Conditions for Coherent Synchrotron Radiation Microbunching Gain Suppression

Cheng-Ying Tsai

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IPAC'16 at Busan, Korea

TUOAB02

Outline

- Introduction and Overview
- Theoretical formulation of CSR microbunching in a single-pass system
- Conditions for CSR microbunching gain suppression
- Examples
- Summary and Conclusion

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  for test particle receiving radiation within dipoles.
- Longitudinal field acting on the head particle
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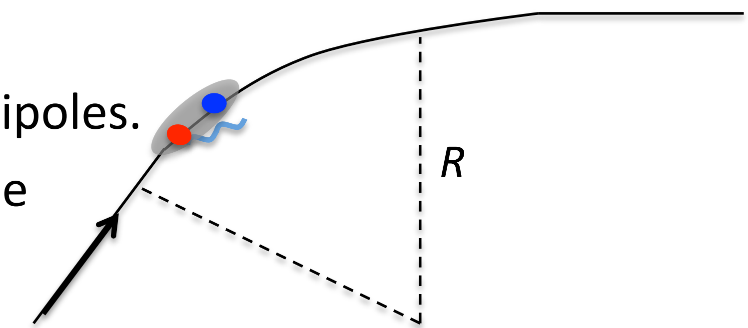

$$E_s(z)=\frac{2e^2}{4\pi\varepsilon_0\left(3R^2\right)^{1/3}}\int_{-\infty}^{z}\frac{dz'}{(z-z')^{1/3}}\frac{d\lambda(z')}{dz'}$$

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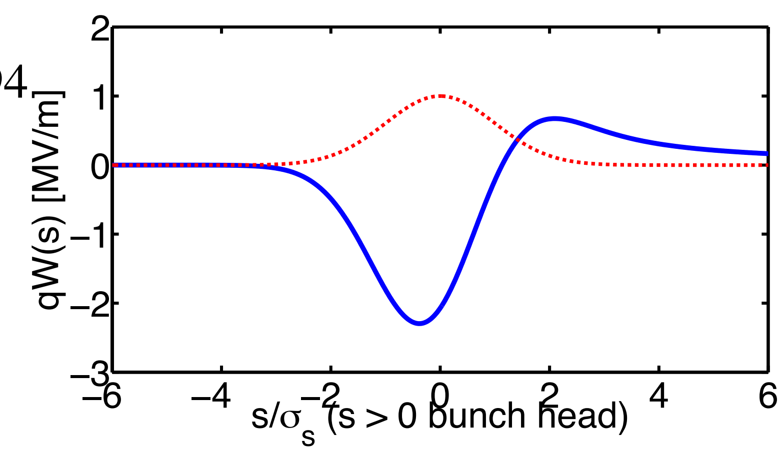

Ya. S. Derbenev et al., DESY Report No. TESLA-FEL 95-05 (1995)
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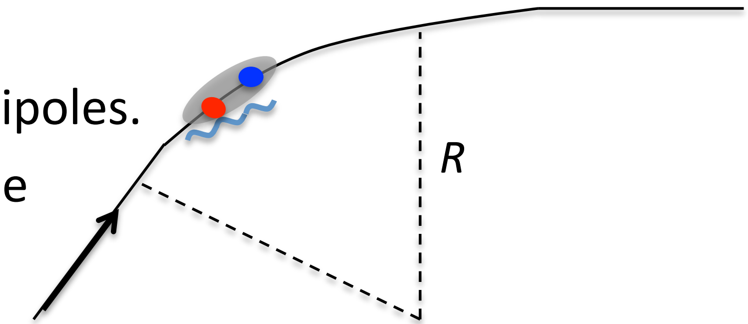

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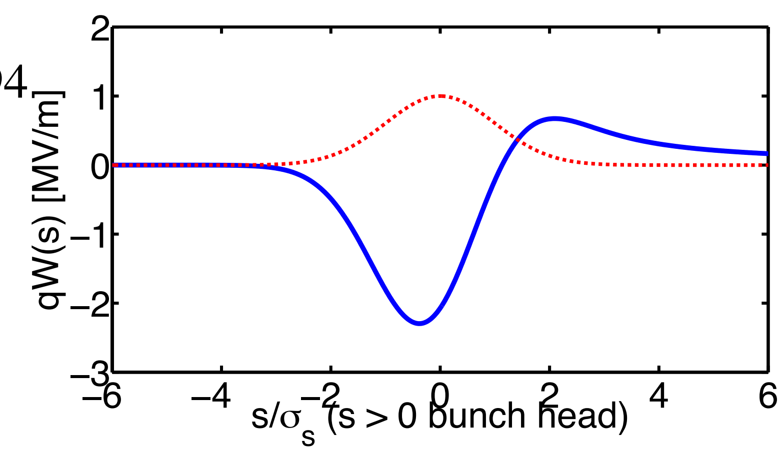

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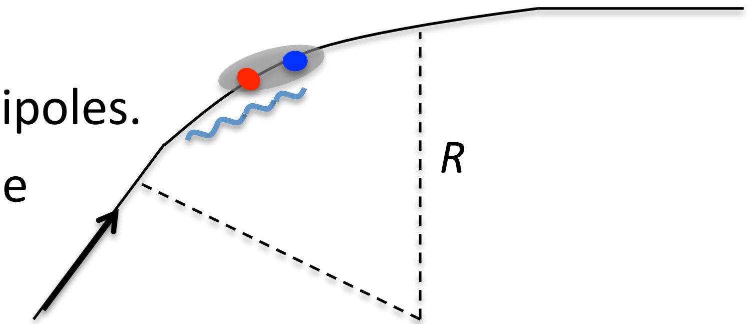

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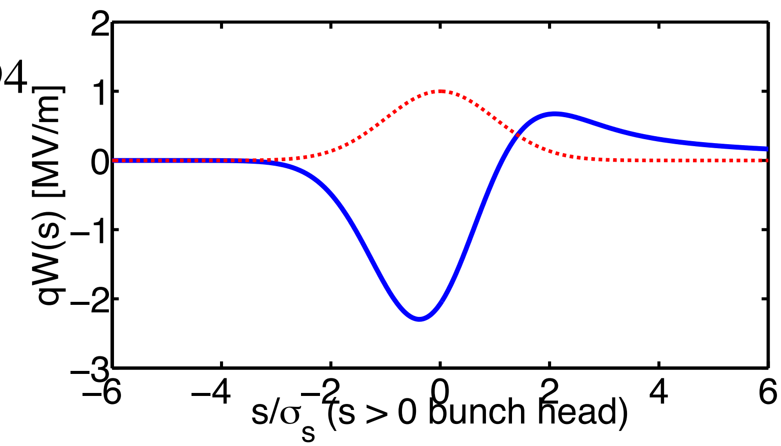

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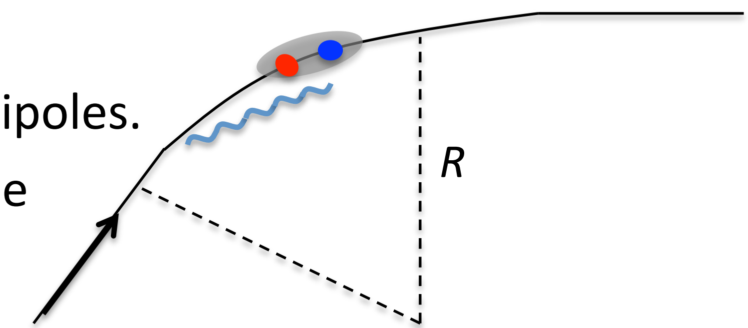

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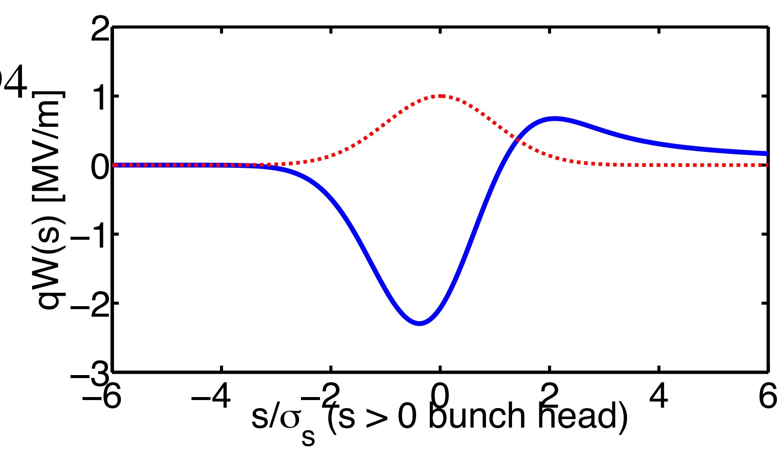

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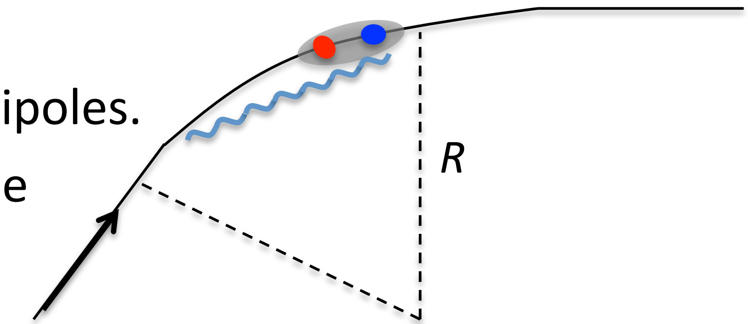

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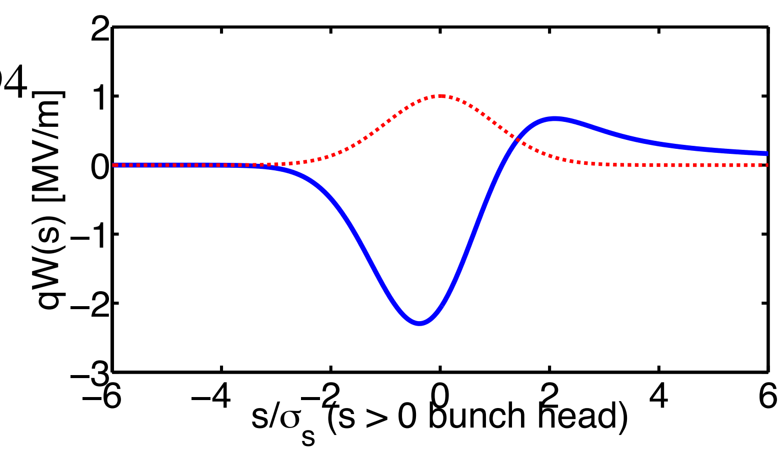

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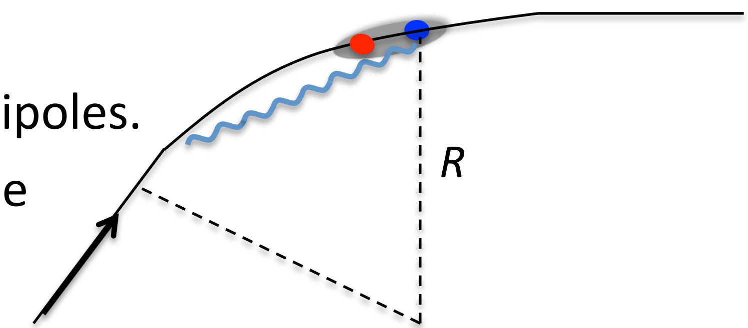

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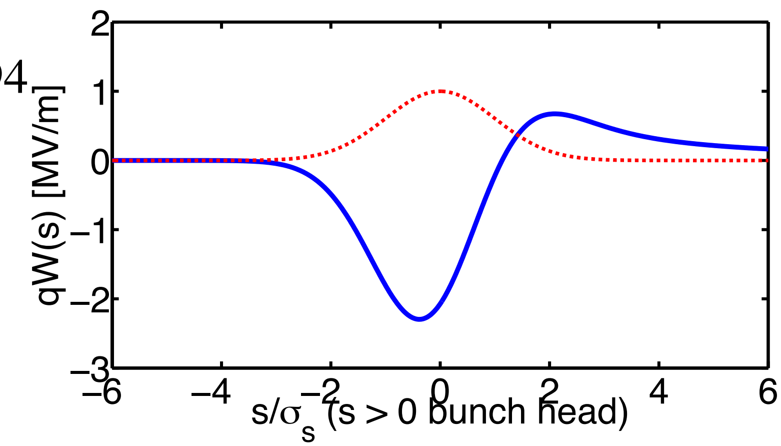

