ESTIMATION OF LONGITUDINAL DIMENSIONS OF SUB-PICOSECOND ELECTRON BUNCHES WITH THE 3-PHASE METHOD*

H. Purwar†, C. Bruni, A. Gonnin, LAL, Univ. Paris-Sud, CNRS/IN2P3, Université Paris-Saclay, Orsay, France
T. Vinatier, DESY, Hamburg, Germany

Abstract

An estimation of the longitudinal dimensions for short electron bunches in an accelerating field is an important diagnostic and can be extremely helpful in evaluating the performance of an accelerator. We investigate a method for close estimation of bunch length for sub-picosecond electron bunches from the measurement of their energy spreads. Three or more measurements for the bunch energy spread are made by varying the phase of the accelerating structure and later a reconstruction of the bunch longitudinal dimensions, namely bunch length, initial energy spread and chirp at the entrance of the accelerating structure are obtained using the least square method. The proposed 3-phase method is very simple from both understanding and experimental point of views.

At first, we present a model for a standing wave accelerating structure, such as a booster, which will later be used for the estimation of bunch longitudinal dimensions. Then, we will evaluate the accuracy of this model. To end, a comparison of the obtained results with ASTRA simulations is made to validate the 3-phase method for sub-ps electron bunches.

BOOSTER MODEL

Assumptions Following assumptions were made in order to derive an expression for the energy gain and transfer matrix of a standing wave (SW) accelerating structure.

- Motion of electrons is purely longitudinal.
- Accelerating fields are perfectly sinusoidal (no fringe fields).
- Energy of the electrons at the entrance is relativistic.

Energy gain The energy gained by the electrons, E_g, passing through a SW structure, like a booster, of length L, with an accelerating field amplitude E_m, frequency f, wave vector k and phase ϕ is modelled as [1],

$$E_g = -\frac{eE_m}{4k}\left[\cos(\phi) - \cos(2kL+\phi) - 2kL\sin(\phi)\right] \quad (1)$$

with e being the elementary charge.

Transfer matrix The longitudinal transfer matrix (R) for a SW accelerating structure is modelled as,

$$\begin{pmatrix} 1 & 0 \\ \pi e f E_m \left(L\cos(\phi) - \frac{1}{k}\cos(kL+\phi)\sin(kL)\right) & 1 \end{pmatrix} \quad (2)$$

Symbols have their usual meanings as indicated before.

3-PHASE METHOD

The 3-phase method allows us to estimate the electron bunch length (σ_t) and energy spread (σ_E) at the entrance of the SW structure from 3 or more measurements of its energy spread for different accelerating conditions achieved by varying ϕ, E_m and/or f. This method doesn't take space charge forces into account which becomes more significant for the low energy short electron bunches [2]. We also assume here that the transverse motion of the electron bunch is completely independent from its longitudinal motion and hence the beam transport is linear. The transfer matrices therefore take the form of a 2×2 matrices in the longitudinal phase-space. Let us define a longitudinal beam matrix by gathering the relevant statistical parameters for the bunch,

$$\begin{pmatrix} \langle \Delta t^2 \rangle & \langle \Delta E \Delta t \rangle \\ \langle \Delta t \Delta E \rangle & \langle \Delta E^2 \rangle \end{pmatrix} = \begin{pmatrix} \sigma_t^2 & \sigma_{Et} \\ \sigma_{Et} & \sigma_E^2 \end{pmatrix} \quad (3)$$

where, $\langle\rangle$ represents mean over all the electrons in the bunch, σ_t is the rms bunch length, σ_E is the rms bunch energy spread and σ_{Et} is the rms bunch time-energy correlation.

Using the transport of this longitudinal beam matrix between the accelerating structure entrance (i) and exit (f), with R_{mn} being the coefficients of the longitudinal transfer matrix for the SW structure, we have,

$$\sigma_{E_f}^2 = R_{21}^2(\phi)\sigma_{t_i}^2 + 2R_{21}(\phi)\sigma_{Et_i} + \sigma_{E_i}^2 \quad (4)$$

The goal now is to measure the final bunch energy spread σ_{E_f} for **n** (at least 3) different values of the matrix R. Varying phase of the accelerating section during experiments is the simplest way of changing the R_{21} element of the matrix R for the SW accelerating structure, Eq. (2). Thus,

$$\underbrace{\begin{pmatrix} \sigma_{E_f\mathbf{1}}^2 \\ \vdots \\ \sigma_{E_f\mathbf{n}}^2 \end{pmatrix}}_{\mathbf{Y}} = \underbrace{\begin{pmatrix} R_{21\mathbf{1}}^2 & 2R_{21\mathbf{1}}R_{22\mathbf{1}} & 1 \\ \vdots & \vdots & \vdots \\ R_{21\mathbf{n}}^2 & 2R_{21\mathbf{n}}R_{22\mathbf{n}} & 1 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} \sigma_{t_i}^2 \\ \sigma_{Et_i} \\ \sigma_{E_i}^2 \end{pmatrix}}_{\mathbf{X}} \quad (5)$$

Solving for $\mathbf{X}$ using the least square method, we obtain $\mathbf{X} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{Y}$. This method has already been validated experimentally on PHIL at LAL, Orsay, France with a RF gun [1] and on PITZ at DESY, Hamburg, Germany [3] with a RF gun and a booster for 1 ps electron bunches.

