DYNAMICS OF SPECTATOR PARTICLES IN SPACE-CHARGE FIELDS OF MISMATCHED BEAMS WITH CROSS-PLANE COUPLING

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Abstract

In accelerators with high beam power, even moderate beam losses must be avoided. These losses are due to particles reaching large transverse amplitudes that form a low density halo orbiting the beam core. To study the beam halo formation, we place a spectator particle outside the beam core and let it interact with the core's electric field. The core, we model by a self-consistent transverse Gaussian beam including non-linear space charge forces and cross-plane coupling.

INTRODUCTION

One of the main challenges for high-power particle accelerators like neutron sources or neutrino factories is to avoid even the smallest particle losses. The European Spallation Source (ESS) will use a proton beam with a design power of 5 MW. With such high power, even minute beam losses will produce unwanted radiation and heat-load to the cryogenic systems of the superconducting sections. A commonly accepted beam loss limit in linear accelerators that allow hands-on maintenance without long cool-down times is below 1 W/m [1]. Space charge-driven resonances due to beam mismatch lead to the generation of a high-amplitude beam halo surrounding the beam core. These halo particles predominantly contribute to beam loss. The formation of the beam halo has been extensively studied in 2D phase space by using KV-distributions with linear space charge forces [2]. Naturally, models that study the beam halo formation can be divided into the beam core and the beam halo, called the particle-core-model. It describes the transverse dynamics of particles in the beam halo [3]. In order to model our beam core we developed a fully analytic, self-consistent transverse Gaussian beam model with non-linear space charge forces and cross-plane coupling. It is based on a covariant form of Bassetti's and Erskine's closed expression for the electric field of a upright, transverse Gaussian beam [4, 5]. In this paper, we place spectator particles around a previously simulated beam core and examine how beam mismatch and cross-plane coupling affects their dynamic behavior.

BEAM CORE

The covariant form of Bassetti's and Erskine's expression for the transverse electric field created by a Gaussian particle distribution is given by

$$F_0\left(x_1, x_3, \sigma\right) = \frac{\sqrt{\pi}}{\sqrt{2(\sigma_{11}-\sigma_{33}+2i\sigma_{13})}} \times \left[w\left(z_1\right) - e^{-\frac{1}{2}\tilde{\sigma}^{-1}_{mn}x_m x_n}\cdot w\left(z_2\right)\right] \tag{1}$$

with

$$\begin{aligned} z_1 &= \frac{x_1 + ix_3}{\sqrt{2\left(\sigma_{11}-\sigma_{33}+2i\sigma_{13}\right)}} \\ z_2 &= \frac{\left(\sigma_{33}-i\sigma_{13}\right)x_1 + i\left(\sigma_{11}+i\sigma_{13}\right)x_3}{\sqrt{\sigma_{11}\sigma_{33}-\sigma_{13}^2}\sqrt{2\left(\sigma_{11}-\sigma_{33}+2i\sigma_{13}\right)}} \end{aligned} \tag{2}$$

and where x_1, x_3 are the real space coordinates in the transverse plane, σ is the 4D beam matrix and $w(z)$ is the complex error function [6]. The covariant form allows for any tilt angle of the beam and covers cross-plane coupling in the transverse plane. With the self-field, we calculate the change in horizontal and vertical beam angle Δx_2 and Δx_4 of the beam as

$$\Delta x_4 + i\Delta x_2 = NKF_0(x_1, x_3, \sigma) \tag{3}$$

where N is the number of particles per bunch and K is a perveance-like parameter, scaling the strength of the space charge kick. Based on Eq. 3, we found a fully analytic and self-consistent solution to calculate the change of all elements of the beam matrix and the consequent increase in emittance and beam size which allows for quick simulations of a beam core propagating through a pre-defined accelerator lattice.

As a test lattice, we use a simple FODO cell with a horizontal and vertical phase advance per cell of 60° and 52.6°, respectively. We prepare the proton beam with 1×10^9 protons per bunch, a bunch length of 4.7 cm and a kinetic energy of 200 MeV. The initial beta-functions are $\beta_x = 0.21$ m and $\beta_y = 0.37$ m. The horizontal and vertical emittance are both 0.364 mm rad. We characterize the space charge tune-shift by treating it as quadrupoles that defocus in both planes and determine their effective focal lengths. For this beam core, we obtain the tune-shifts $\Delta Q_x = -0.055$ and $\Delta Q_y = -0.06$. Applying our model, we first find the equilibrium solution of the beam including non-linear space charge forces by propagating it several thousand turns through the lattice. Then, we apply a mismatch to the equilibrium solution by increasing the beam sizes, resulting in an oscillating beam core once the mismatched beam propagates through the lattice again. The behavior of the spectator particles is investigated using the oscillating beam core.

