OPTIMIZATION OF A SKEW PARAMETRIC RESONANCE IONIZATION COOLING CHANNEL USING GENETIC ALGORITHM

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Abstract

Skew Parametric-resonance Ionization Cooling (Skew PIC) is designed for the final 6D cooling of a high-luminosity muon collider. Tracking of muons in such a channel has been modeled in MADX and matter-dominated simulation tool G4beanline in previous studies. In this work, we developed an optimization code based on Genetic Algorithm (GA). We optimized the cooling channel and increased the acceptance of the channel by using the GA code.

INTRODUCTION

A high-luminosity muon collider requires extreme 6D cooling, which conventional methods can not reach. Parametric-resonance Ionization Cooling (PIC) [1] is promising to cool the muon beam in 6D to reach the emittance requirement of a muon collider. It strongly focuses the muon beam periodically along the channel, so that the multiple Coulomb scattering in the ionization cooling process is overcome by the large angular spread.

In order to attain simultaneous focusing in both planes at regular locations, the horizontal and vertical betatron oscillation periods must be commensurate with each other and with the channel's period. A channel with such optical properties, called a Twin helix channel [2], has been designed and simulated in previous studies. It successfully focus the beam and introduces the required angular spread. However, the multipole fields with correlated optics introduce strong non-linear resonances and beam instability. The Skew PIC channel [3] is designed to overcome this problem. The canonical betatron tunes are shifted from their resonant values and the point to point focusing is preserved.

Such a channel has been designed and simulated in MADX [4] and G4beamline [5]. In order to increase the acceptance of the channel and enhance the total performance, we have developed an optimization code based on Genetic Algorithm (GA). In this report, we first use GA to optimize the Skew PIC optics. Then we use the GA code to improve the channel to accept a beam source with angular spread up to 100 mrad in RMS. Successful ionization cooling requires good control of particles with such large angular spread.

GENETIC ALGORITHM

GA optimization techniques have been widely applied to an extremely broad range of fields. It has been used in the accelerator field since the early 1990s, and has quickly become a fundamental tool in multi-dimensional optimization problems. The idea of GA is to mimic biological reproduction, the theory of evolution, and the behavior of biological mutation to solve mathematical optimization problems. A general GA optimization process includes the following steps:

1. Produce the first generation of individuals. This generation is a random sample of the N-dimensional parameter space. Specifically in the Skew PIC optimization, one "individual" is a set of magnet strengths in the channel, which we want to optimize, and we randomly generate 100 individuals in the first step.

2. Evaluate each individual by a fitness function, which indicates the relative strength of each individual. It is a function of the objectives and can include constraints. For the Skew PIC case, we use G4beamline to simulate the beam transport in the channel with each setting of magnets, which gives the average beam size at the focal points and a total transmission. We then evaluate the fitness of the individual as follows: if the beam transmission (T) is lower than 90%, the fitness (F) is a function of T:

$$F = (1 - T)^4 * 10^5 \tag{1}$$

   if the beam transmission is higher or equal to 90%, the fitness is a function of the average beam size (R):

$$F = R^4/10^4 \tag{2}$$

   While we minimize the fitness we increase the transmission first and then minimize the beam size. The power law function controls the speed of the convergence, and coefficients are used to have a relatively smooth connection between the 2 optimization zones.

3. Produce the next generation: select, cross-over and mutate.

   Select: according to the fitness function, we give a weight to each individual, and then randomly select 100 individuals (individuals can be selected multiple times) and put them into the selected queue.

   Cross-over: we take 2 individuals from the queue at one time to do the cross-over. We convert the parameters into binary numbers to represent the "gene" of the individuals, then we randomly select the point on the genes for the 2 individuals to cross-over.

   Mutation: we randomly choose the position and number of genes on the newly produced gene to flip.

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4. Go back to step 2 with the new generation of individuals.

5. Terminate the program if optimization goal or time limit is reached. We then find out the best individual in all generations.

We developed such a code to cooperate with both G4beamline and MADX in order to compare the results. There are several advantages of using GA technique: it can search in a wide range even if the number of parameters is large; it converges fast; it is modular and separate from applications and it is inherently parallel and can be easily distributed on super computers.

OPTIMIZATION OF THE SKEW PIC OPTICS

We use high order multipoles to compensate the nonlinear effects in the Skew PIC channel. Controlling the beam with large angular and momentum spread needs very accurate design of the multipoles. Currently the analytic design of the channel includes magnetic fields expansion up to octupoles. It accepts a beam with an angular spread up to 82 mrad and momentum spread $dp/p = 2\%$. We use GA to optimize the design to obtain a smaller beam size at the focal points. Twelve parameters including dipole strength, dipole radius, quadrupoles, sextupoles, and octupoles are set as independently varying parameters. Figure 1 shows the smallest fitness in one generation as a function of generation number. Because the first generation is generated randomly, the initial fitness is large. The F value decreases rapidly and in 7 generations it converges to a stable value. The minimum fitness we found in this optimization process is 0.00588, which corresponds to an average beam radius of 2.7 mm. The straight line in Fig. 1 shows the original design value of the average beam size along the channel. The GA optimization improves the result by about 12% in this case.

