

Transverse Emittance Exchange for



Improved Injection Efficiency



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- I** **INJECTION PROCESS** — *WHAT CAN WE GAIN*

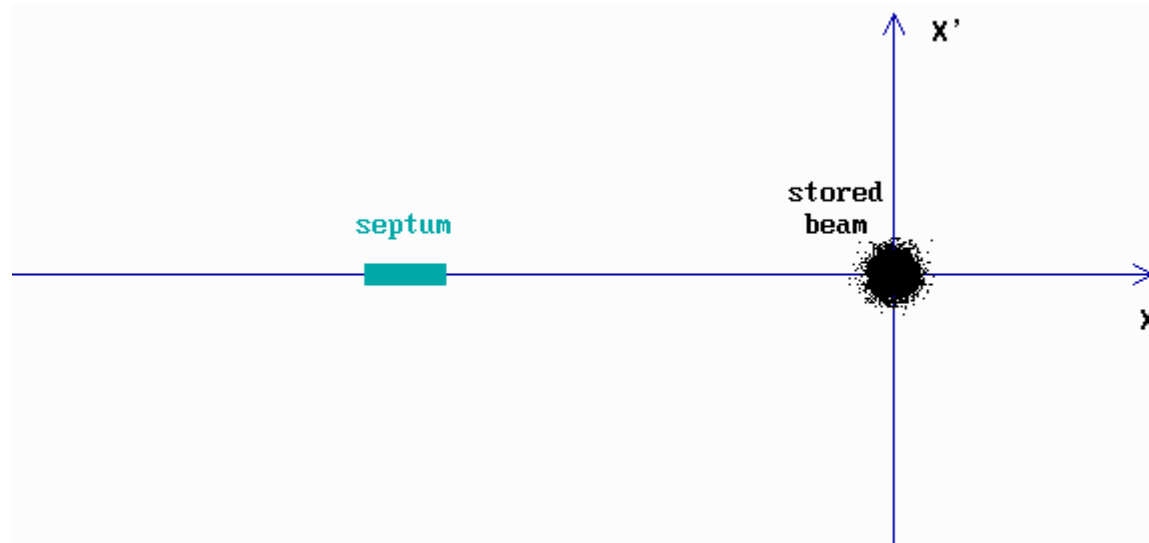
- II** **EMITTANCE EXCHANGE** — *HOW FAR CAN WE GO*

- III** **EMITTANCE SHARING** — *MODERATE REDUCTION*
 - III. 1** **ON THE COUPLING RESONANCE**
 - III. 2** **EXCITATION OF THE RESONANCE**

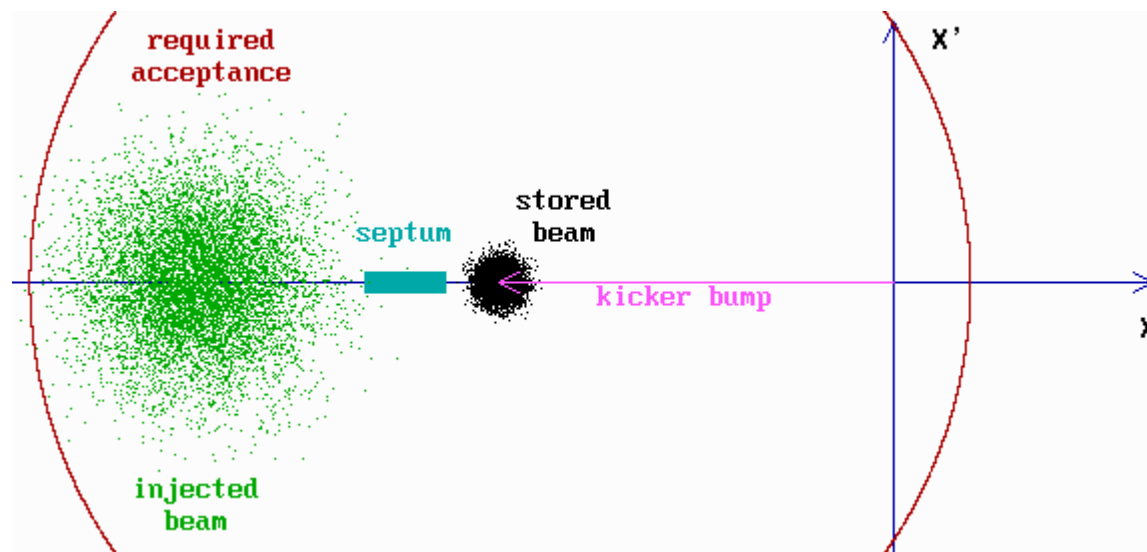
- IV** **EMITTANCE EXCHANGE** — *FULL REDUCTION*
 - IV. 1** **RESONANCE CROSSING**
 - IV. 2** **π -PULSE EXCITATION**
 - IV. 3** **IN THE TRANSFER LINE**

- V** **SUMMARY**

before injection

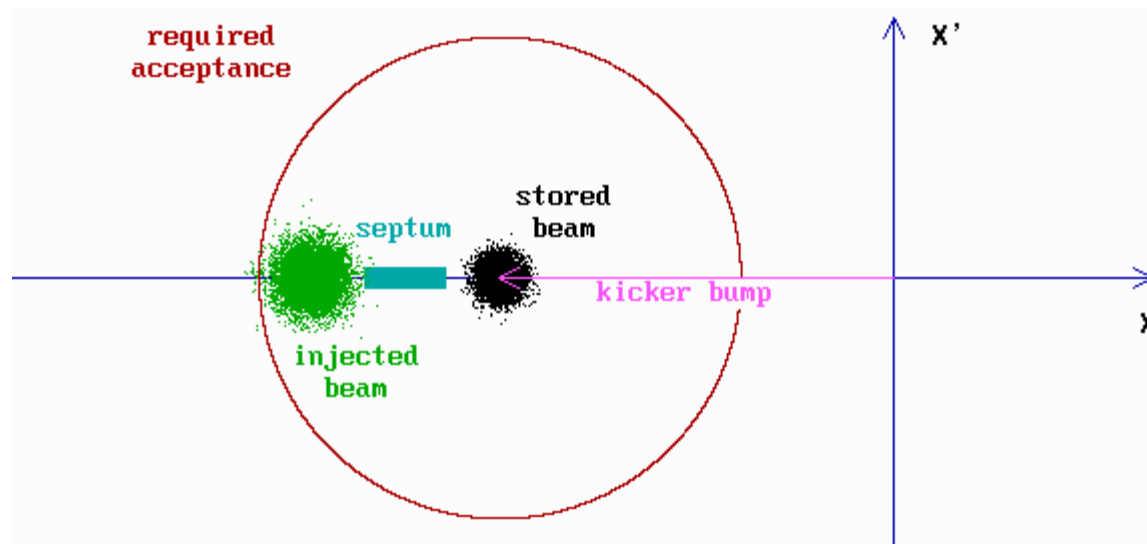


at injection – stored beam kicked close to the septum
injected beam arrives at the septum



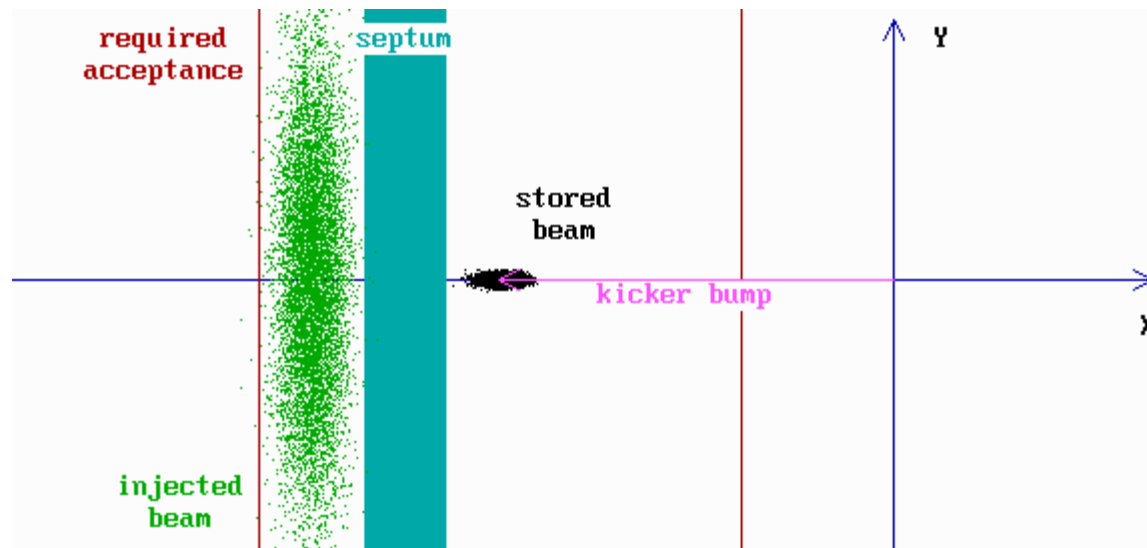
required acceptance $> 6 \cdot \sigma_{inj} + 6 \cdot \sigma_{sto} +$ **effective septum thickness**
dominated by large emittance of injected beam