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SPECTATOR PARTICLES

The initial spectator particles are placed on circles with different radii in normalized phase space. Each circle contains 1000 particles which share a common action J. We place particles with orbits close to the center of the beam, around the maximum of the space charge field, and further outside where the field has already reasonably decreased. Figure 1 shows the initial placement in real phase space and an overlay of the beam's space charge field (magnitude not to scale). After placing the circles, we transform them to ellipses in real phase space according to the equilibrium solution of the beam envelope obtained from the core simulation. The spectator particles are tracked while propagating through the lattice and are simultaneously influenced by the field of the beam. We investigate the behavior of the particles for matched and mismatched beams.

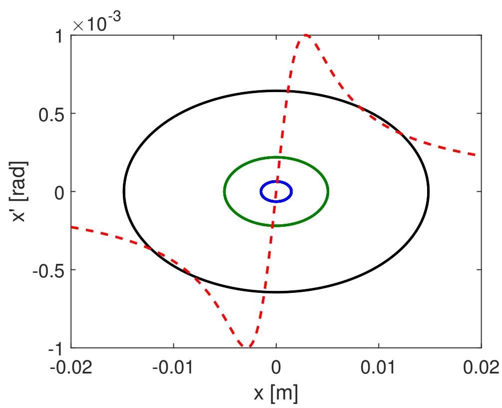

Figure 1: Initial distribution of spectator particles in horizontal phase space. Each ellipse contains 1000 equidistantly distributed particles. The dashed line is an overlay of the beam's self-field.

SIMULATION RESULTS

First, we look at the horizontal phase space of a matched beam. Figure 2 shows the ellipses after ten turns through the lattice, being affected by both, the external magnetic forces and the self-field of the beam. Clearly, the ellipses differ in angle and are partially deformed after ten turns. Particles in an orbit where the gradient of the self-force is positive experience a negative tune-shift. As a result, these particles have a lower tune compared to the bare tune of the lattice and consequently a smaller phase advance. Thus, they cannot complete the full rotation within a single turn through the FODO cell. Particles that see a negative gradient receive a positive tune-shift, have thus a bigger phase advance and complete more than a full rotation. Letting the matched beam complete 1000 turns though the lattice, we obtain Fig. 3. We observe, that the ellipses generally appear to be better aligned horizontally like they were in the beginning. However, the particles exhibit slightly different actions and orbits and ellipses have smeared. The outermost ellipse stays visibly deformed.

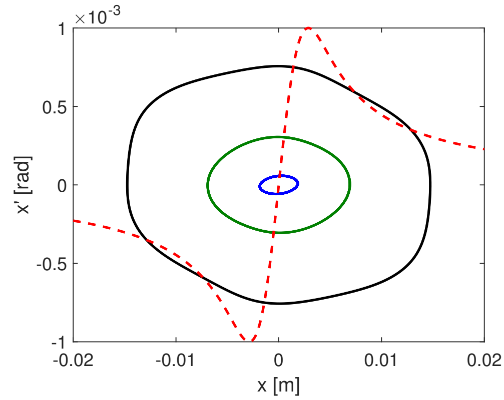

Figure 2: Matched beam after ten turns. Initial ellipses exhibit different angles, depending on whether particles see a positive or a negative self-field gradient and the respective incoherent tune-shifts.

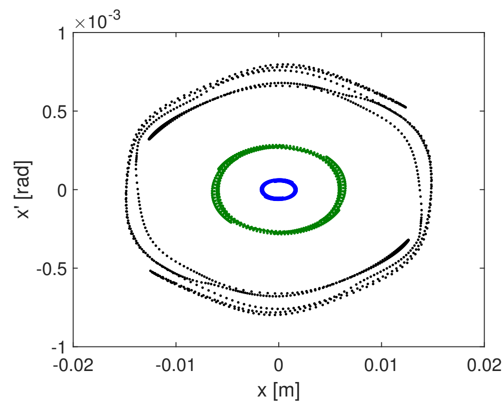

Figure 3: Phase space ellipses for of a matched beam after 1000 turns. Smearing of ellipses is observed.

In order to investigate particle-core resonances, we introduce a mismatch to the equilibrium solution. In contrast to using the equilibrium solution, the spectator particles now see a self-field where both, the amplitude and the position of the field maximum can strongly vary over time. Potentially, this leads to beam oscillations resonating with the oscillation of some single particles particles. We quantify the beam mismatch with the B_{mag} parameter from reference [7].