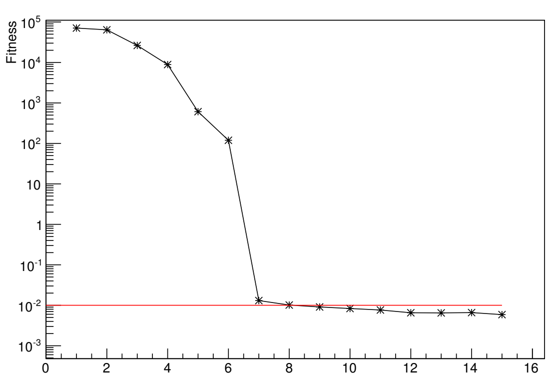

Figure 1: The smallest fitness in the generation as a function of generation number. The red line represents the original design value of the average beam size corresponding to the fitness function.

Figure 2 shows the beam radius along the channel with the original design (black curve) and after (red curve) the GA optimization. The beam size in the optimized channel is stabilized in a few meters, while in the original channel it is slowly increasing, representing the beam is not well controlled.

The optimization also increases the beam transmission from 94% to 98%. The optimized channel has a smaller bending angle for the dipole magnets, which not only reduces the complexity of building the geometry but also significantly reduces the total length of the channel by about 40%, so as to greatly reduce the beam loss from muon decay.

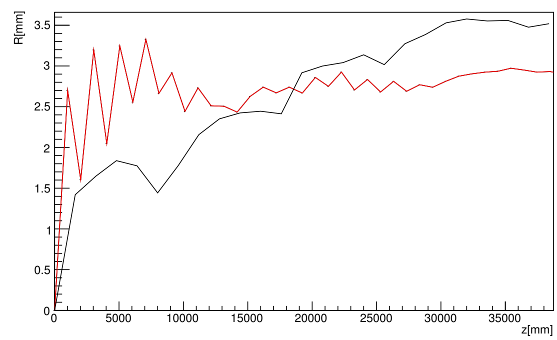

Figure 2: Beam radius along the channel before (black curve) and after (red curve) the GA optimization.

INCREASING THE ACCEPTANCE

Skew PIC channel need to focus the beam with a large angular and momentum spread. We use GA to increase the acceptance of the Skew PIC channel to accept a beam with an RMS angular spread of 100 mrad and an RMS momentum spread of 5 MeV/c. We search in a wide range for each parameter in order to find the global optimum of the channel.

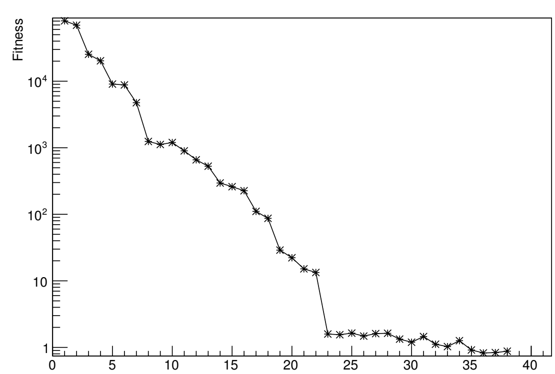

Figure 3: The smallest fitness in the generation as a function of generation number.

Figure 3 shows the optimization process of such a channel. Due to the large divergence of the beam, the converging process is much slower than the previous case. The minimum beam size it reached is 10.5 mm.

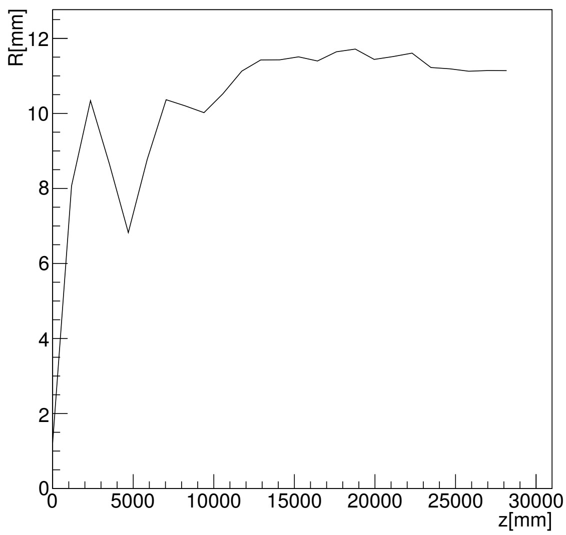

Figure 4: Beam radius along the channel. The source beam has an RMS angular spread of 100 mrad and momentum spread of 5 MeV/c.

Applying the best reached settings, we show the beam radius along the channel in Fig. 4. The beam size increases quickly to about 10 mm and becomes stable. Although this beam size and the corresponding angular spread can not yet satisfy the requirement of ionization cooling, the transmission of this channel is over 90%, which means a good control of the beam.

SUMMARY AND FURTHER WORK

Genetic Algorithm is a powerful tool for optimizing accelerator systems. We applied GA in improving the design of the Skew PIC channel optics. The beam size is reduced and the beam is stabilized in the channel. At the same time the total channel length is shorter after the optimization so that the beam loss due to muon decay is less.

We also applied GA to increase the acceptance of the channel. We use GA to search in a wide range of strengths for each magnet and eventually find settings that can accept the objective beam with a transmission over 90%.

Further work needs to be done to reduce the beam size in such a channel. We will introduce higher order magnetic fields and extra driving force for inducing the parametric resonance. We will also improve the GA code to have a dynamic mutation rate to better control the convergence of the optimization process.

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