smaller emittance of injected beam



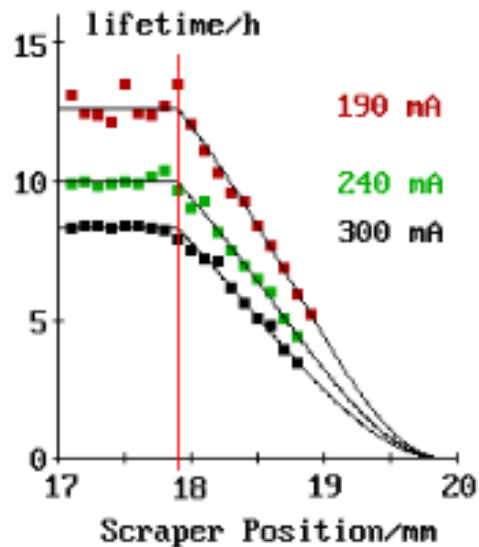
Swiss light source – synchrotron and storage ring in same tunnel
Spring8 – horizontal collimation in the transfer line
MAXIV – injection LINAC

Can we take advantage of the flat beam delivered by the synchrotron and use emittance exchange to reduce the horizontal emittance?



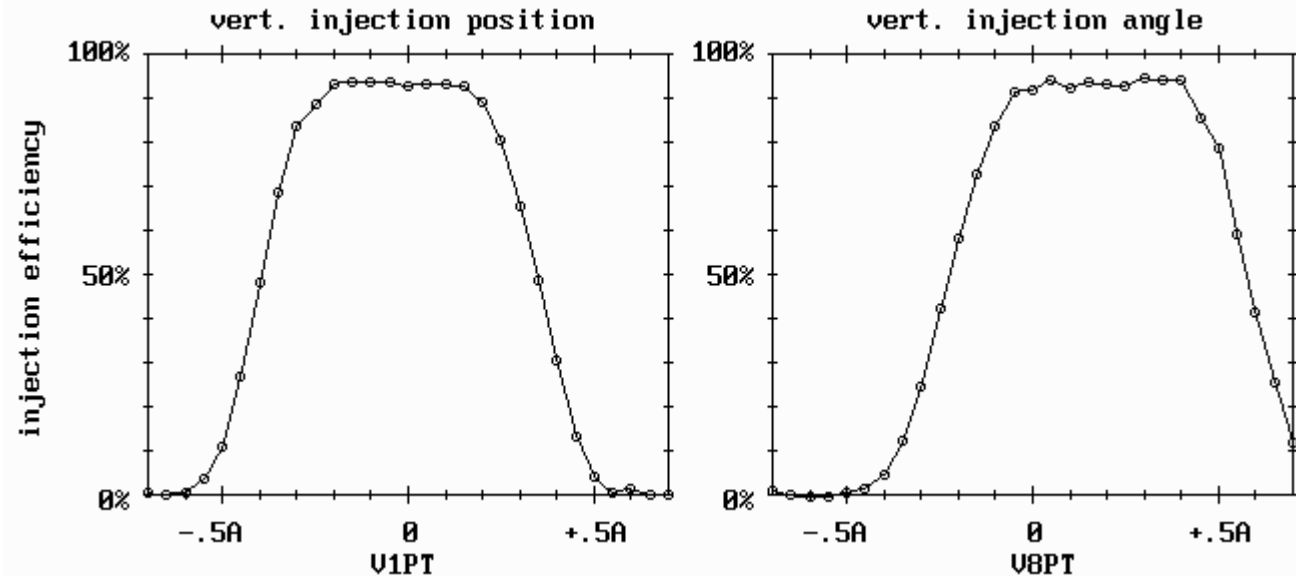
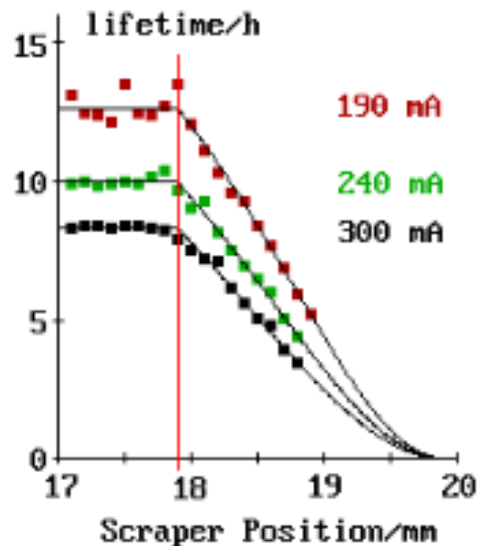
scraper measurements at
 $\beta_y = 1.25\text{m}$ and $\alpha_y = 0$ yield
 $A = 2\text{mm}$:

vertical acceptance $> 3 \cdot \varepsilon_{\text{inj}}$



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vertical acceptance $> 3 \cdot \varepsilon_{\text{inj}}$



measurements of injection efficiency as functions of vertical injection angle and position yield:

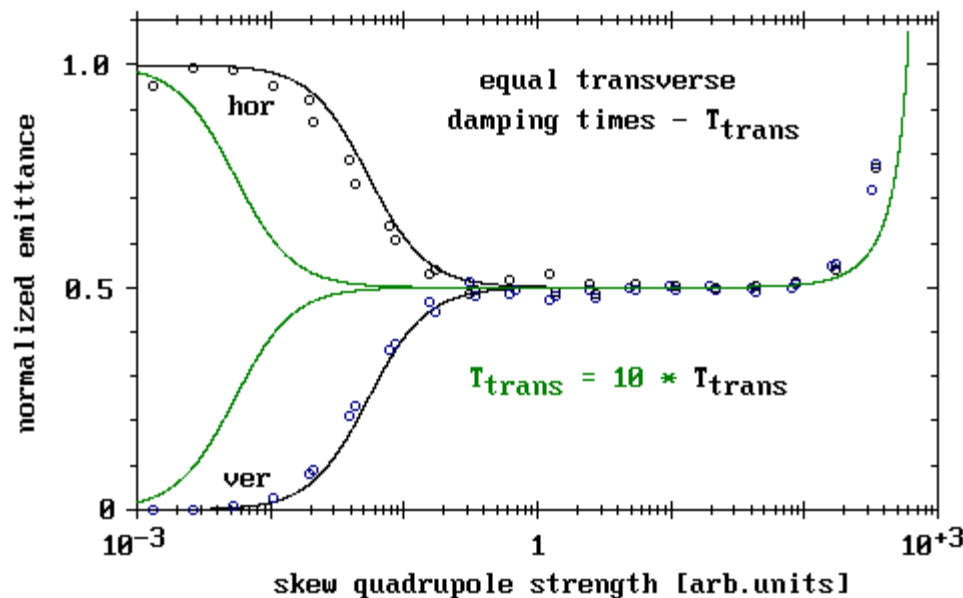
vertical acceptance $\sim \varepsilon_{\text{inj}}$

At Bessy II full emittance exchange feasible – considerable reduction of horizontal emittance

linear coupling due to skew quadrupole gradient:

$$Q_x - Q_y = n, \quad n = \text{integer}$$

on resonance emittance sharing - $\varepsilon_y = \varepsilon_x = \varepsilon_0/2$