Figure 4 shows the horizontal phase space after 1000 turns for a beam with $B_{mag} = 1.083$. During beam propagation, particles close to the core form small spiral arms which fold back into the original ellipse. As a result, the ellipse broadens. We account the formation of spiral shapes to the amplitude-dependent tune-shift of the Gaussian beam. The effect is small because the particles move in the linear regime of the self-field. Particles orbiting close to the field maximum (green) split into two separate ellipses. In the process of splitting, the creation of spiral shapes was observed which we account to strong, non-linear space charge forces. The spiral arms completely detach from the main body. The original shape of the ellipse does not recover

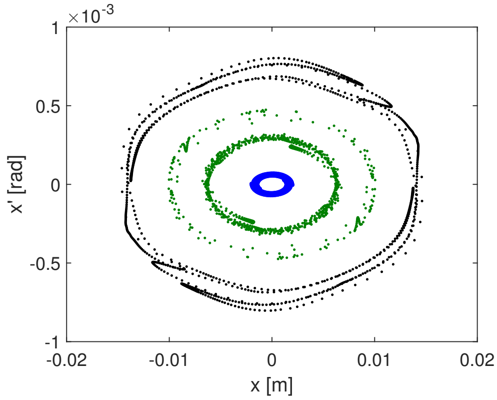

Figure 4: Hor. phase space after 1000 turns with B_{mag} = 1.083. Particles close to field maximum experience strong filamentation and split into two ellipses.

and the secondary ellipse persists. The outer ellipse is deformed and its orbit is less affected, which is expected due to the reduced self-field.

Increasing the mismatch corresponding to B_{mag} = 1.25 shown in Fig. 5, distortions in phase space are more pronounced, even close to the center where eye-shape structures appear. The pumping of the beam core is strong enough to affect particles close to the core, resulting in distinct structures in indicating a formation of stability islands where particles start to accumulate.

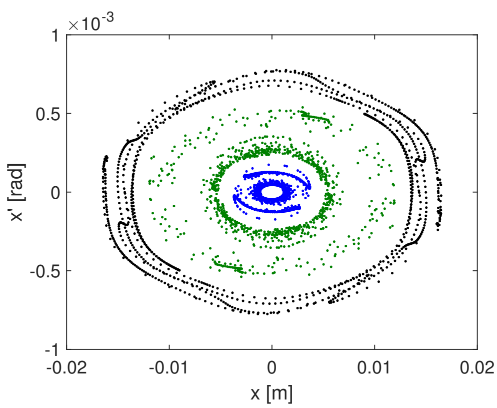

Figure 5: Hor. phase space after 1000 turns with B_{mag} = 1.25. Particles close to beam core form eye-shaped structures.

Single Particle Dynamics with Coupled Lattice

We repeat the core simulations with B_{mag} = 1.083 and couple the lattice by giving one of the quadrupoles a skew angle of 1°. Consequently, both the beam core and the spectator particles see full cross-plane coupling. Comparing Fig. 4 with Fig. 6, we observe similar eye-shaped structures for the inner particles as well as a split ellipse for particles close to the field maximum. However, the secondary ellipse appears to have a lower orbit in exchange for being more diffuse and the observation of the dynamic process shows the ellipse partially recovering its initial shape. The outer particles are pushed into a high orbit and show excursions towards higher amplitude, compared to Fig. 4.

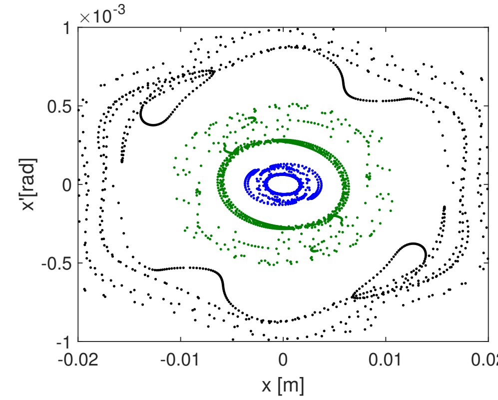

Figure 6: Beam mismatch with B_{mag} = 1.083 after 1000 turns. Planes are coupled by a quadrupole with a 1° skew angle.

CONCLUSIONS

We investigate the dynamics of spectator particles co-propagating with matched and mismatched beam cores with uncoupled and coupled lattices. In order to simulate the underlying beam core, we have developed a fully analytic, self-consistent transverse Gaussian beam model that includes non-linear space charge forces and cross-plane coupling. We observe a filamentation of the phase space ellipses due to amplitude-dependent tune-shifts. The effect is enhanced with a mismatched beam which drives particle-core resonances, pushing particles to higher amplitudes. There are indications that suggest the formation of islands in phase space using mismatched beam cores. Simulations introducing coupling in addition to beam mismatch show that the outer particles are pushed to very high amplitudes. This indicates that coupling is a contributor to beam halo formation.

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