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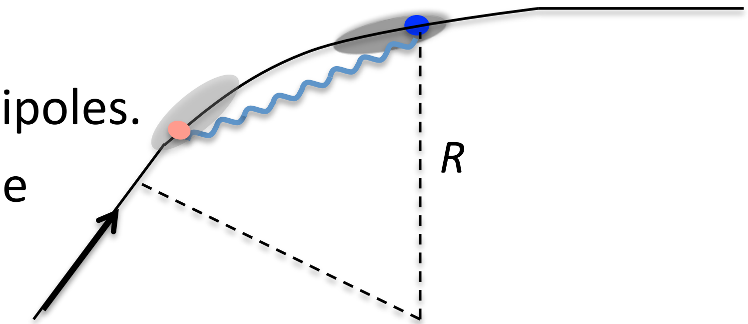

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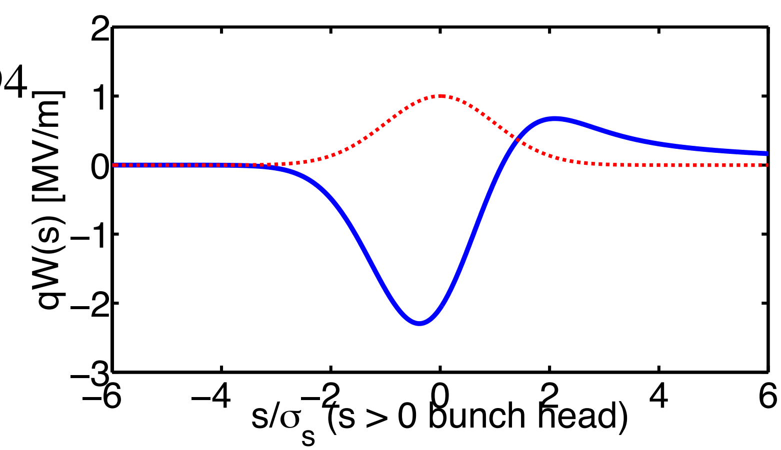

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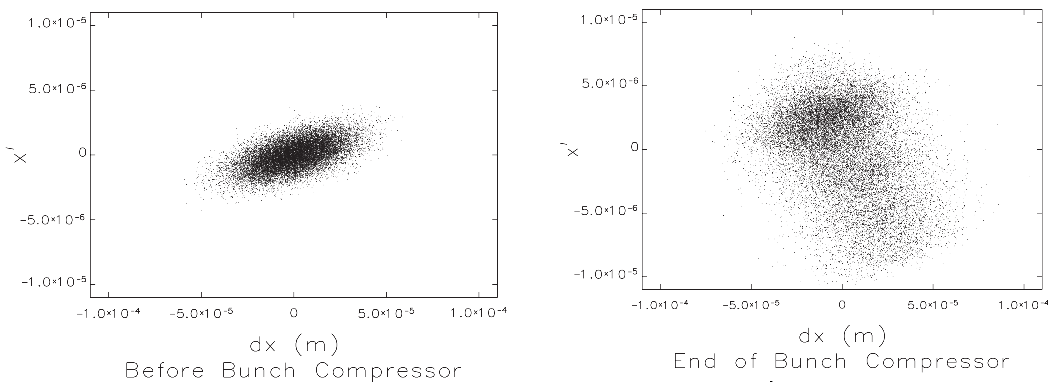
CSR effects on the beam

- **Transverse: Emittance Growth**



Y. Jing et al., PRSTAB 16, 060704 (2013)

CSR kick $\Delta\delta_{CSR}(z)$

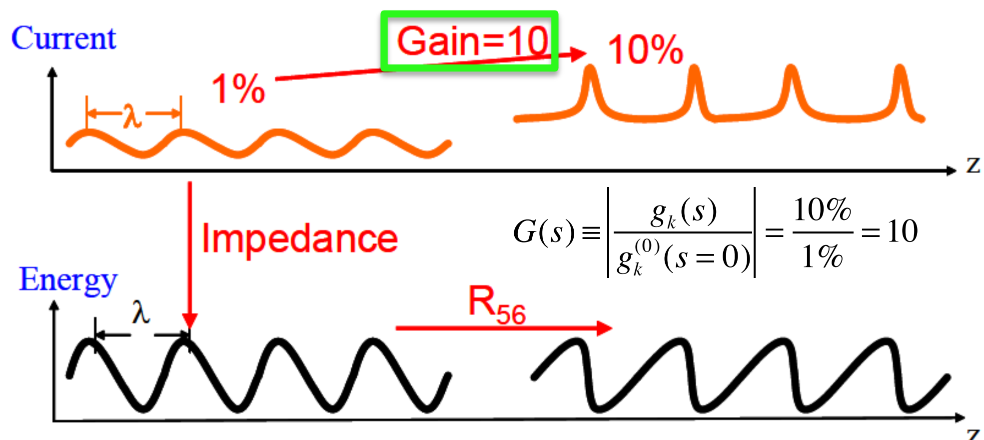
$\Downarrow$ R_{16} or R_{26}

$\Delta x_{CSR}, \Delta x'_{CSR}$

$\Downarrow$

possible ε-growth

- **Longitudinal: Microbunching Instability (MBI)**



Z. Huang and J. Wu, Beam Dynamics Newsletter, No. 38

density modulation

$Z_{CSR}(k) \Downarrow$ $\Uparrow$ R_{56}

energy modulation

Overview of mitigation schemes

Mitigation of CSR effects on beam dynamics		
Dimension	Mitigation schemes	Note
Transverse	**Cell-to-cell phase matching** (Douglas, Di Mitri *et al.*)	optics adjustment
	Beam **envelope matching** (Hajima)	
	Combination of the above concepts, application to DBA/TBA (Jiao *et al.*) or bunch compressor system (Jing *et al.*)	
	Longitudinal **bunch shaping** (Mitchell *et al.*)	tailoring initial conditions
Longitudinal	Laser **heating** (Saldin *et al.*, Huang *et al.*)	Landau damping enhancement via σ_δ
	Magnetic **mixing** chicane (Di Mitri *et al.*)	
	Reversible electron beam **heating** (Behrens *et al.*)	
	Insertion of dipole pair in an accelerator system (Qiang *et al.*)	take advantage of ε_x via R_{51} and R_{52}

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	(this presentation)	**optics adjustment**

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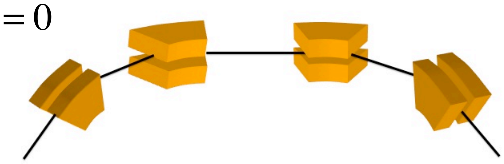
Vlasov treatment - a kinetic model

- Particle tracking: straightforward, subject to numerical noise (posing computational load).
- Vlasov method: more efficient in numerical simulation, free from numerical noise.
- Vlasov equation + single-particle equations of motion:

$$\frac{\partial f}{\partial s}+\left(\frac{dz}{ds}\right)\frac{\partial f}{\partial z}+\left(\frac{d\delta}{ds}\right)\frac{\partial f}{\partial \delta}+\left(\frac{dx}{ds}\right)\frac{\partial f}{\partial x}+\left(\frac{dy}{ds}\right)\frac{\partial f}{\partial y}+\left(\frac{d\theta_x}{ds}\right)\frac{\partial f}{\partial \theta_x}+\left(\frac{d\theta_y}{ds}\right)\frac{\partial f}{\partial \theta_y}=0$$

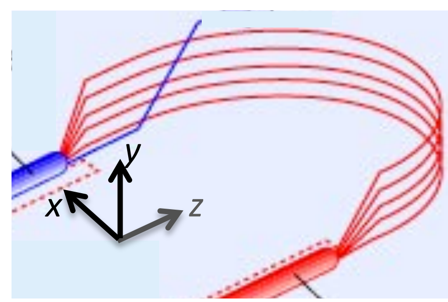


$$\frac{dz}{ds}=-\frac{x}{\rho}$$

$$\frac{d\delta}{ds}=-\frac{r_e}{\gamma}\int_{\infty}^{\infty} dz' W_{\parallel}(z-z',s)\,n(z',s)$$

$$\frac{dx}{ds}=\theta_x \qquad \frac{dy}{ds}=\theta_y$$

$$\frac{d\theta_x}{ds}=-k_{\beta x}^2(s)x+\frac{\delta}{\rho_x} \qquad \frac{d\theta_y}{ds}=-k_{\beta y}^2(s)y+\frac{\delta}{\rho_y}$$

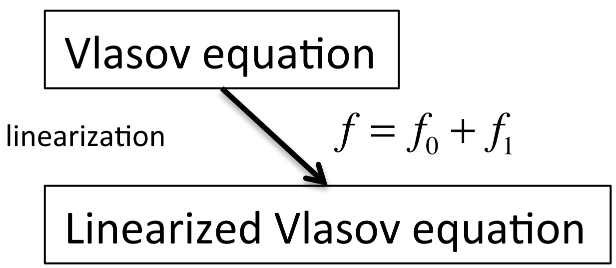


Including **vertical** bending is particularly useful for recirculation machines because such lattices usually contain **spreader** and **recombiner** parts.

S. Heifets et al., PRSTAB **5**, 064401 (2002).
Z. Huang and K. Kim, PRSTAB **5**, 074401 (2002)

Vlasov treatment - a kinetic model

- Linearization of Vlasov equation
- Transform this problem **into frequency domain**
  - modulation of a bunch (i.e. bunching factor) is Fourier component of its bunch distribution
- Track the evolution of the **bunching factor**, which is used to characterize MBI
- Take into account the relevant collective effects (**impedances**)



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Vlasov equation

linearization

$$f = f_0 + f_1$$

Linearized Vlasov equation

transform to frequency domain

$$g_k(s) \equiv \int d\mathbf{X} f_1 e^{-ikz}$$

$$g_k(s) = g_k^{(0)}(s) + \int_0^s K(s,s') g_k(s') ds'$$

$$G(s) = \left| \frac{g_k(s)}{g_k^{(0)}(s=0)} \right|$$ (microbunching gain)