EVALUATION OF 3-PHASE METHOD

To evaluate the performance of this method, the values used for the energy spread were the ones obtained using

* Work supported by Labex P2IO.
† purwar@lal.in2p3.fr

ASTRA [4] simulations for different values of the phase (ϕ). A booster with maximum energy gain of about 4 MeV was used for these simulations. Such a booster will be used with the photo-injector based electron linear accelerator at LAL, Orsay. Figure 1 shows, on the left, a comparison of the standing wave electric field inside the booster as considered in the Booster model (perfectly sinusoidal field of length 0.15 m), with the field obtained through a simulation using SUPERFISH taking into account the actual geometry of the booster, 3 cell, operating at 3 GHz and 50 MV/m. This allows us to study the effect of fringe fields separately (case 2 of this section).

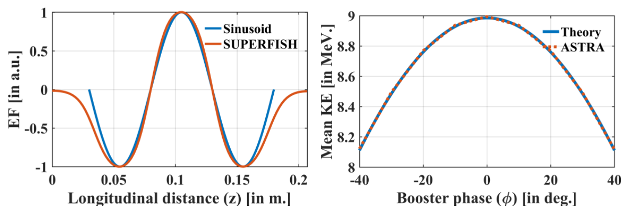

Figure 1: Left: Comparison of theoretical & 'realistic' (simulated with SUPERFISH) electric fields inside the booster. Right: Mean kinetic energy obtained from ASTRA and the theoretical model (Eq. (1) + 5.25 MeV) with booster phase (ϕ).

An electron bunch with total charge of 10 pC was generated and tracked using ASTRA at the entrance of the booster with the mean bunch kinetic energy of 5.25 MeV, and an energy spread of 13 keV (all with Gaussian distributions). The transverse dimensions of the bunch were 0.5 mm (rms) with small normalized transverse emittance. All magnets (solenoids, etc.) were ignored in the simulations. The relative energy spread (0.2%) is small but even smaller energy spreads have been measured for the AlphaX RF gun with PHIL photoinjector at LAL, Orsay, France [5].

Figure 1, on the right, shows a comparison of the mean bunch kinetic energy curves versus the booster phase obtained using Eq. (1) with the curve obtained from ASTRA simulations. The discrepancy is less than 0.5%.

Case 1: Without Fringle Field and Space Charge

The only accelerating cavity was a booster operating with perfectly sinusoidal electric field, as shown in Fig. 1, to minimize the effect of the fringe fields which is not taken into account in the SW model and thus, we expect to achieve a very good reconstruction of the bunch longitudinal characteristics. The space charge effects were ignored in this case.

Energy spread from ASTRA The obtained rms bunch energy spread curves at the exit of the booster for different initial bunch lengths are shown in Fig. 2 as a function of the booster phase. These values were then used for estimation of the bunch length (σ_{t_i}) and energy spread (σ_{E_i}) using the 3-phase method. As expected, the amplitude of the curves is dependent on the initial bunch lengths, which is the core of the 3-phase method.

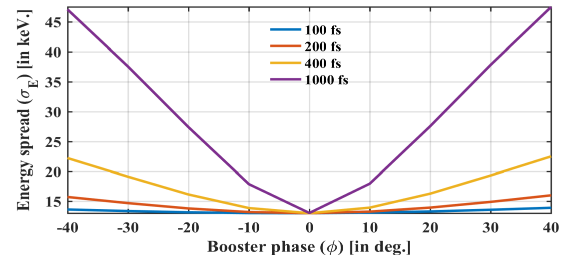

Figure 2: Bunch energy spread with booster phase, without considering space charges and fringe fields.

Estimation of σ_{t_i} with 3-phase method The rms bunch energy spread values at the exit of the booster obtained from ASTRA simulations was used to estimate the σ_t and σ_E at the entrance of the booster. The discrepancy in the estimation of σ_{t_i} for this case with the perfect sinusoidal standing wave inside booster and without considering the space charge effects is less than 0.5% showing the robustness of the model.

Case 2: with Fringle Field but without Space Charge

This case focuses on understanding the difference in the estimated longitudinal bunch characteristics if the fringe fields of the accelerating structures are also taken into account. The same initial electron bunch as in Case 1 was generated at the entrance of the booster. The more 'realistic' electric field as shown in Fig. 1, including fringe field is considered in this case. The space charge effects were ignored again.