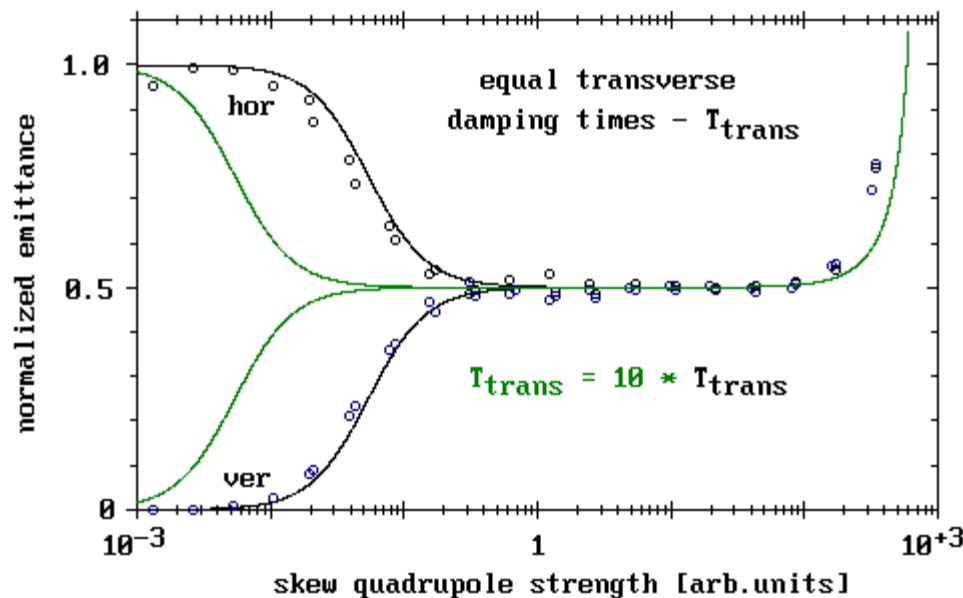


Comparison of solutions from multi particle tracking and first modelling attempts with analytical solutions based on moment mapping

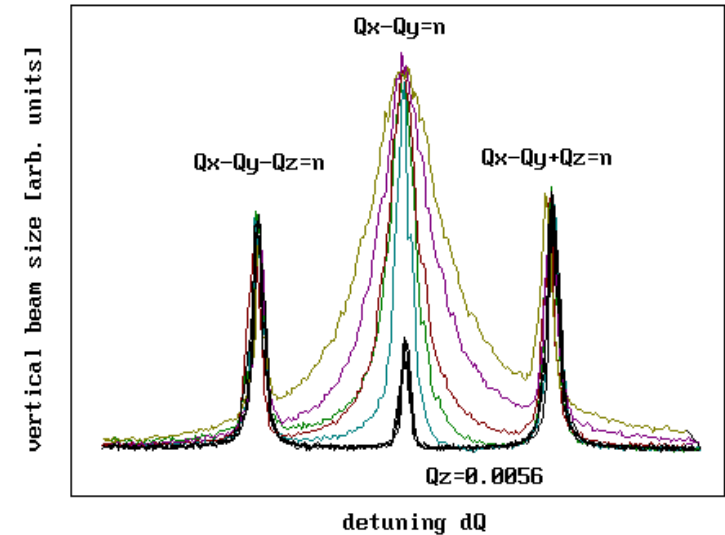
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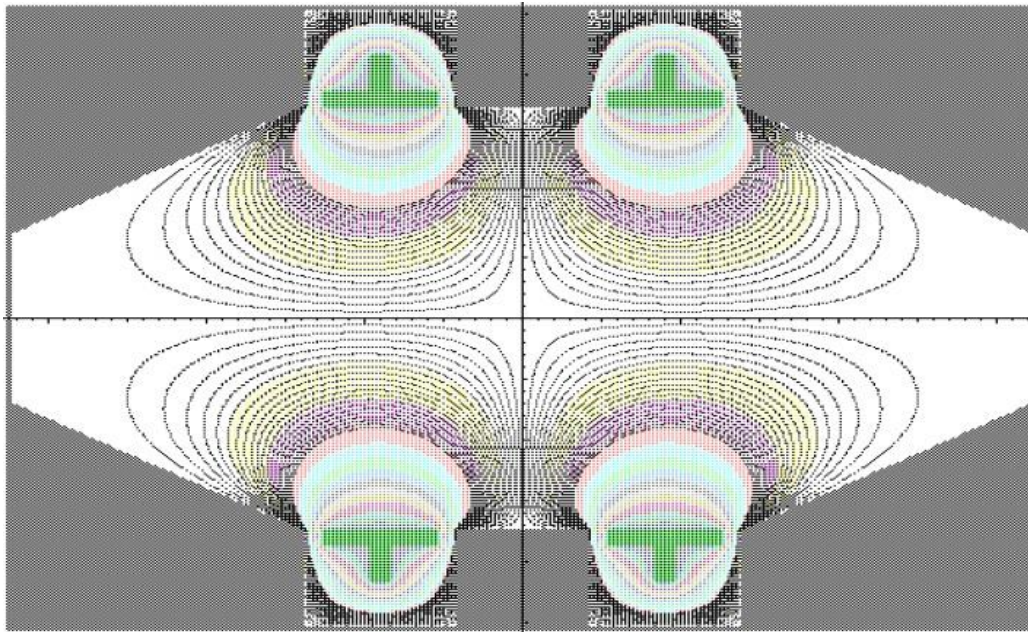
Compensation of the coupling resonance in the BESSY II storage ring – as expected: damping dominates for very small coupling coefficients, and width depends on coupling strength: „power broadening“, will be helpful later on

emittance sharing on resonance: $\varepsilon_y = \varepsilon_x = \varepsilon_0/2$, already valuable reduction

In case the synchrotron can not be operated at the coupling resonance the creation of an artificial resonance could help. With a time dependent sinusoidal varying skew gradient the resonance condition is:

$$Q_x - Q_y = n \pm F_{sq}/F_0,$$

with the revolution frequency, F_0 , and of the skew gradient, F_{sq} .



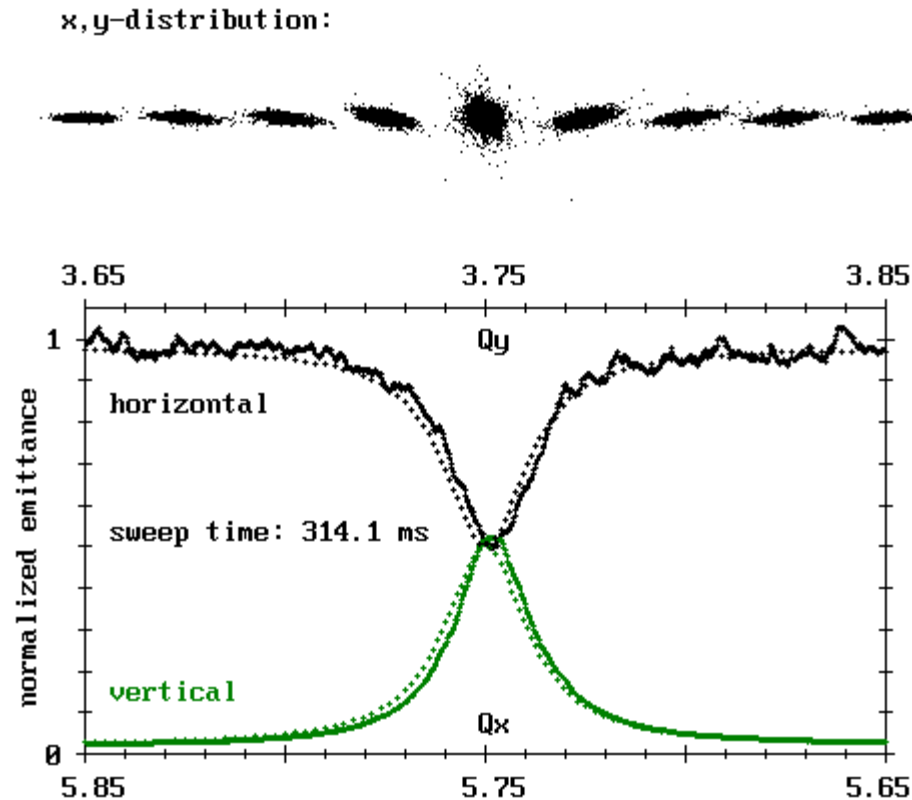
Skew quadrupole-like field distribution in the centre of the stripline arrangement.

Neighboring currents in opposite directions.

Full coupling and equal horizontal and vertical emittances achievable – little power broadening, sensitivity to tune jitter.

Could produce round beams compatible with horizontal off-axis injection.