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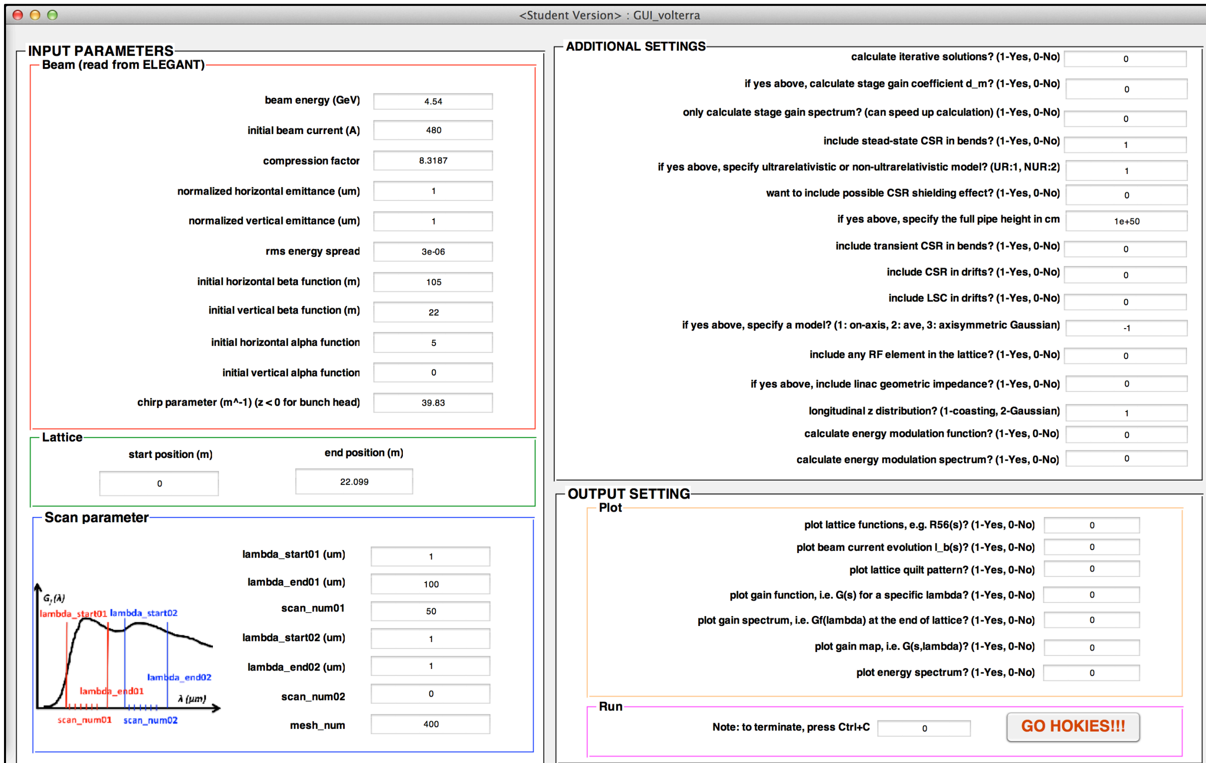
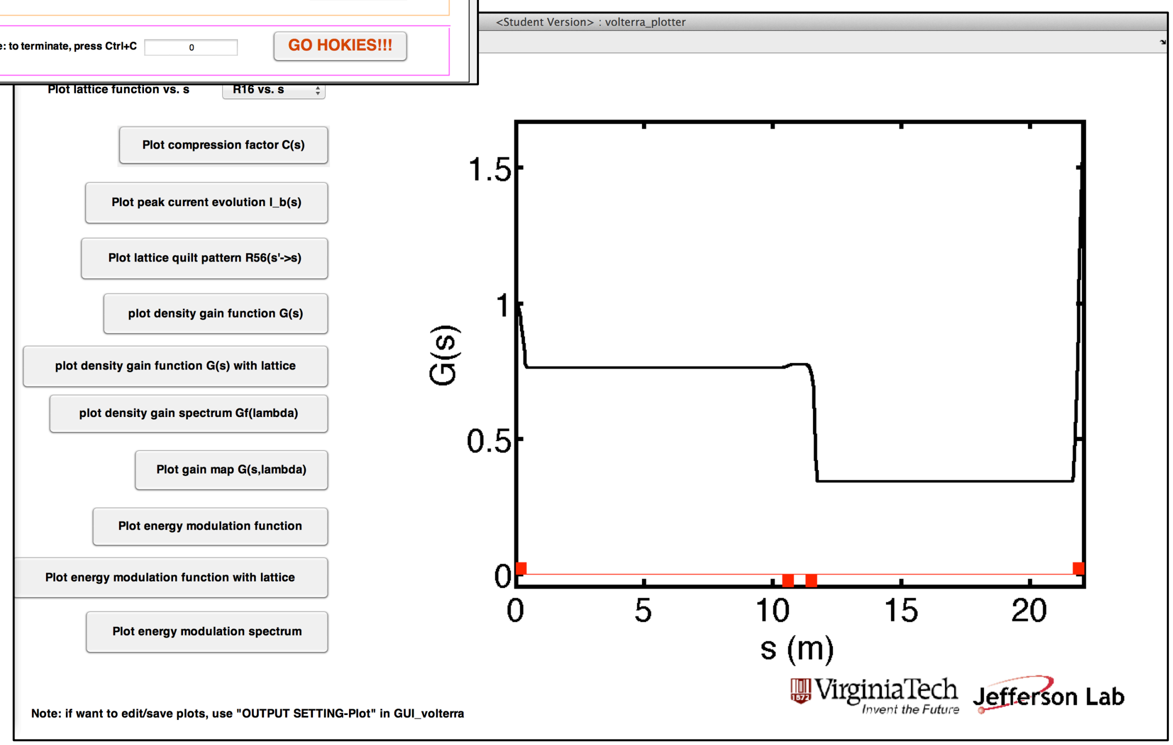
GUI: `volterra_mat`



<u>Input</u>: ELEGANT files (*.ele, *.lte)
<u>Output</u>: gain curves

A numerical code has been developed for the study and was benchmarked against ELEGANT. See, for detail, **JLAB-TN-14-016** and **JLAB-TN-15-019**.



<u>Features</u>:

1. general (linear) lattice
2. fast
   (can be used for systematic study, or for lattice optimization if microbunching gain is of particular concern)
3. graphical user interface

Summary of mathematical formulas

- Integral form of the linearized Vlasov equation:

$$g_k(s) = g_k^{(0)}(s) + \int_0^s K(s,s')g_k(s')ds'$$

$$G(s) = \left| \frac{g_k(s)}{g_k^{(0)}(s=0)} \right|$$

$$K(s,s') = \frac{ik}{\gamma}\frac{I(s)}{I_A}C(s')R_{56}(s'\to s)Z\left(kC(s'),s'\right)\times[\text{Landau damping}]$$

$$[\text{Landau damping}] = \exp\left\{\frac{-k^2}{2}\left[\varepsilon_{x0}\left(\beta_{x0}R_{51}^2(s,s') + \frac{R_{52}^2(s,s')}{\beta_{x0}}\right) + \varepsilon_{y0}\left(\beta_{y0}R_{53}^2(s,s') + \frac{R_{54}^2(s,s')}{\beta_{y0}}\right) + \sigma_\delta^2 R_{56}^2(s,s')\right]\right\}$$

$$R_{56}(s'\to s) = R_{56}(s) - R_{56}(s') + R_{51}(s')R_{52}(s) - R_{51}(s)R_{52}(s') + R_{53}(s')R_{54}(s) - R_{53}(s)R_{54}(s')$$

$$R_{5i}(s,s') = C(s)R_{5i}(s) - C(s')R_{5i}(s') \text{ for } i = 1, 2, 3, 4, 6$$

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intrinsic beam spread: {transverse emittances} {energy spread}

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We aim to make this relative momentum compaction small around isochronous arc.
Small $R_{56}(s' \to s) \Rightarrow$ **small** $K(s,s') \Rightarrow$ **small** g_k

$$R_{56}(s' \to s) = R_{56}(s) - R_{56}(s') + R_{51}(s')R_{52}(s) - R_{51}(s)R_{52}(s') + R_{53}(s')R_{54}(s) - R_{53}(s)R_{54}(s')$$

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Linear optics analysis

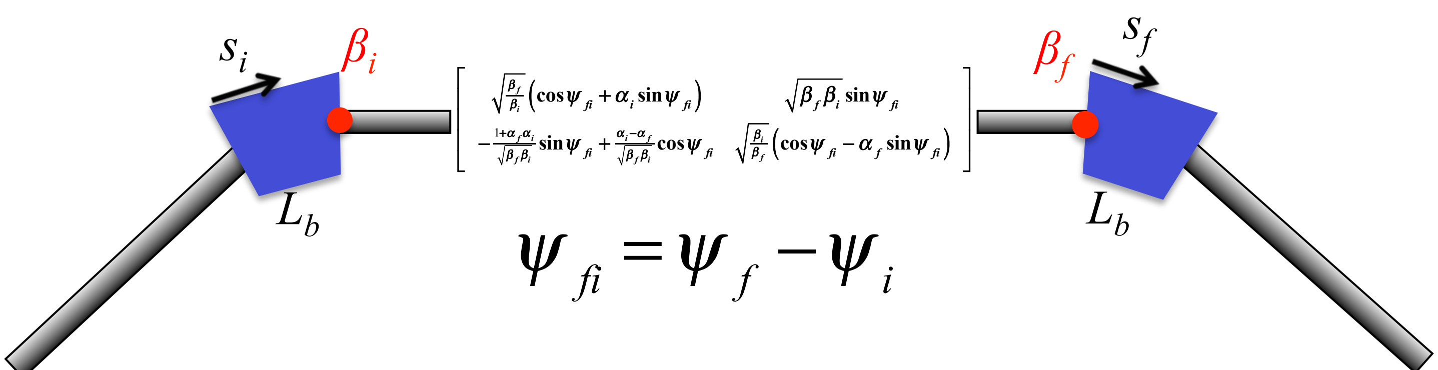
- Linear transport matrix from emission site (s_i) to receiving site (s_f)

$$\mathbf{R}_{6\times6}^{s_i\to s_f} = \mathbf{R}_{6\times6}^{0\to s_f}\left(\mathbf{R}_{6\times6}^{0\to s_i}\right)^{-1}$$

- Consider the simplest case: **{dipole-achromat/straight-dipole}**
- The momentum compaction term

$$R_{56}(s_i \to s_f) \simeq \left[\left(\frac{s_i - L_b}{\rho_b^2}\sqrt{\beta_i\beta_f} + \frac{s_i L_b \alpha_i}{\rho_b^2}\sqrt{\frac{\beta_f}{\beta_i}}\right)\sin\psi_{fi} + \left(\frac{s_i L_b}{\rho_b^2}\sqrt{\frac{\beta_f}{\beta_i}}\right)\cos\psi_{fi}\right] s_f$$

where we have made thin-dipole approximation and assumed achromaticity of the in-between section.



$$\begin{bmatrix} \sqrt{\frac{\beta_f}{\beta_i}}\left(\cos\psi_{fi} + \alpha_i \sin\psi_{fi}\right) & \sqrt{\beta_f\beta_i}\sin\psi_{fi} \\ -\frac{1+\alpha_f\alpha_i}{\sqrt{\beta_f\beta_i}}\sin\psi_{fi} + \frac{\alpha_i - \alpha_f}{\sqrt{\beta_f\beta_i}}\cos\psi_{fi} & \sqrt{\frac{\beta_i}{\beta_f}}\left(\cos\psi_{fi} - \alpha_f\sin\psi_{fi}\right) \end{bmatrix}$$

$$\psi_{fi} = \psi_f - \psi_i$$

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- Consider the simplest case: **{dipole-achromat/straight-dipole}**
- The momentum compaction term

$$R_{56}(s_i \to s_f) \simeq \left[\left(\frac{s_i - L_b}{\rho_b^2} \sqrt{\beta_i \beta_f} + \frac{s_i L_b \alpha_i}{\rho_b^2} \sqrt{\frac{\beta_f}{\beta_i}} \right) \sin\psi_{fi} + \left(\frac{s_i L_b}{\rho_b^2} \sqrt{\frac{\beta_f}{\beta_i}} \right) \cos\psi_{fi} \right] s_f$$

where we have made thin-dipole approximation and assumed achromaticity of the in-between section.

