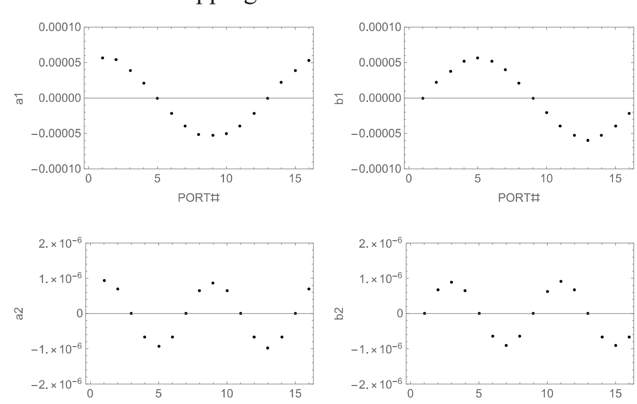

Figure 5: Column elements of the response matrix M' measured with wire mapping. Only the 1st and 2nd moments are plotted here.

RESULTS FROM THE BEAM TEST

Installation

One 16-electrode monitor was installed in the address #15 of J-PARC MR. The left photograph of Fig. 6 shows the inner structure of the monitor. The right photograph of Fig. 6 shows the monitor after installation. Sixteen coaxial cable are connected to the common mode choke coil and then connected to the connectors (vacuum feed-through, N-type) of the monitor.

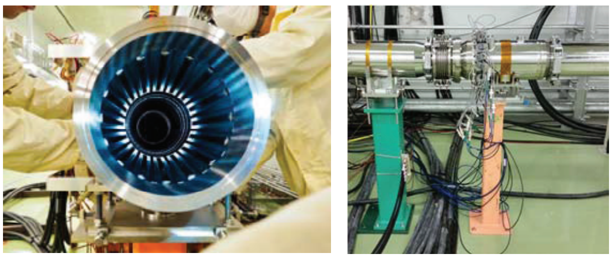

Figure 6: Inner structure and the installed monitor.

Table 1: Beam Parameters

	Quantities	Units
Intensity	2.8E13	Protons/bunch
# of bunches	2	-
Beam energy	3	GeV

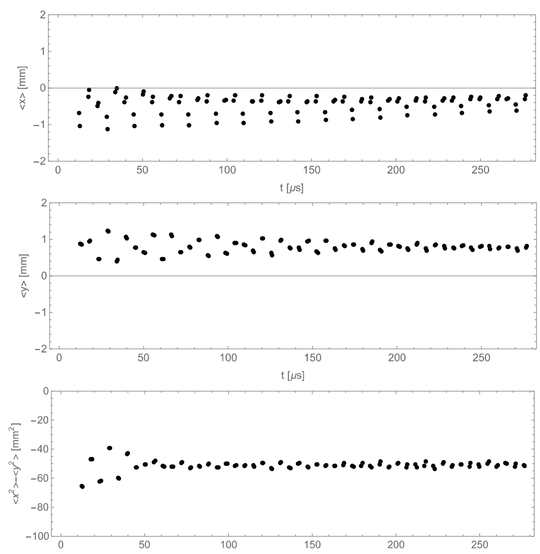

Figure 7: The reconstructed positions <x>, <y> and the quadrupole moment <x^2>-<y^2> just after injection into MR.

Beam Test and Results

Several dedicated beam test have been performed from Nov. 2016. We present the preliminary results from the analysis on the data acquired in Dec. 19, 2016. In Table 1 the main beam parameters are tabulated. Figure 7 shows the reconstructed positions <x>, <y> and the quadrupole moment <x^2>-<y^2> just after two bunches injected into MR. Each bunch quantities are plotted in the figure. The frequency spectra in Fig. 8 suggest the fractional part of the betatron tunes ν_x=0.333 and ν_y=0.413 (zero padding was used for Fourier transform). Although the twice of the tunes are indicated in the lower plot, they do not match the peaks. This might indicate the space charge tune shift of the quadrupole moment.

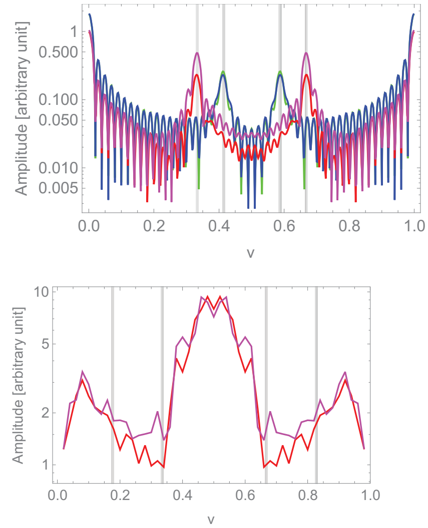

Figure 8: Frequency spectra of the <x> (red: 1st, magenta: 2nd bunch) and <y> (blue: 1st, green: 2nd bunch) in the upper plot and <x^2>-<y^2> (red: 1st, magenta: 2nd bunch) in the lower plot.

SUMMARY

The 16-electrode beam monitor has been developed. The BEM simulation and wire mapping was performed. Preliminary measurements suggest possibility of the quadrupole moment. Additional examination of the error and higher order moments in future. A specific electronic circuit is under development for real time observation.

ACKNOWLEDGEMENT

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