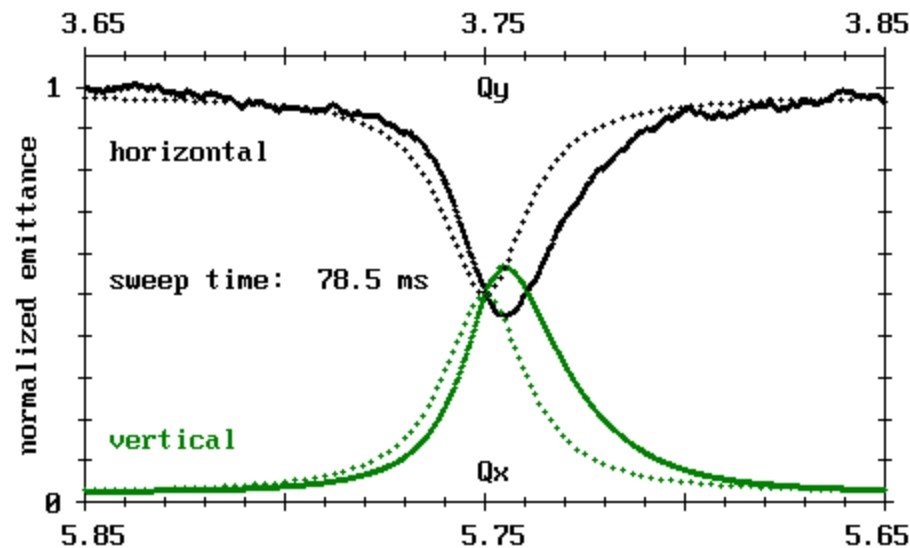
Slow crossing of the coupling resonance approaches the steady state solution for the beam emittances – dotted lines



Horizontal axis is also a time axis. Tunes are swept within a sweep time of 314.1 ms across the resonance. Transverse damping time of the BESSY synchrotron is 9.8 ms.

The faster the resonance is crossed the more the result deviates from the steady state solution and the better is the emittance exchange

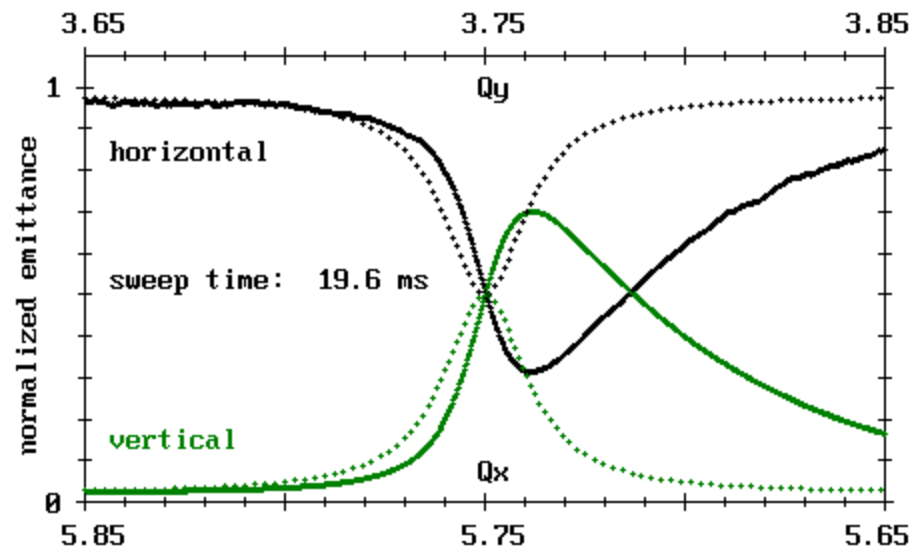
x,y-distribution:



Horizontal axis is also a time axis. Tunes are swept within a sweep time of 78.5 ms across the resonance. Transverse damping time of the BESSY synchrotron is 9.8 ms.

The faster the resonance is crossed the more the result deviates from the steady state solution and the better is the emittance exchange

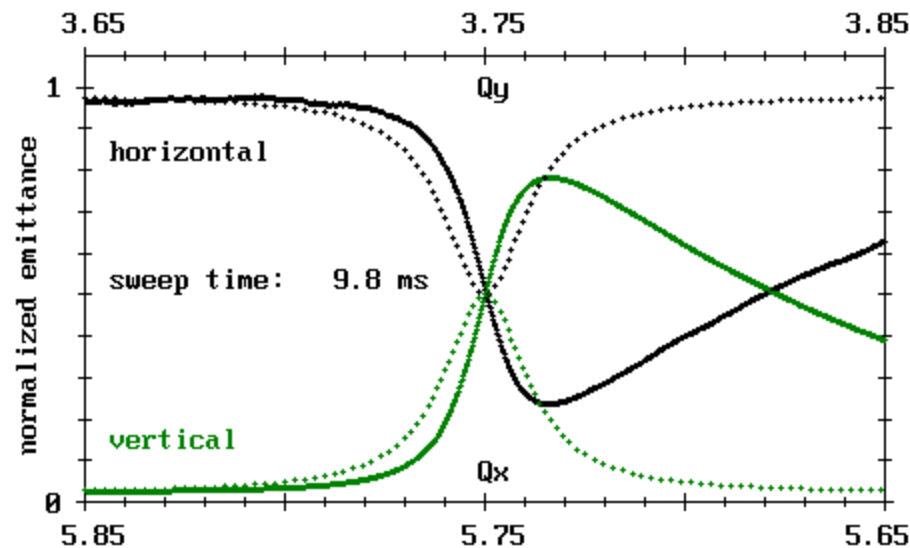
x, y -distribution:



Horizontal axis is also a time axis. Tunes are swept within a sweep time of 19.6 ms across the resonance. Transverse damping time of the BESSY synchrotron is 9.8 ms.

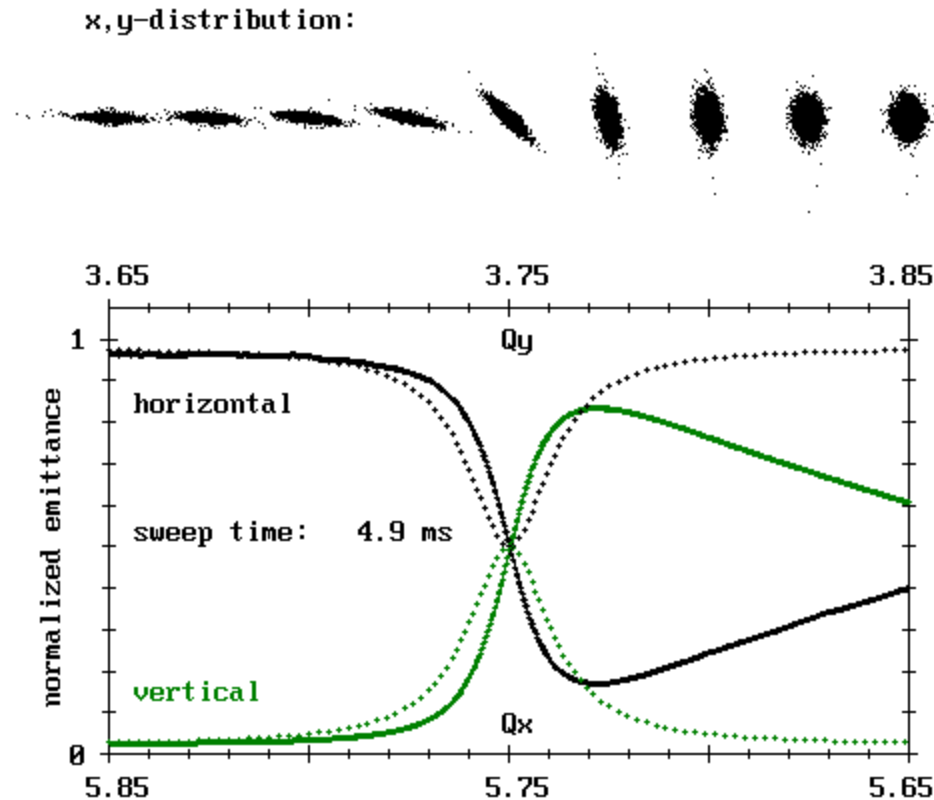
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x,y-distribution:



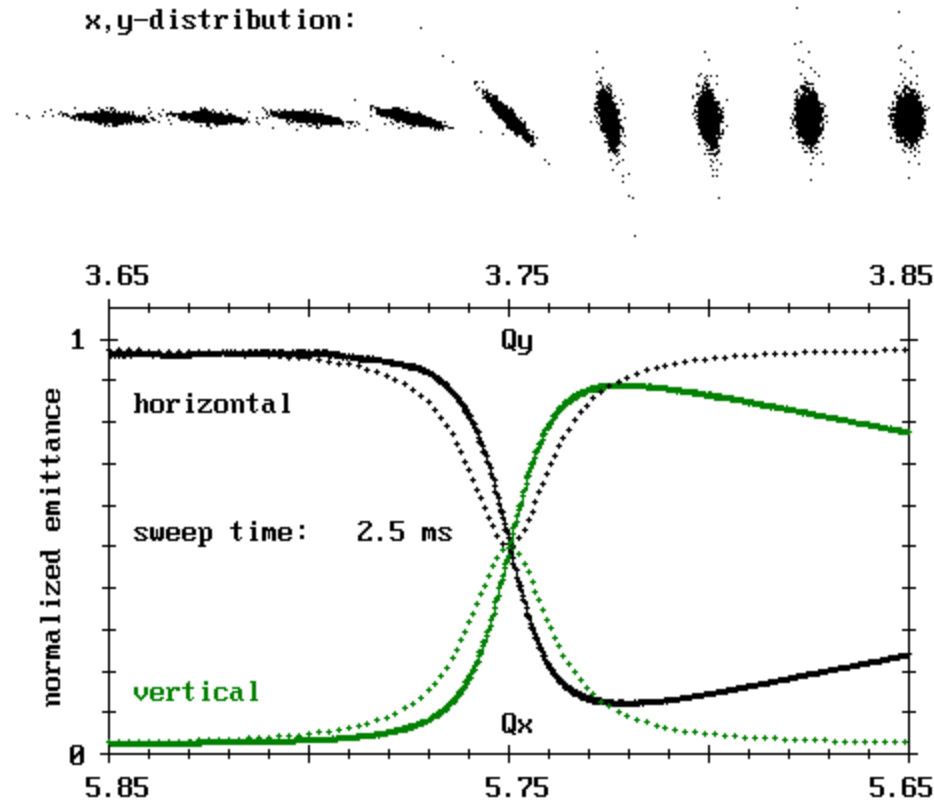
Horizontal axis is also a time axis. Tunes are swept within a sweep time of 9.8 ms across the resonance. Transverse damping time of the BESSY synchrotron is 9.8 ms.