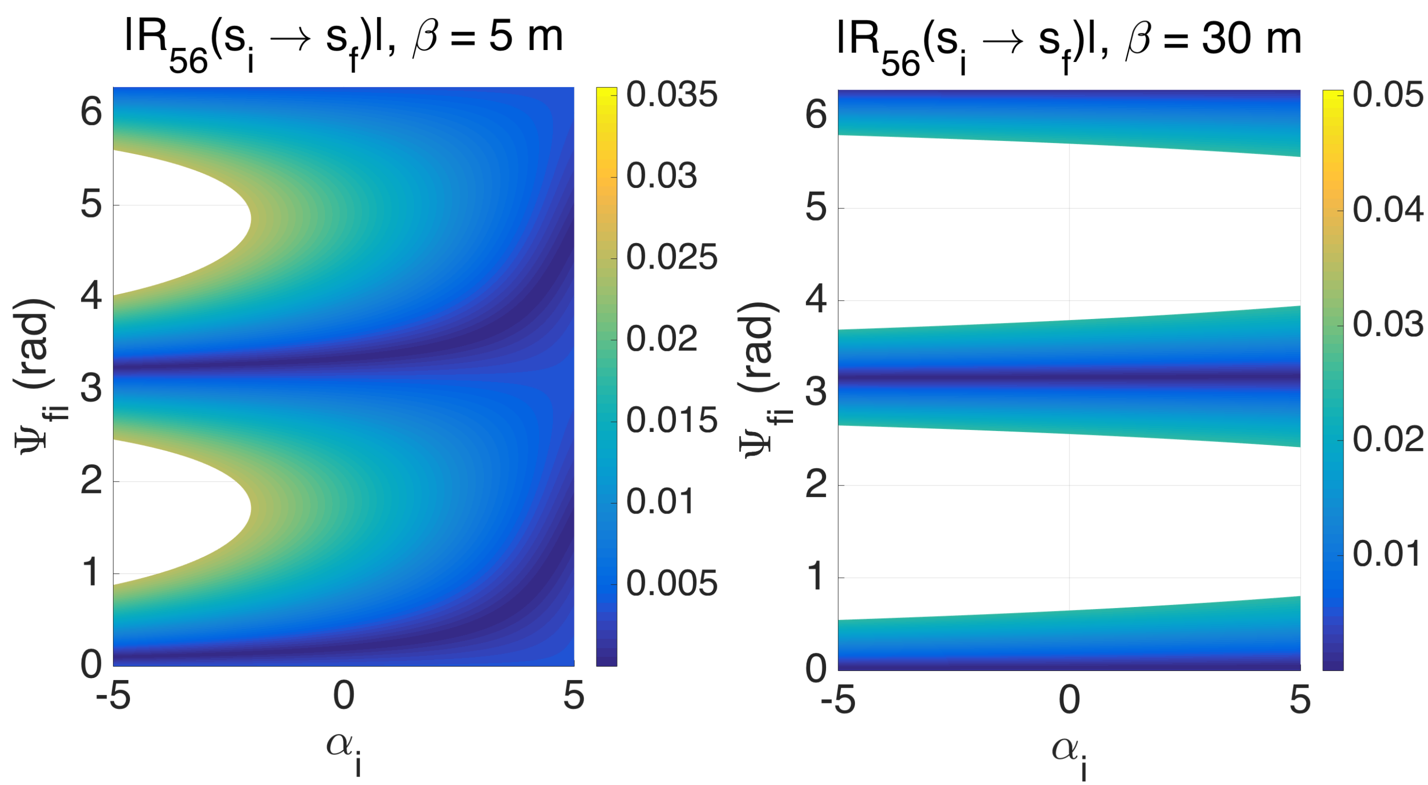
- **Our goal is to make the momentum compaction small along isochronous arc.**
- Sufficient conditions to achieve the goal:
  - (1) small β functions are preferred within dipoles
  - (2) but try to avoid small α functions within dipoles
  - (3) choose ψ_{fi} close to $\sim\pi$ (or its integer multiple)

Outline

- Introduction and Overview
- Theoretical formulation of CSR microbunching in a single-pass system
- Conditions for CSR microbunching gain suppression
- Examples
- Summary and Conclusion

Conditions for CSR Gain Suppression

$$R_{56}(s_i \to s_f)(\alpha,\beta,\psi_{fi}) \simeq \left[\left(\frac{s_i - L_b}{\rho_b^2}\sqrt{\beta_i\beta_f} + \frac{s_i L_b \alpha_i}{\rho_b^2}\sqrt{\frac{\beta_f}{\beta_i}}\right)\sin\psi_{fi} + \left(\frac{s_i L_b}{\rho_b^2}\sqrt{\frac{\beta_f}{\beta_i}}\right)\cos\psi_{fi}\right]s_f$$

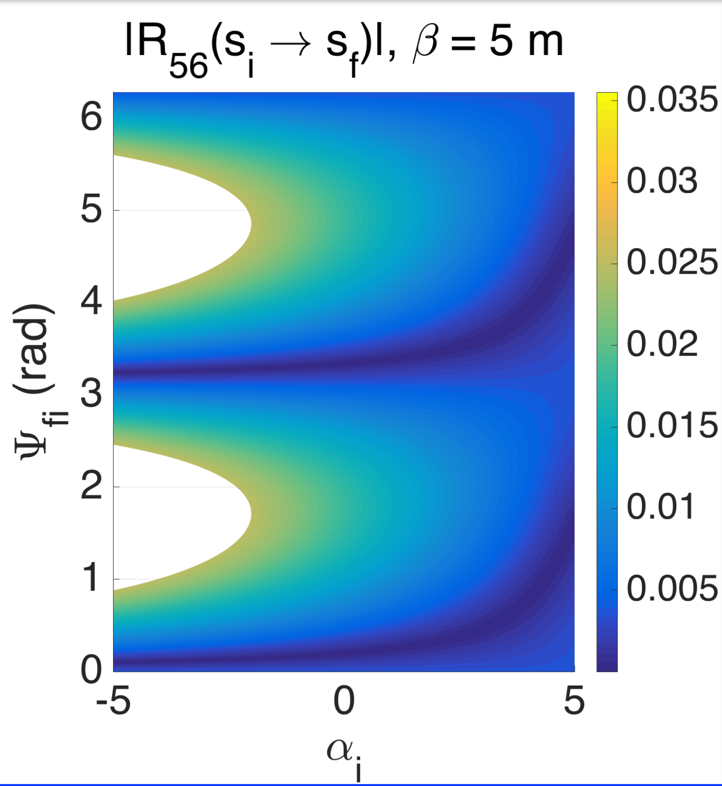


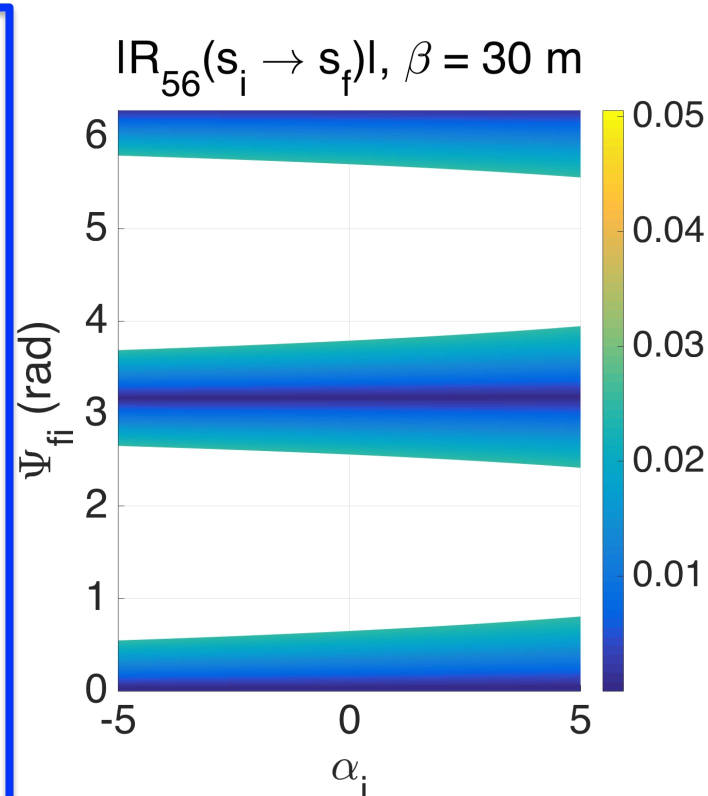
(1) small β functions are preferred (within dipoles)

Note: shaded area for $\left|R_{56}(s_i \to s_f)\right| < 0.025$ m. More shaded area give more flexibility for arc design.

Conditions for CSR Gain Suppression

$$R_{56}(s_i \to s_f)(\alpha,\beta,\psi_{fi}) \simeq \left[\left(\frac{s_i - L_b}{\rho_b^2}\sqrt{\beta_i\beta_f} + \frac{s_i L_b \alpha_i}{\rho_b^2}\sqrt{\frac{\beta_f}{\beta_i}}\right)\sin\psi_{fi} + \left(\frac{s_i L_b}{\rho_b^2}\sqrt{\frac{\beta_f}{\beta_i}}\right)\cos\psi_{fi}\right] s_f$$





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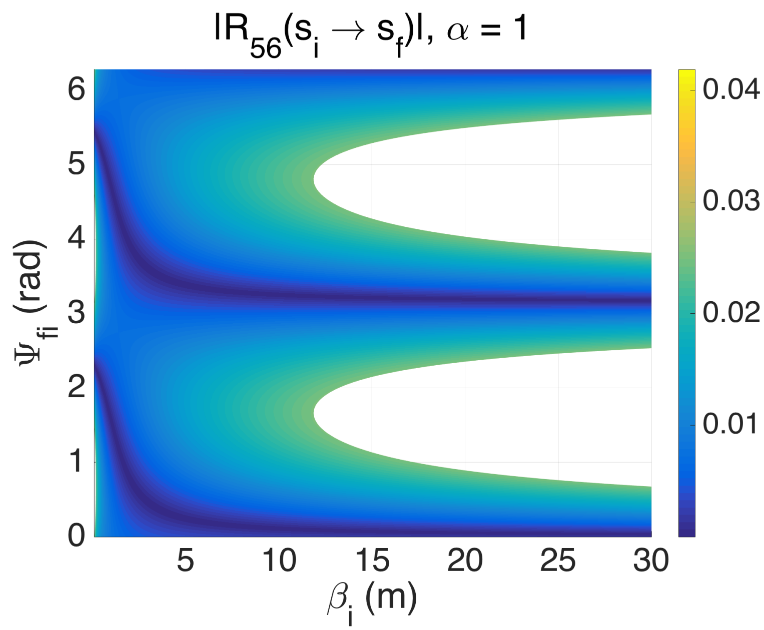
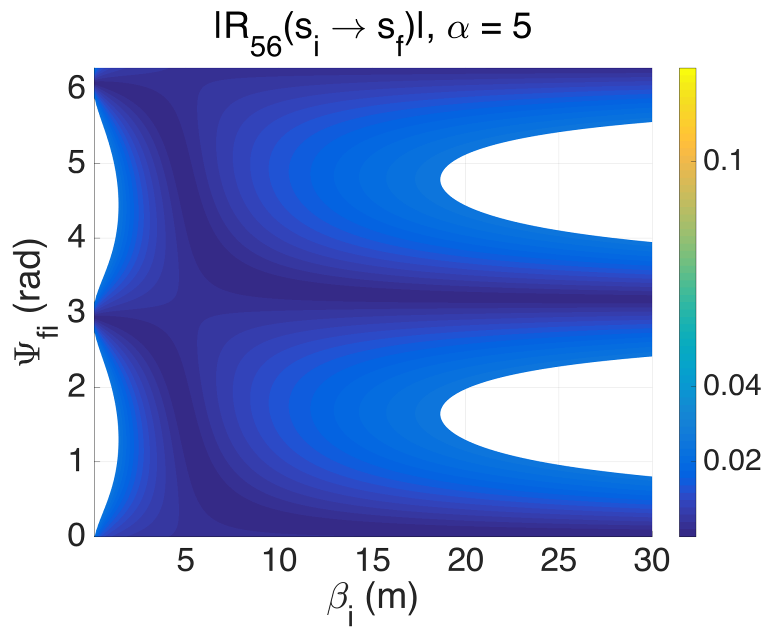
Conditions for CSR Gain Suppression

$$R_{56}(s_i \to s_f)(\alpha,\beta,\psi_{fi}) \simeq \left[\left(\frac{s_i-L_b}{\rho_b^2}\sqrt{\beta_i\beta_f} + \frac{s_i L_b \alpha_i}{\rho_b^2}\sqrt{\frac{\beta_f}{\beta_i}}\right)\sin\psi_{fi} + \left(\frac{s_i L_b}{\rho_b^2}\sqrt{\frac{\beta_f}{\beta_i}}\right)\cos\psi_{fi}\right] s_f$$





(1) small β functions are preferred (within dipoles)
(2) small α functions should be avoided (within dipoles)
(3) choose ψ_{fi} close to $\sim\pi$ (or its integer multiple)

Note: shaded area for $\left|R_{56}(s_i \to s_f)\right| < 0.025$ m. More shaded area give more flexibility for arc design.