Energy spread from ASTRA The obtained rms bunch energy spread curve for an initial bunch length of 100 fs for this case is shown in Fig. 3 as a function of the booster phase. There is a small increment in the values of the energy spread

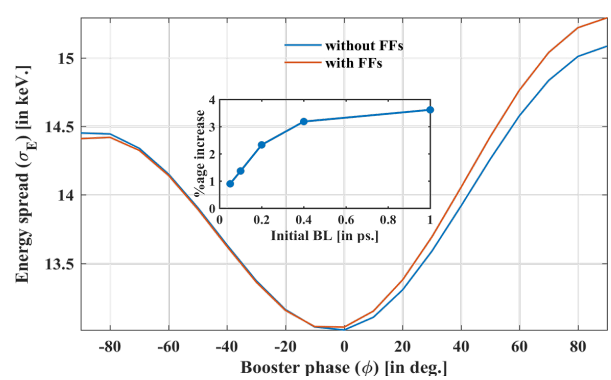

Figure 3: Bunch energy spread with booster phase, with and without the fringe fields for an initial bunch length of 100 fs at the entrance of the booster. FFs: fringe fields.

with fringe fields. The difference in the energy spread values, with and without the fringe fields is due to the difference in the electric fields near the entrance/exit of the booster in the two cases (see Fig. 1). The inset in Fig. 3 shows the percentage increase in the bunch energy spread at 90 deg. phase for various initial bunch lengths. Nevertheless, σ_t and σ_E at the booster entrance was estimated with a discrepancy of less than 1% using the proposed 3-phase method even with the error introduced by the assumption of the sinusoidal field.

Case 3: With Fringle Field and Space Charge

In this case we include the space charge effects as well as the fringe fields.

Estimation of σ_{t_i} with 3-phase method The results obtained are summarized in Table 1. The discrepancy in the estimation of σ_{t_i} for the short initial bunch lengths shorter than 200 fs, is much higher in this case, due to the stronger repulsion amongst the electrons of the bunch. In other words, we observe a stronger effect of the space charge forces for the ultra-short electron bunches. This is due to the fact that we are at low energy: 5.25 MeV at the booster entrance and less than 10 MeV at the exit. As mentioned before the effect of space charges is not included in the Booster model [Eq. (1) & Eq. (2)] and thus we observe a larger discrepancy in the estimated σ_{t_i} for ultra-short electron bunches of about 17%.

Case 4: With Fringle Field & Space Charge (High Energy)

A similar electron bunch as in the above cases was generated at the entrance of the booster with the mean bunch kinetic energy of 100 MeV, and an σ_{E_i} of 300 keV. The effects of space charge and fringe fields were both taken into account. Due to the high mean bunch energy, we expect to minimize the effect of space charges and thus expect to obtain a good estimation of the bunch longitudinal dimensions.

Estimation of σ_{t_i} with 3-phase method The results are summarized in Table 1. We observe that the estimation of both σ_{t_i} and σ_{E_i} is quite good in the high energy case. The discrepancy in the estimation of σ_{t_i} is less than 2%.

Table 1: Estimation of σ_{t_i} using 3-phase method for low and high energy electron bunches. KE: mean bunch kinetic energy, low KE: 5.25 MeV and high KE: 100 MeV.

σ_{t_i}	Estimated σ_{t_i}		Discrepancy	
	low KE	high KE	low KE	high KE
100 fs	117 fs	102 fs	17%	2%
200 fs	215 fs	204 fs	7.5%	2%
400 fs	401 fs	402 fs	0.2%	0.5%
1.0 ps	1.0 ps	1.0 ps	0%	0%

CONCLUSIONS

An easy approach for the estimation of bunch length at the entrance of an accelerating cavities is presented and its performance is evaluated in different conditions of bunch kinetic energy, longitudinal characteristics, etc. We found that for an ideal case, without considering the effect of fringe fields and space charges, the estimation using the proposed 3-phase method is quite accurate, errors in the estimation of bunch lengths are less than 0.5% showing the accuracy of the standing wave model and the 3-phase method.

Including the fringe fields does not have a large impact on the energy spread values for the ultra-short electron bunches. Even though this difference increases with σ_{t_i}, the bunch length could be successfully estimated using the 3-phase method with a maximum error of 1%. This shows that not considering the fringe fields in the SW model is not a significant limitation.

Including the space charges for the low kinetic energy case had a significant effect on the estimation of bunch length using the 3-phase method. For the same total charge, in case of 1.0 ps electron bunch the space charge effects were minimum and hence the estimation of σ_{t_i} had a similar accuracy. On the other hand, for shorter electron bunches, the space charge effects become influential and thus cannot be neglected. A more complex model for a SW accelerating structure is required to increase the accuracy of the 3-phase method in estimating the bunch lengths on the order of 100 fs or less. For the same extracted charge, another way to minimize the effect of space charges is to increase the bunch kinetic energy and we showed that the method works very well for the high-energy ultra-short electron bunches with errors in the estimation of bunch lengths to be less than 2%.

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