The faster the resonance is crossed the more the result deviates from the steady state solution and the better is the emittance exchange



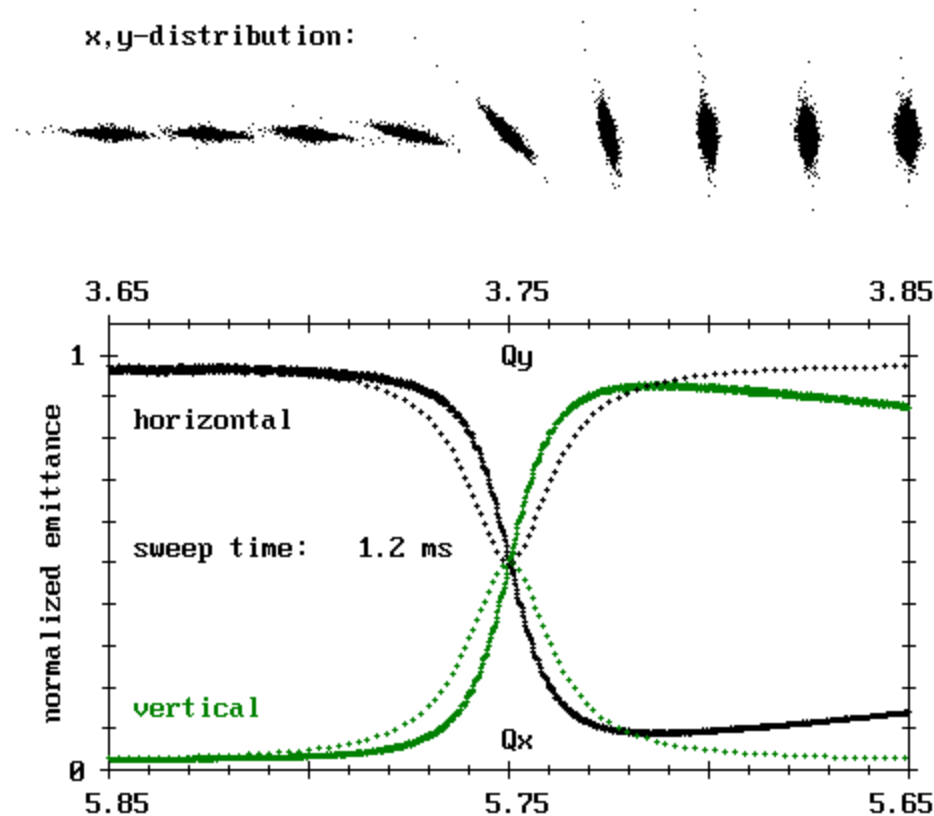
Horizontal axis is also a time axis. Tunes are swept within a sweep time of 4.9 ms across the resonance. Transverse damping time of the BESSY synchrotron is 9.8 ms.

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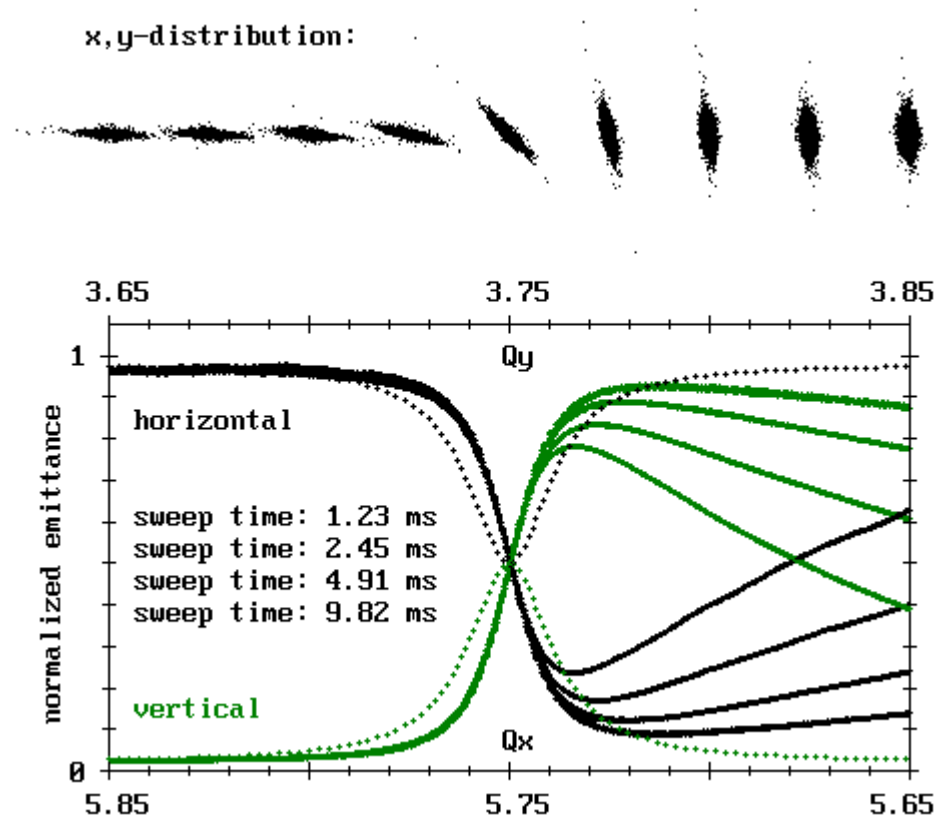
Horizontal axis is also a time axis. Tunes are swept within a sweep time of 2.5 ms across the resonance. Transverse damping time of the BESSY synchrotron is 9.8 ms.

The faster the resonance is crossed the more the result deviates from the steady state solution and the better is the emittance exchange



Horizontal axis is also a time axis. Tunes are swept within a sweep time of 1.2 ms across the resonance. Transverse damping time of the BESSY synchrotron is 9.8 ms.

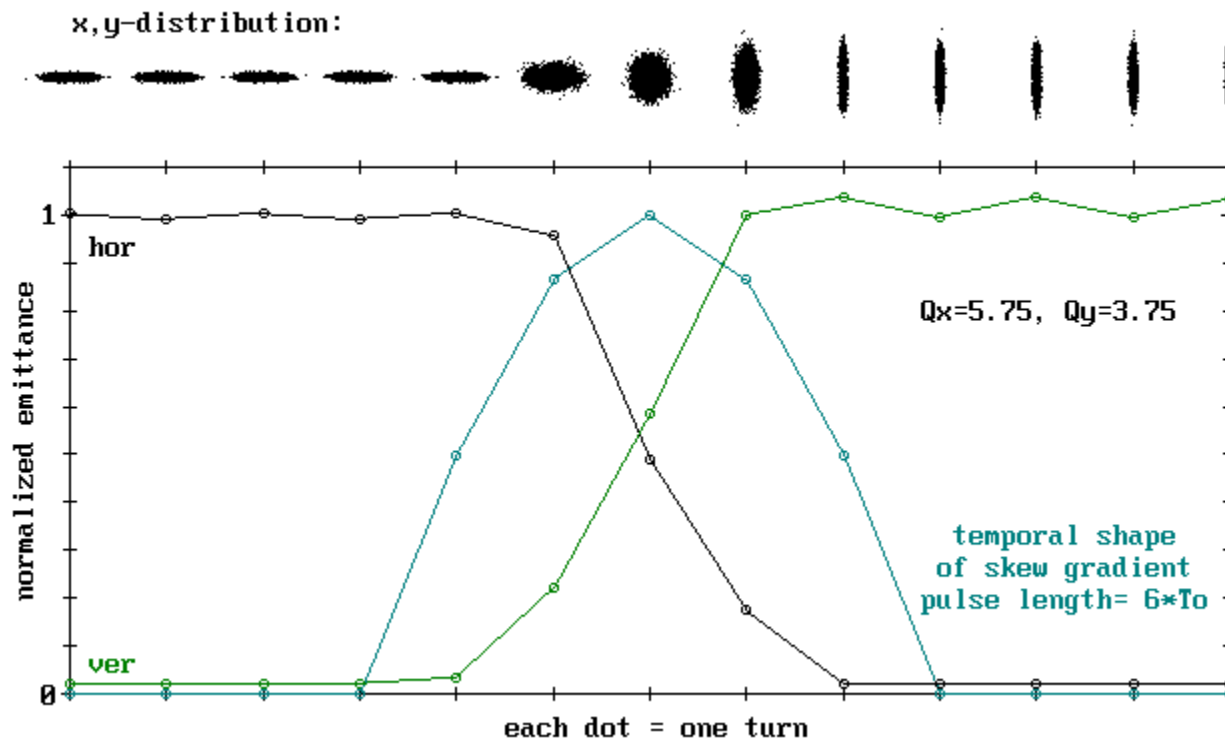
Crossing of the coupling resonance fast compared to the transverse damping times results in an attractive emittance exchange and 10 times smaller horizontal emittance