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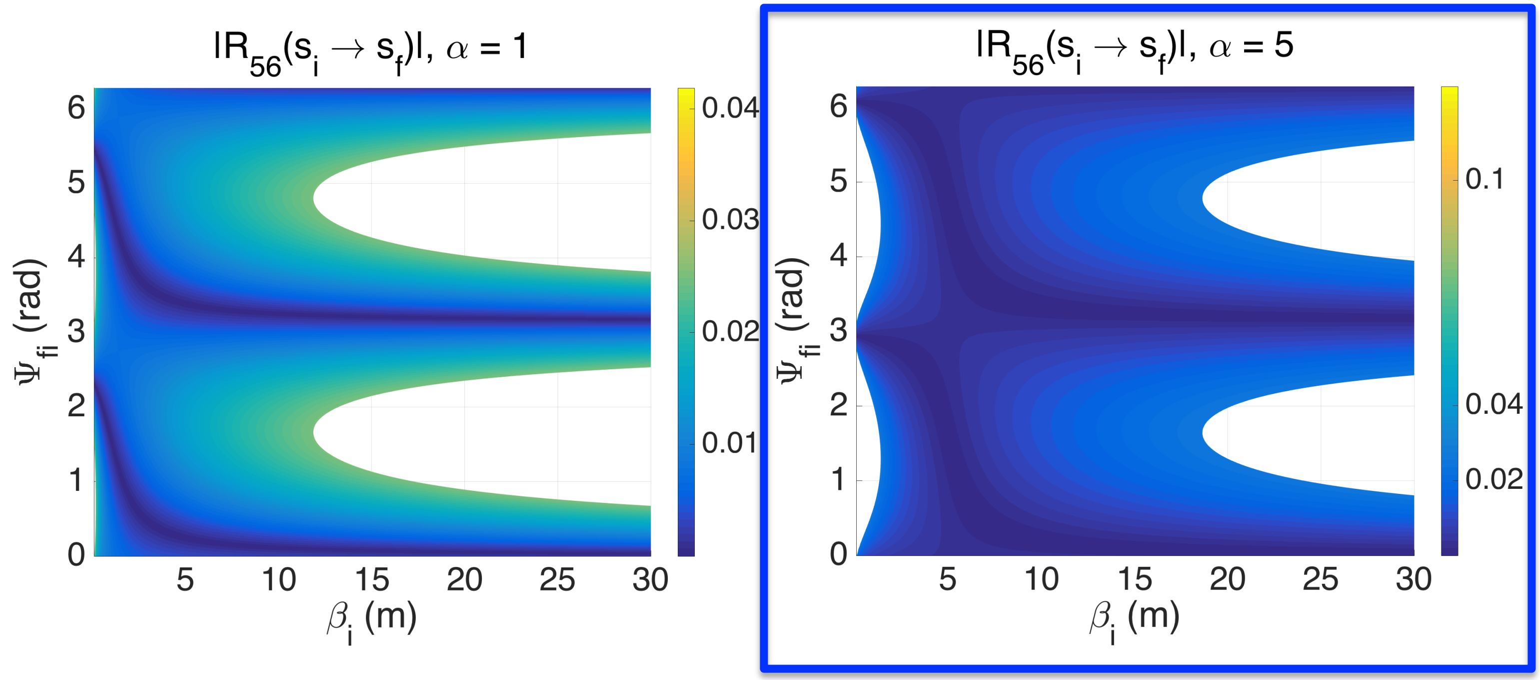
Conditions for CSR Gain Suppression

$$R_{56}(s_i \to s_f)(\alpha,\beta,\psi_{fi}) \simeq \left[\left(\frac{s_i-L_b}{\rho_b^2}\sqrt{\beta_i\beta_f}+\frac{s_iL_b\alpha_i}{\rho_b^2}\sqrt{\frac{\beta_f}{\beta_i}}\right)\sin\psi_{fi}+\left(\frac{s_iL_b}{\rho_b^2}\sqrt{\frac{\beta_f}{\beta_i}}\right)\cos\psi_{fi}\right]s_f$$



(1) small β functions are preferred (within dipoles)
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Note: shaded area for $\left|R_{56}(s_i \to s_f)\right| < 0.025$ m. More shaded area give more flexibility for arc design.

Conditions for CSR Gain Suppression

- At the moment we limit ourselves to the **special** case of **{dipole-achromat/straight-dipole}**.
- The relation between the proposed conditions and the suppression of MBI for **general** beamline lattice still needs further investigation.
- A heuristic connection comes from our previous work on multi-stage behavior of MBI: the microbunching gain always develops and gets amplified from lower-stage interactions.
  - MOP087, FEL 15; TUICLH2034, ERL Workshop 2015
- Work is underway.

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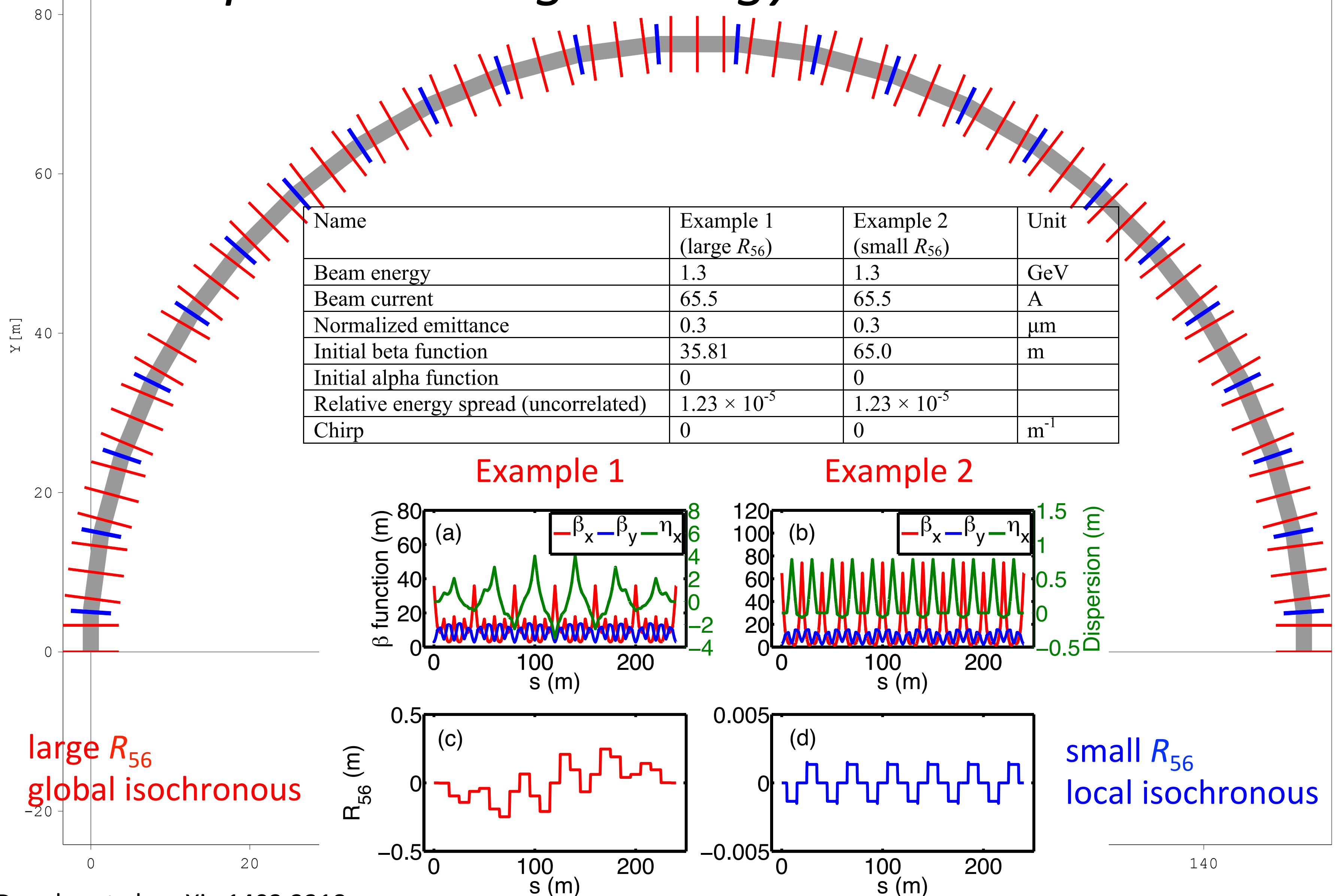
Examples

- Below we examine the proposed conditions by the following two sets of comparative example lattices.

	Example 1	Example 2	Example 3	Example 4
ψ_{fi} description	(see next slides)	**~0 or ~ π** between dipoles	**~ $\pi/2$** between dipoles	**~0 or ~ π** between dipoles
R_{56} description	**larger** R_{56} **global** isochronous	**smaller** R_{56} **local** isochronous	**larger** R_{56} **local** isochronous	**smaller** R_{56} **local** isochronous

Example 1 & 2: high-energy recirculation arcs



Name	Example 1 (large R_{56})	Example 2 (small R_{56})	Unit
Beam energy	1.3	1.3	GeV
Beam current	65.5	65.5	A
Normalized emittance	0.3	0.3	μm
Initial beta function	35.81	65.0	m
Initial alpha function	0	0	
Relative energy spread (uncorrelated)	1.23×10^{-5}	1.23×10^{-5}	
Chirp	0	0	m^{-1}

Example 1

Example 2

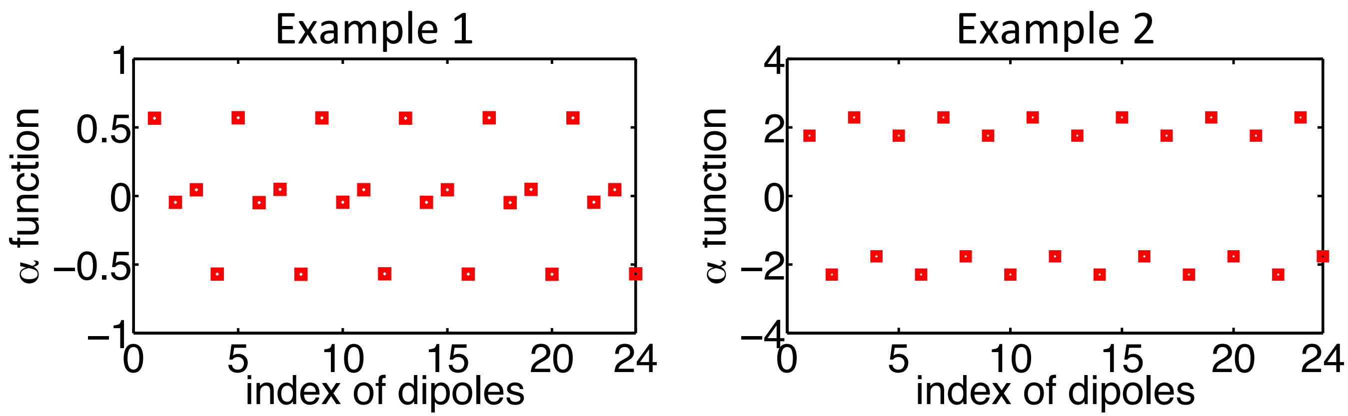
large R_{56}
global isochronous

small R_{56}
local isochronous

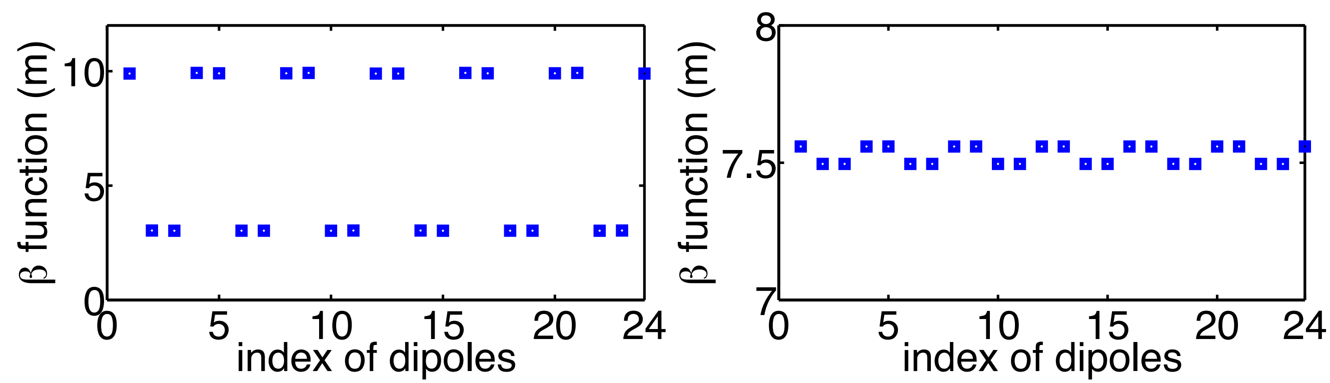
D. Douglas et al., arXiv:1403.2318
D. Douglas et al., IPAC'15 (TUPMA038)

Example 1: bad ε_{nx} preservation, bad gain suppression
Example 2: good ε_{nx} preservation, good gain suppression

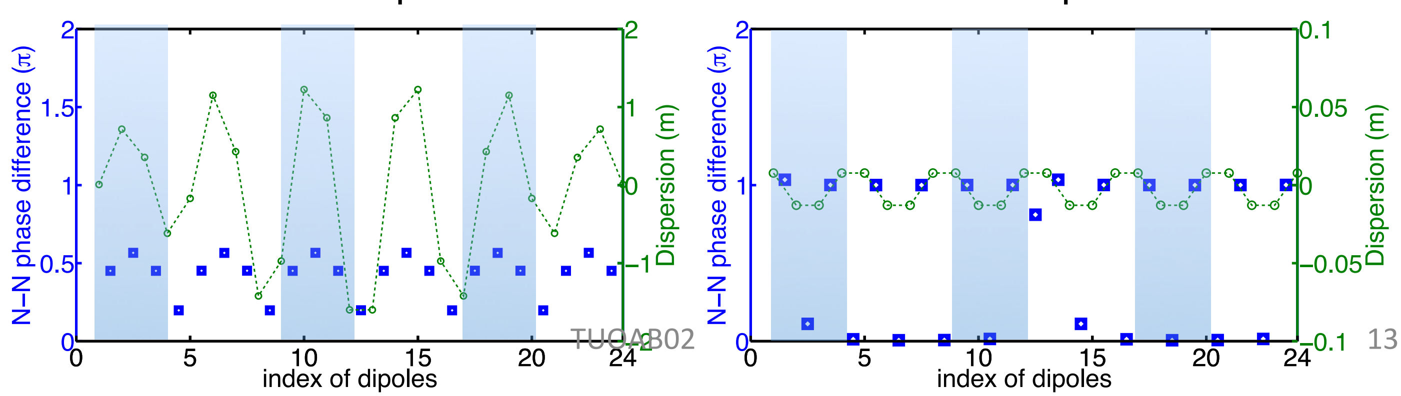
avoid small $|\boldsymbol{\alpha}|$ function



$\boldsymbol{\beta}$ functions as small as possible

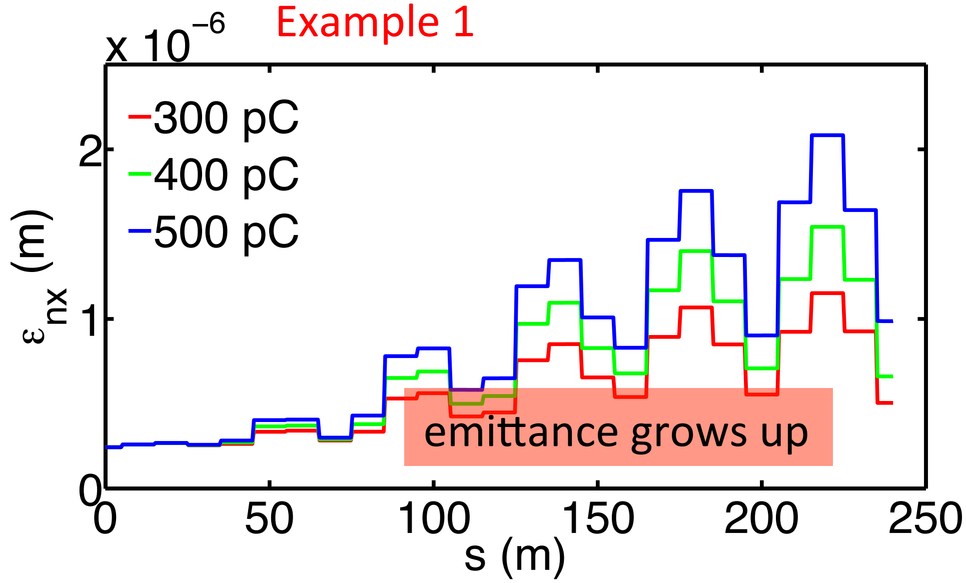


phase difference close to $m\pi$



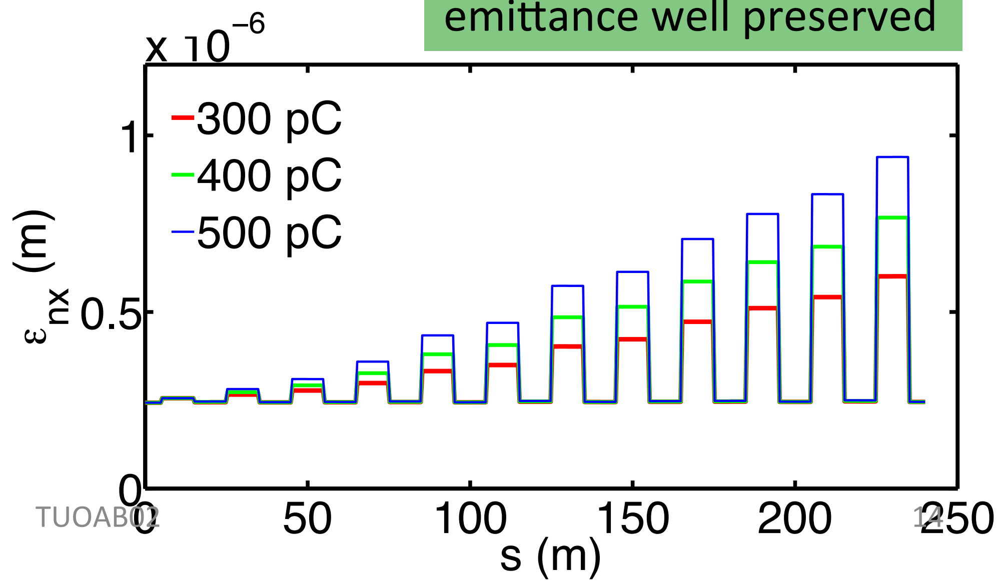
Example 1: bad ε_{nx} preservation, bad gain suppression
Example 2: good ε_{nx} preservation, good gain suppression

Example 1



Example 2

emittance well preserved



Example 1: bad ε_{nx} preservation, bad gain suppression
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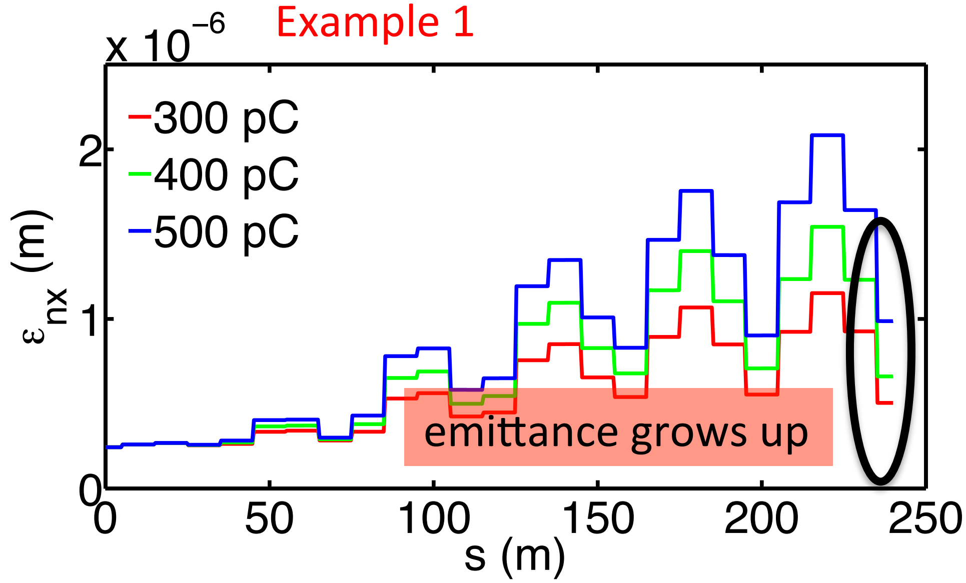
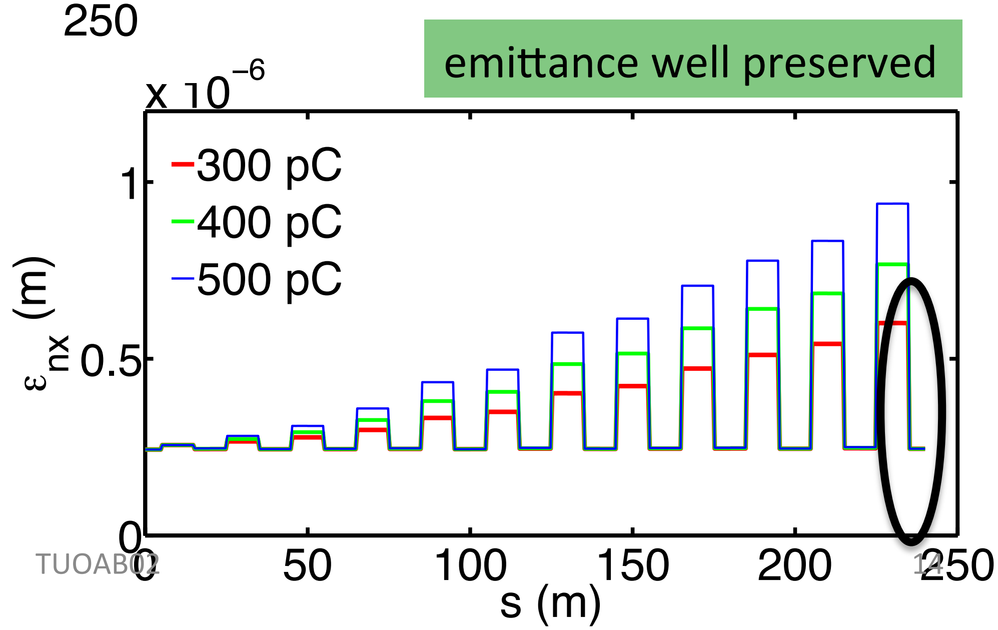
Example 1