Quadrupole magnets and power supplies of ramped synchrotrons can produce the desired tune shifts shortly before the beam is extracted. The White-circuits of the BESSY synchrotron do not allow this.

C. Carli, et al., "Emittance Exchange by Crossing a Coupling Resonance", EPAC 2002 **19**

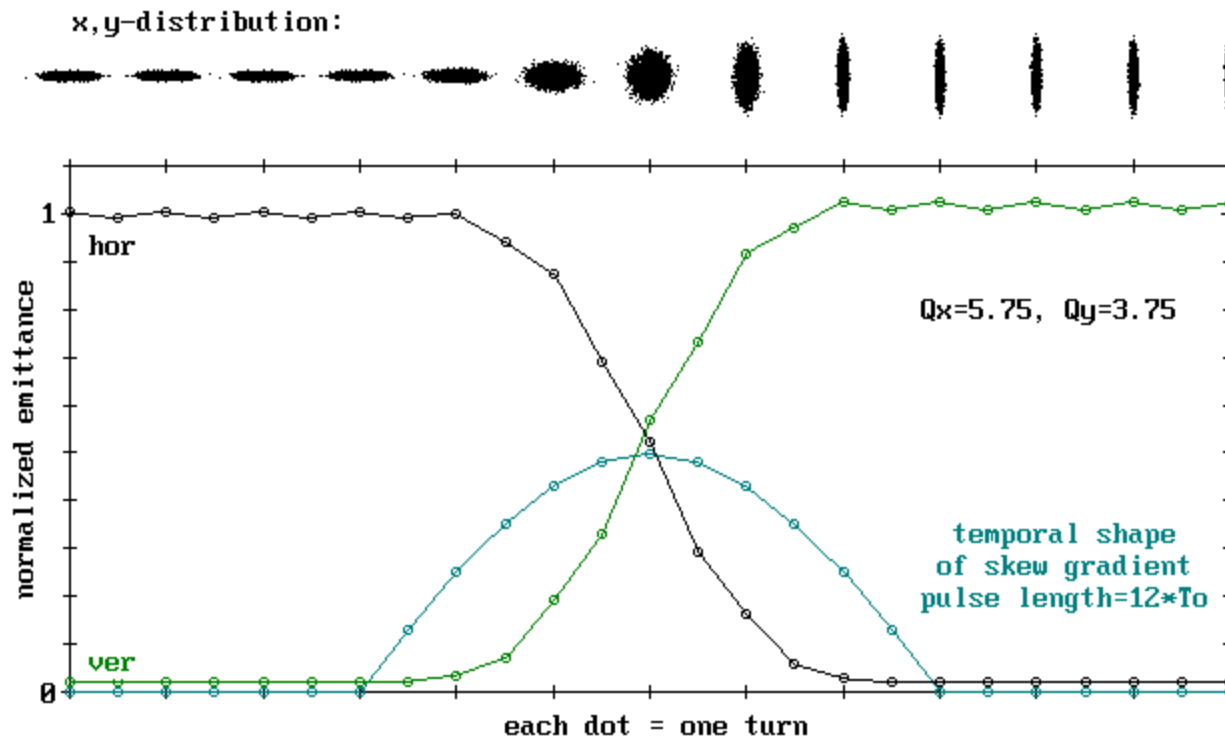
Like in many two-level systems (NMR, atomic transitions, spin=1/2-system, ...) the inversion of the states is achieved with π -pulse. This can be applied to create also an emittance exchange. On the coupling resonance we need a pulsed skew quadrupole magnet with a duration short compared to the transverse damping times.



For π -pulse the time integrated gradient is constant:
 peak skew gradient, $\frac{\partial B_x}{\partial x}$, pulse length $n \cdot T_0$

$$\frac{\partial B_x}{\partial x} \cdot n \approx 4.94 \frac{B\rho}{\sqrt{\beta_x \beta_y} \cdot L_{skew}}$$

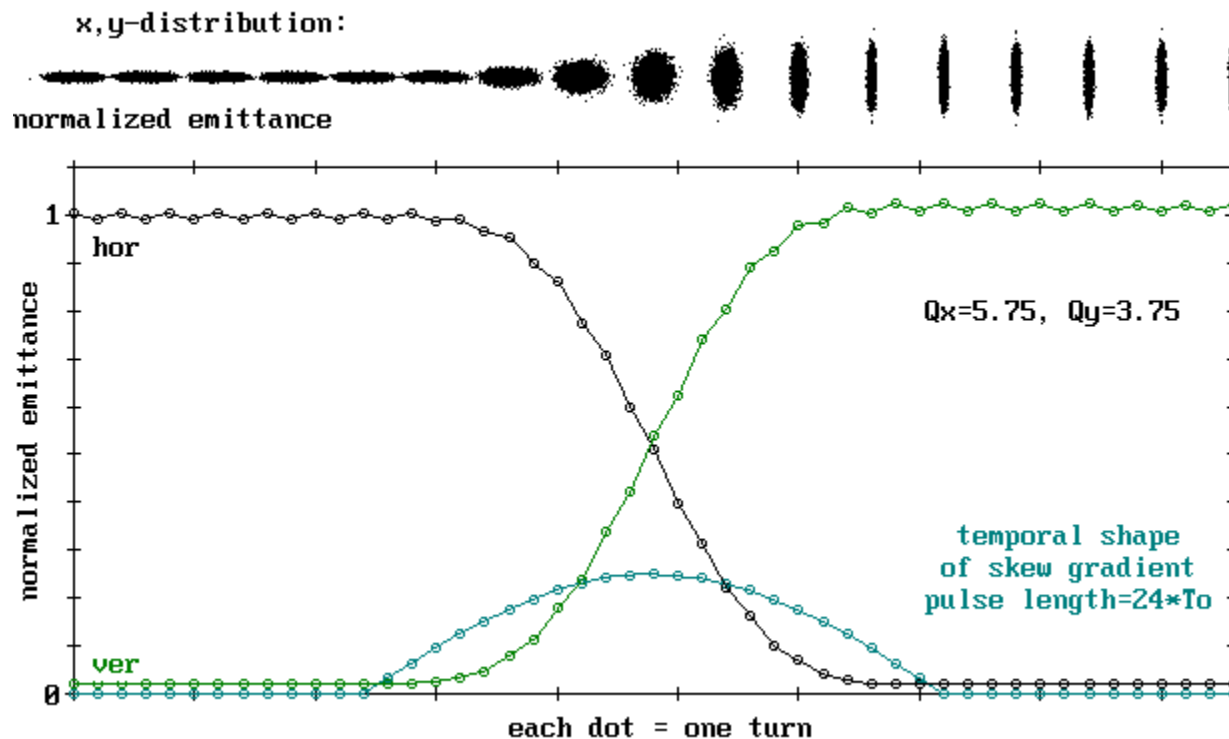
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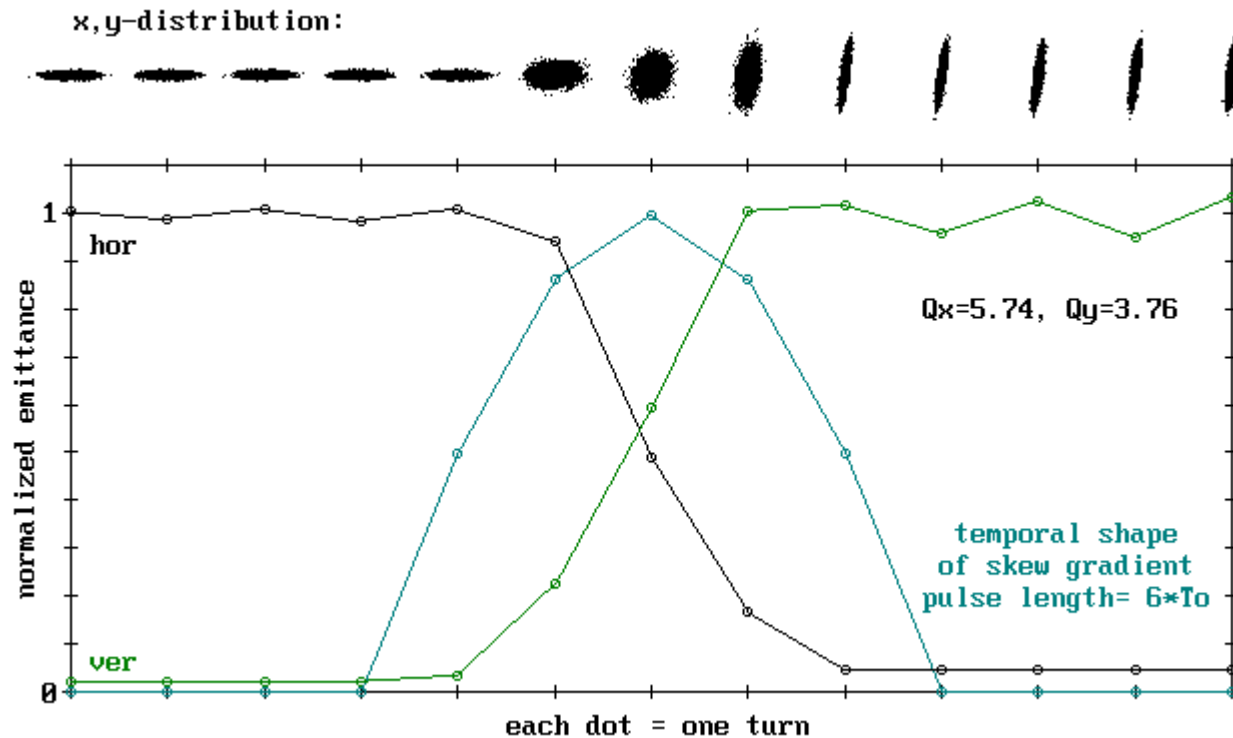
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With a short and strong skew gradient pulse the power broadening helps to exchange the emittance even slightly off-resonance. Tunes still close to the coupling resonance – resonance needs to be decoupled.



The necessary peak field for a half sinusoidal π -pulse can be determined by mapping the moments of the particle distribution turn-by-turn with a time dependent skew quadrupole. Damping and excitation can be ignored because of the short pulse length.

Design of a pulsed skew quadrupole magnet

At BESSY a 6 turn long π -pulse requires a peak skew gradient of:

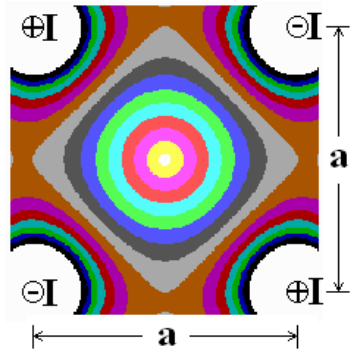
$$\frac{\partial B_x}{\partial x} \approx 4.94/n \frac{B\rho}{\sqrt{\beta_x \beta_y} \cdot L_{skew}} \approx 4.94/6 \frac{5.7 \text{ Tm}}{\sqrt{4.7 \text{ m} \cdot 5.0 \text{ m} \cdot 0.3 \text{ m}}} \approx 3.2 \text{ T/m}$$

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skew quadrupole - 4 wire arrangement
with currents in alternating directions



$$\left| \frac{\partial B_x}{\partial x} \right| = \frac{4 \cdot \mu \cdot I}{\pi \cdot a^2} = \frac{1.6 \cdot 10^{-6} \cdot I [A]}{a^2 [m^2]} [T/m]$$

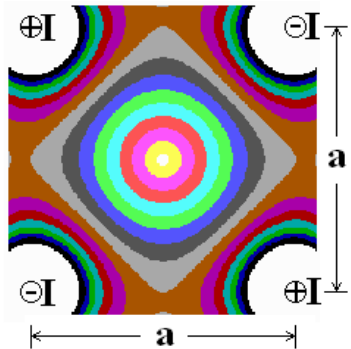
$$\left| \frac{\partial B_x}{\partial x} \right| = 4 T/m \quad \text{for } a=2 \text{ cm and } I=1 \text{ kA}$$

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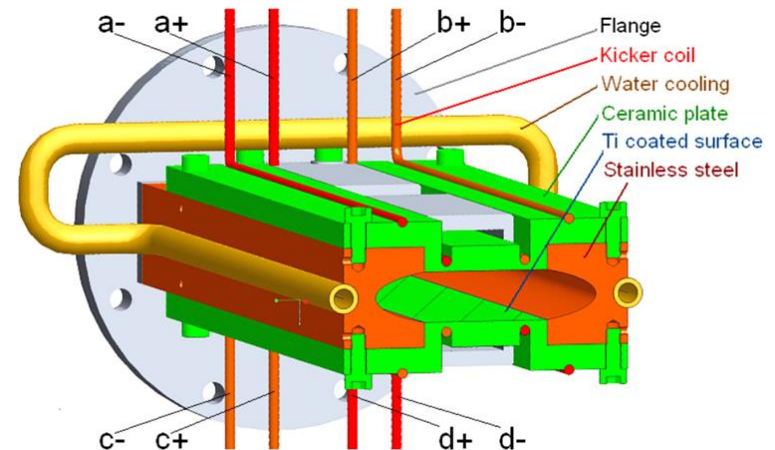
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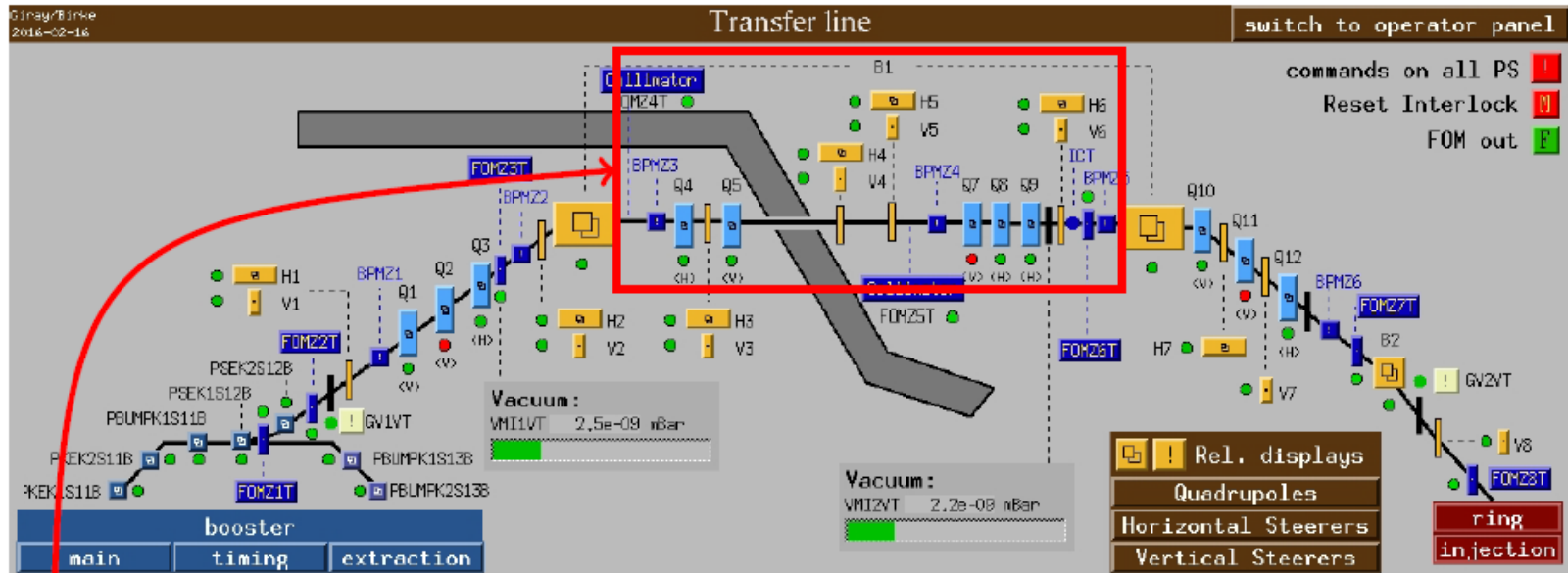
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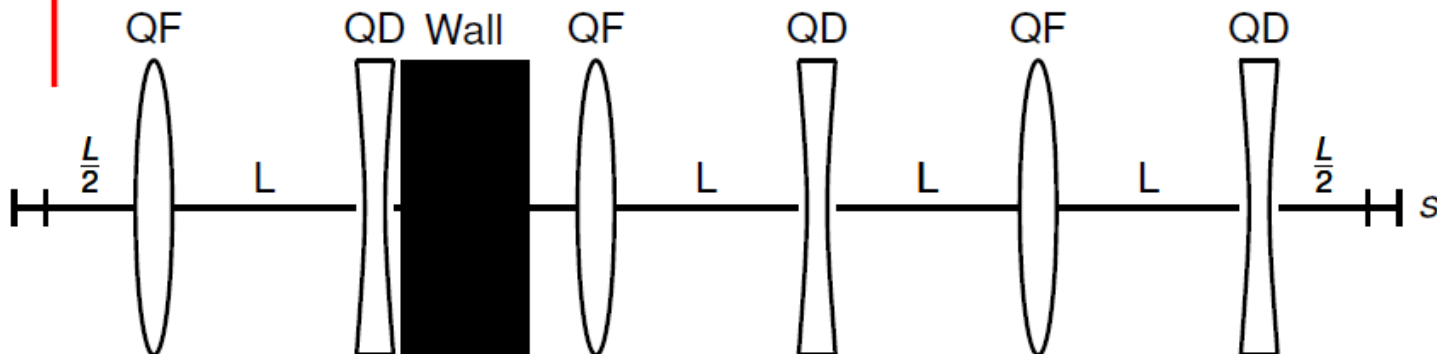