Example 2



Example 1: bad ε_{nx} preservation, bad gain suppression
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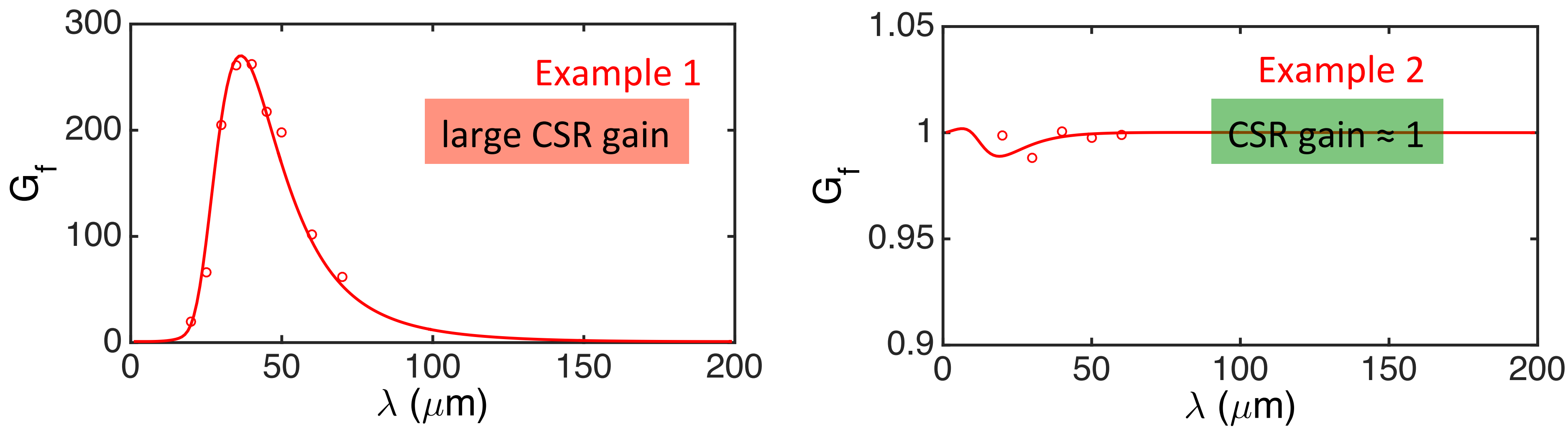
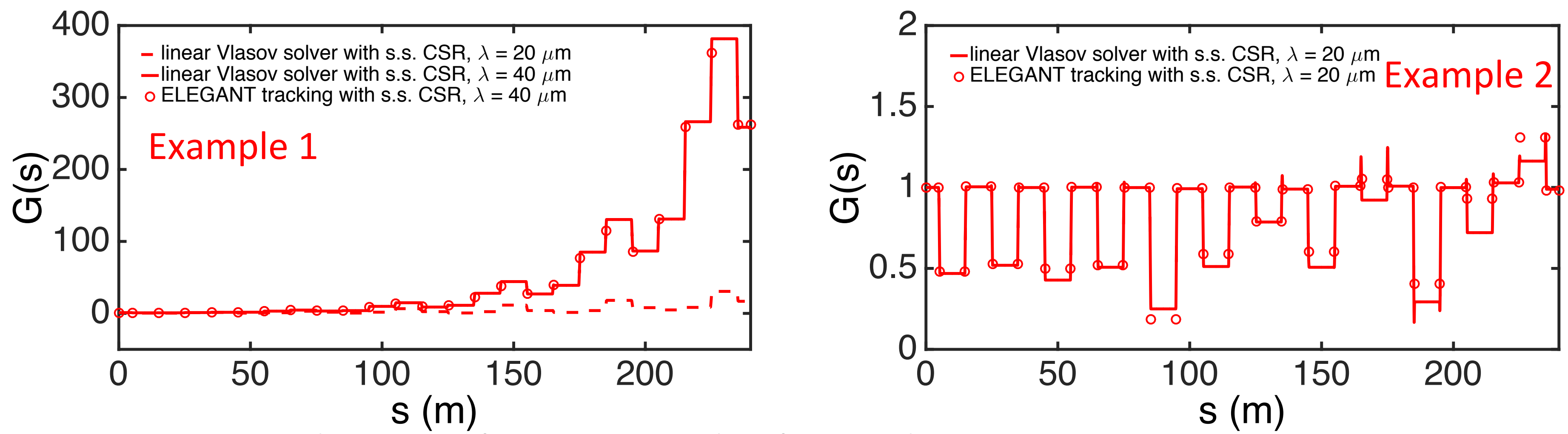
- Microbunching gain spectrum at the end of the arc

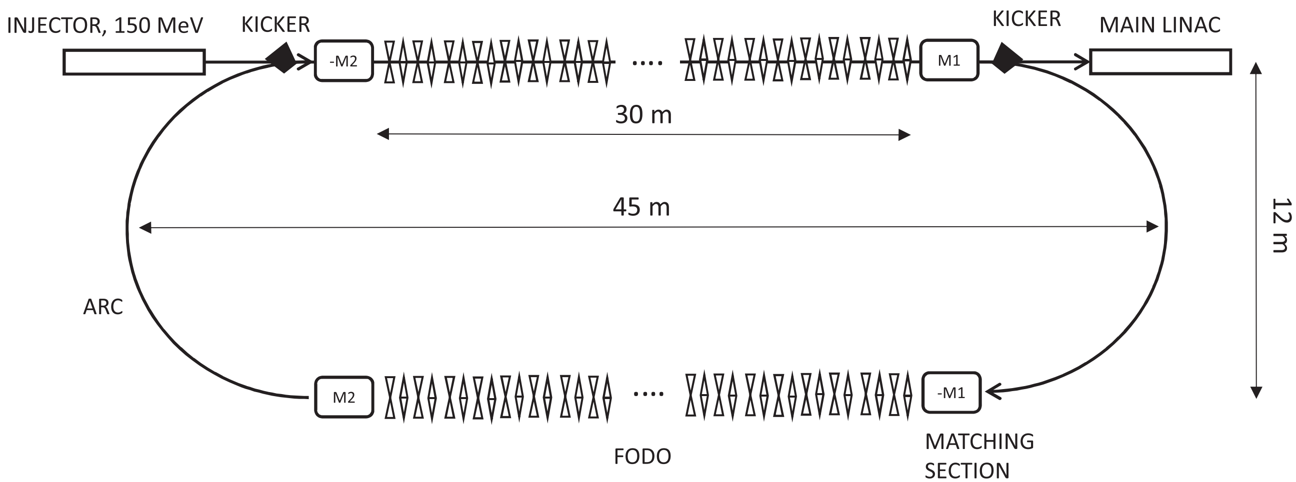


- Microbunching gain development along the arc

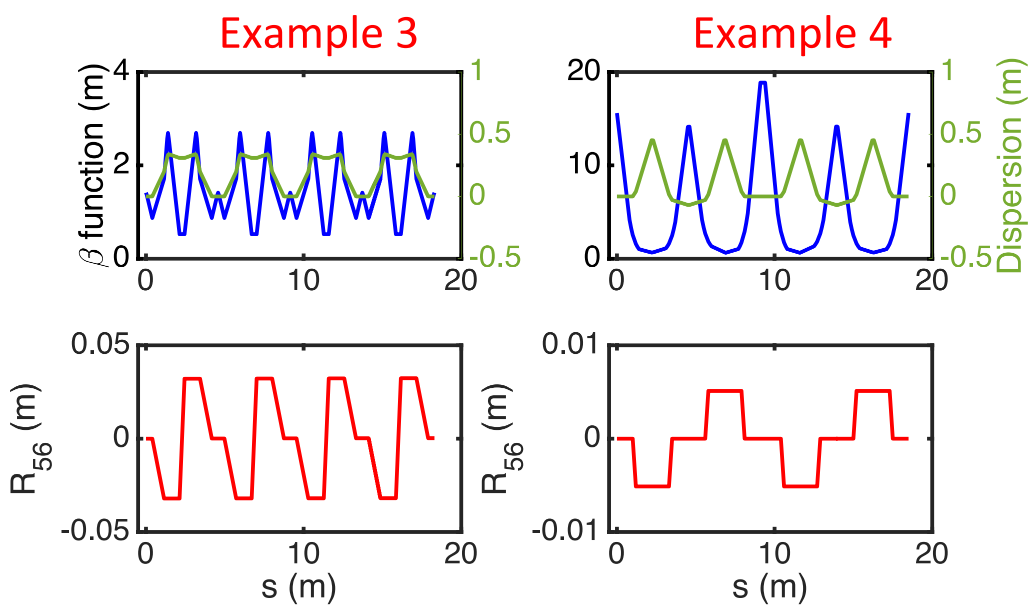


Up to **70M** macroparticles are used for ***p***ELEGANT tracking for Example 1.
Each dot in G_f even takes several hours after careful numerical convergence is obtained.

Example 3 & 4: mid-energy recirculation arcs



S. Di Mitri, PRSTAB **17**, 074401 (2014)



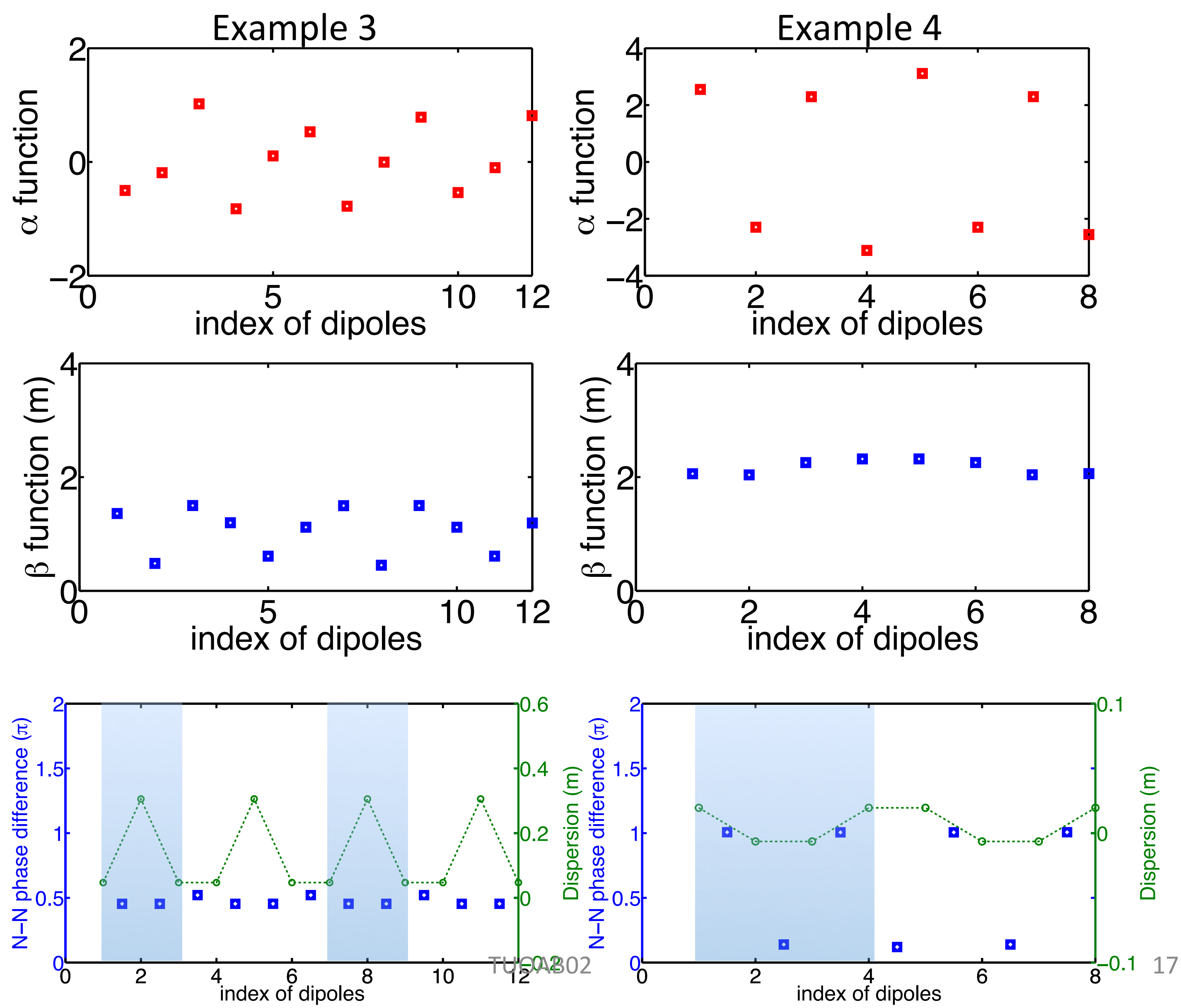
Name	Example 3	Example 4	Unit
beam energy	150	120	MeV
chirp	0	0	m^{-1}
bunch current (peak)	80	80	A
normalized emittance (H/V)	0.4/0.4	0.4/0.4	μm
relative rms energy spread	1.33×10^{-5}	1.33×10^{-5}	
rms bunch length	~2.5	~2.5	ps
bending radius	1.5	0.5	m