quite similar to our non-linear
injection kicker magnet

BESSY II transferline overview in control system



Possible transverse emittance exchange sequence, available space: 9.42551m, used: 9m

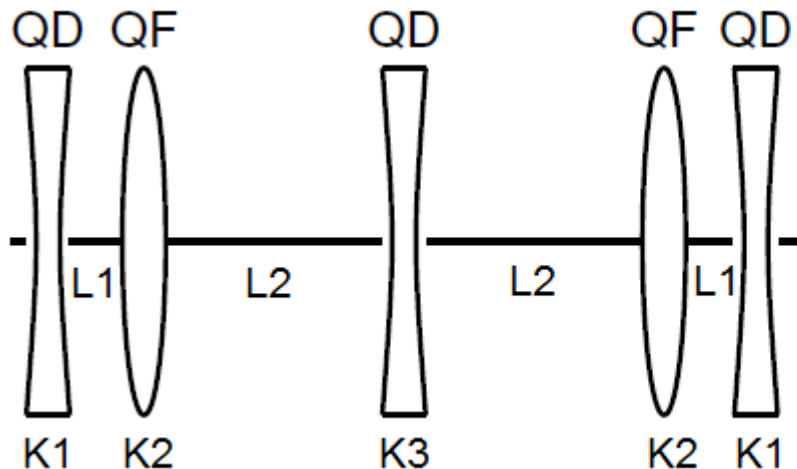


$$\begin{aligned}
 Lq &= 0.25m \\
 L &= 1.25m \\
 k_F &= 4.7586 \\
 k_D &= 3.3879
 \end{aligned}$$

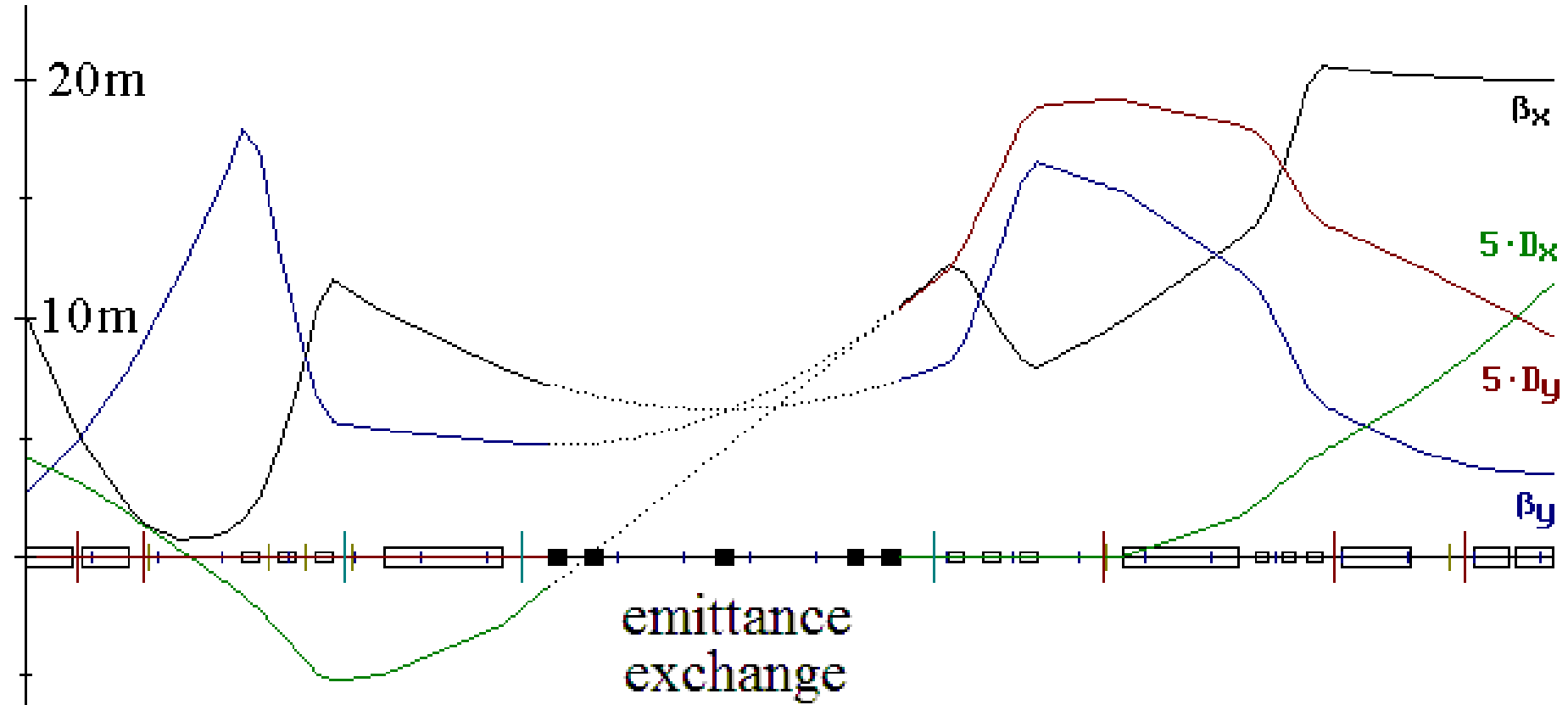
Transfer matrix of emittance exchange structure – with 6 skew quadrupole magnets:

$$R = \begin{pmatrix} 0 & D \\ D & 0 \end{pmatrix} \quad \text{with: } D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad L_{\text{tot}} = 9 \text{ m, too long for any optics matching}$$

Transfer matrix of emittance exchange structure – with 5 skew quadrupole magnets:



$$D = \begin{pmatrix} 1 & L_{\text{tot}} \\ 0 & 1 \end{pmatrix}$$

$\beta_x=20.000\text{m}$ $\alpha_x=0.001$ $\beta_y=3.499\text{m}$ $\alpha_y=-0.000$ $D_x=2.289\text{m}$ $D_x'=0.546$ $D_y=1.859\text{m}$ $D_y'=-.687$


No control of the dispersion – contribution from the energy spread of the injected beam spoils gain from emittance exchange completely

Smaller emittance of injected beam eases injection

Beam delivered by synchrotrons usually have quite large horizontal to vertical emittance ratios – horizontal injection can profit from emittance sharing or exchange – if vertical acceptance is sufficiently large

Emittance sharing on linear coupling resonance or excited by oscillating skew gradient

Emittance exchange by:

- 1) fast crossing the coupling resonance – operating close to the coupling resonance
- 2) π -pulse excitation – operating very close to the coupling resonance
- 3) skew quadrupole arrangement in transfer line – dispersion is very critical

Which option to choose depends on the actual boundary conditions of the facility