Example 3: good ε_{nx} preservation, bad gain suppression
Example 4: good ε_{nx} preservation, good gain suppression

avoid small $|\alpha|$ function

β functions as small as possible

phase difference close to $m\pi$



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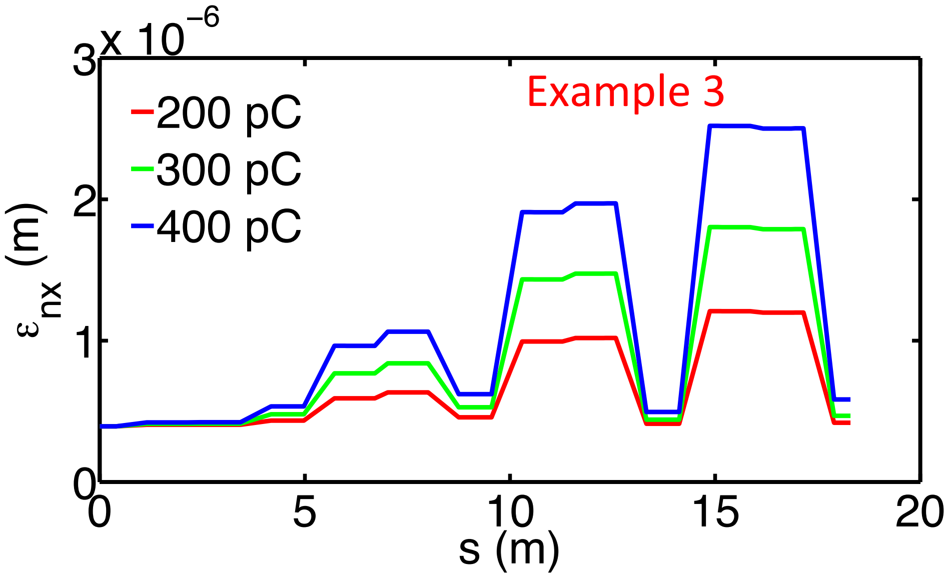
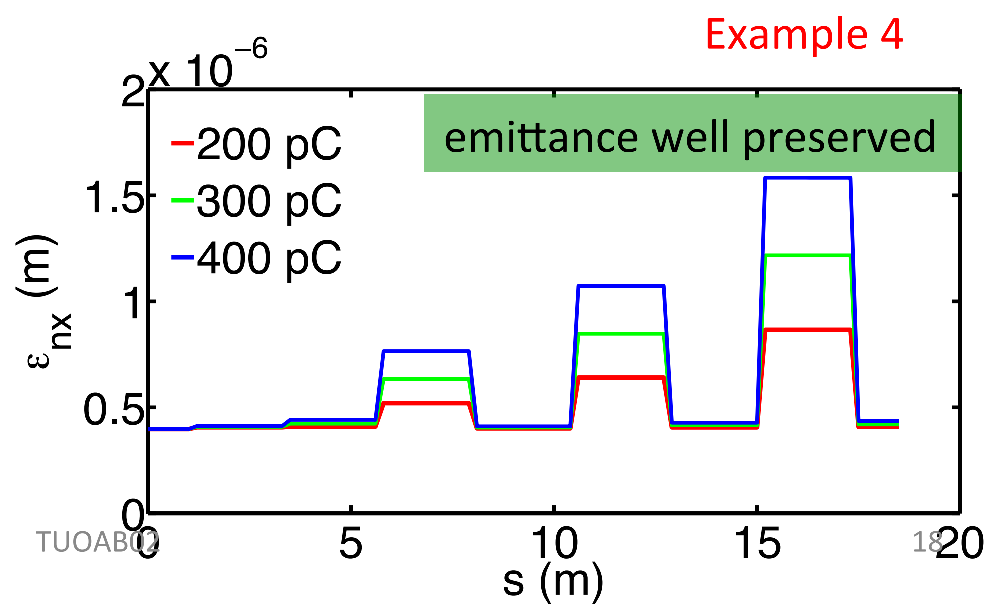
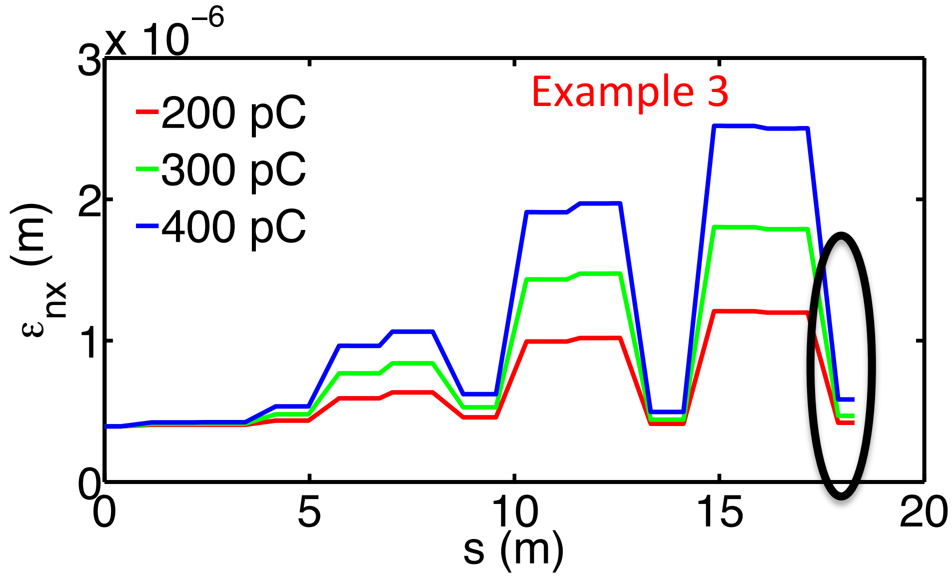


emittance preservation seems ok

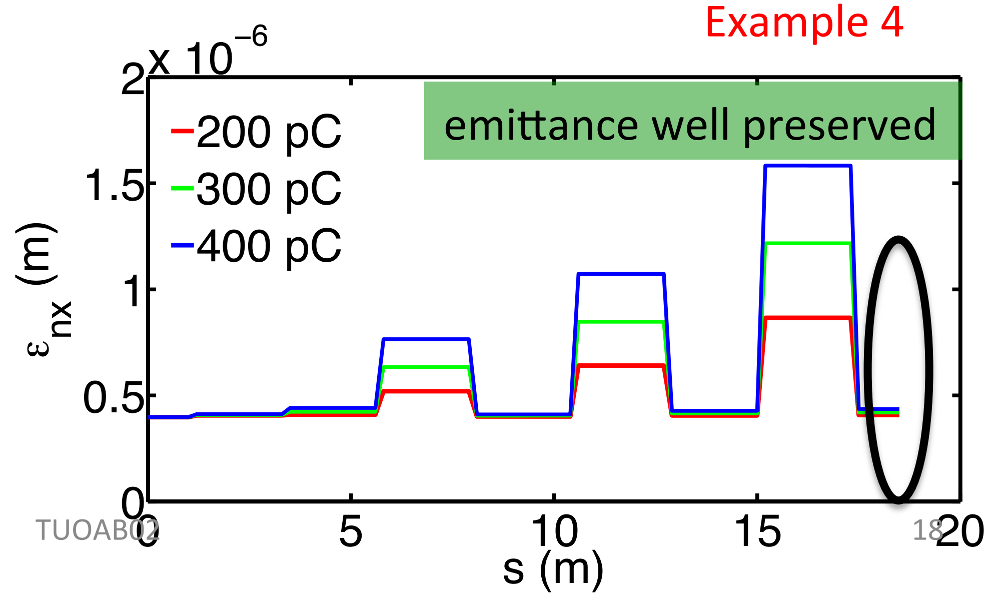


S. Di Mitri, PRSTAB 17, 074401 (2014)

Example 3: *good ε_{nx} preservation, bad gain suppression*
Example 4: *good ε_{nx} preservation, good gain suppression*



emittance preservation seems ok



S. Di Mitri, PRSTAB 17, 074401 (2014)

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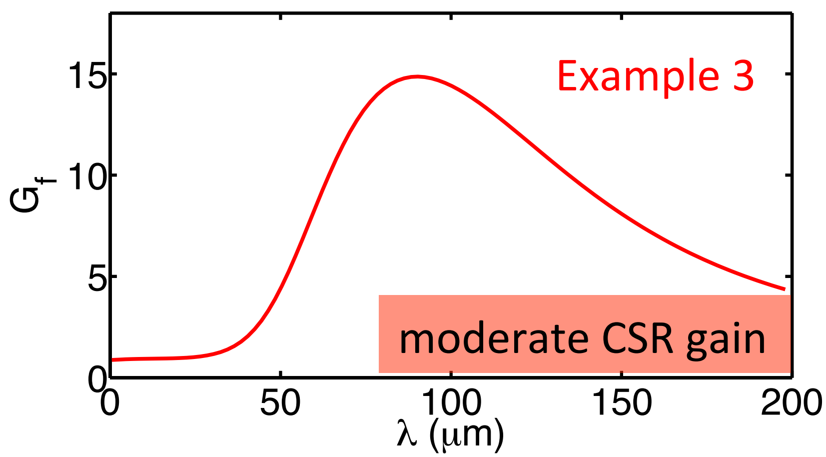
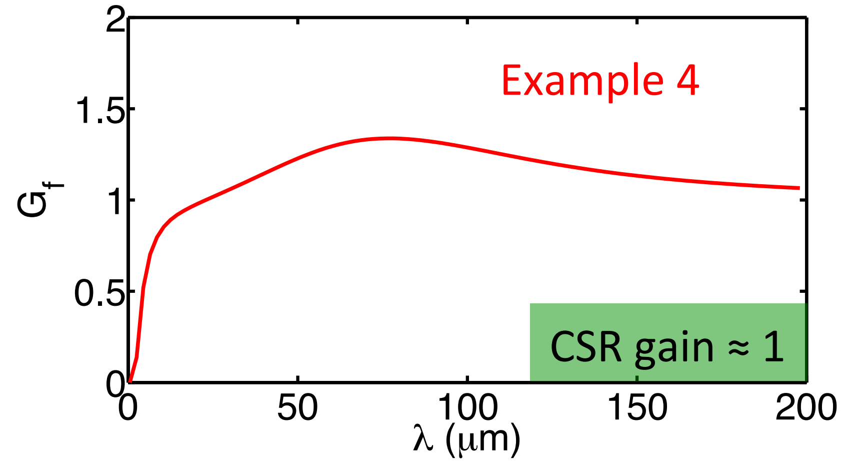
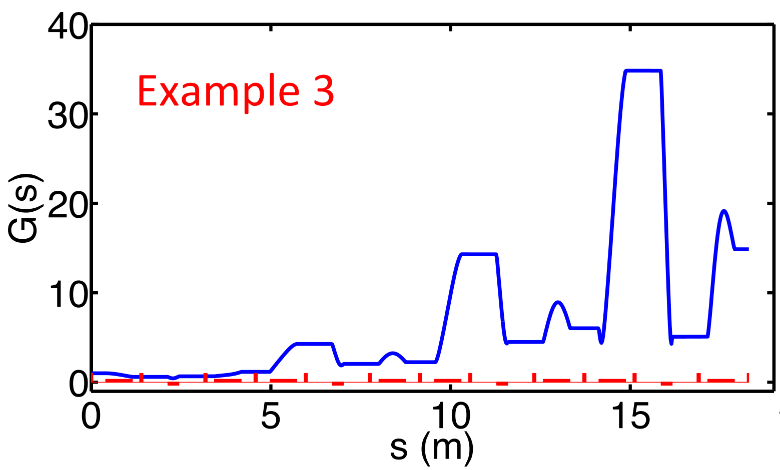
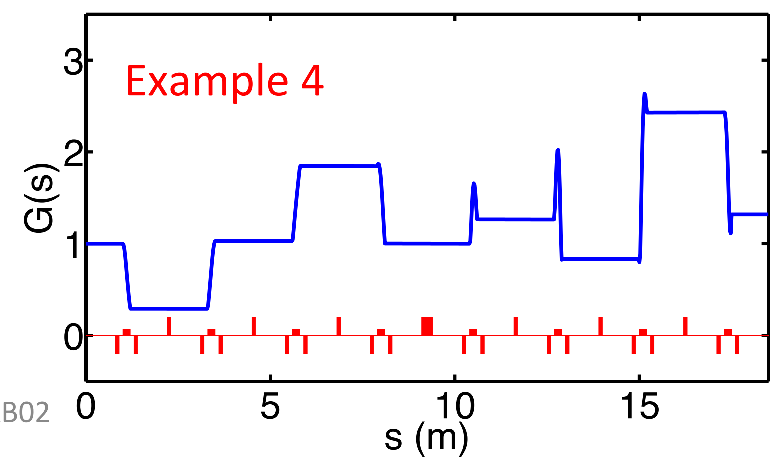




- Microbunching gain development along the arc





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transverse emittance	bad	good	good	good
longitudinal microbunching gain	bad	good	bad	good

Summary and Conclusion

- ✓ Transverse: emittance growth; Longitudinal: MB gain enhancement
- ✓ Linear Vlasov solver for study of MBI for general linear beamline lattices
- ✓ Sufficient conditions for CSR microbunching suppression
  - ➢ prefer small β (within dipoles)
  - ➢ avoid small α (within dipoles)
  - ➢ keep ψ close to $m\pi$ (between dipoles)
- ✓ Illustration of two sets of comparative examples to confirm the conditions
- ✓ Optics impact on microbunching development
- ✓ More systematic study under way

Thank you for your attention

고맙습니다

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