Design and optimisation strategies of the non-linear beam dynamics in diffraction limited synchrotron light sources

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Outline

Diffraction limited light sources

motivations present landscape

Optics design strategies

TME and (quasi-)diffraction limited rings linear optics and nonlinear optics deterministic algorithms numerical algorithms

Examples

MBA, Hybrid MBA, DDBA/DTBA, Antibends (MAX IV, Pep-X, ALS U, ESRF-EBS, APS U, Diamond-II, SLS-II)





Diffraction limited light sources

Photon flux and brilliance and coherent fraction

$$flux = \frac{N_{ph}}{\Delta T \cdot \Delta \omega / \omega} \qquad brilliance = \frac{flux}{4\pi^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}} \qquad F = \frac{\lambda^2 / (4\pi)^2}{\Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}}$$

 Σ 's are the convolution of electrons and photon beam size and divergence

$$\Sigma_{\rm x} = \sqrt{\sigma_{\rm x,e}^2 + \sigma_{\rm ph}^2} \qquad \Sigma_{\rm x'} = \sqrt{\sigma_{\rm x',e}^2 + \sigma_{\rm ph}^2}$$

Brilliance and coherent fraction are maximised for smaller electron beam emittances until the diffraction limit is reached

 $\epsilon_{e^-} \le \epsilon_{ph} = \frac{\lambda}{4\pi}$ ~10 pm for diffraction limit at ~1 Angstron (12.4 keV) ~100 pm for diffraction limit at ~ 1 pm (1.24 keV) ~100 pm for diffraction limit at ~ 1nm (1.24 keV)





Survey of low emittance lattices for light sources







From Theoretical Minimum Emittance (TME) to MBA cells



These conditions provide the theoretical minima for the emittance generated in dipoles (for constant B)





from TME to MBA cells



S.Y. Lee, Accelerator Physics, World Scientific

DIFfraction Limited light source (DIFL)



- ~20 years from the first proposal
- D. Einfeld et al. NIMA 1993
 PAC1995

to the first beam

• M. Eriksson et al. IPAC 2016

About 20 new / upgrade projects Renaissance of storage ring LS due to

- science case improved
- technology of subsystems
 success of 3rd GLS and
- beam dynamics optimisation





e.g. MAX IV-like TME unit cell



The natural chromaticity increses sharply for low emittance (chromaticity wall)

Nonlinear dynamics optimisation



Injection efficiency, beam lifetime (Touschek), control of beam losses Off axis injection

~ 10 mm Dynamic Aperture

few per cent Momentum Aperture

On axis (swap-out) injection

allows more aggressive desing – (like Top Up) reduced DA requirements but still needs a good MA

Optimisation tools

Based on perturbative theory of betatron motion

- resonance driving terms (and detuning terms)
- cancellation rules symmetries

Based on numerical tracking

- detuning with amplitude, momentum, driving terms, FMA, ...
- direct evaluation of DA, MA
- direct evaluation of injection efficiency
- effect of errors (magnets, misalignment, ...) and IDs

Tracking tools: MADX-PTC, elegant, AT, Tracy-II(-III), OPA, ...





Deterministic algorithms

Based on perturbative theory of Hamiltonian of betatron motion

- sextupole resonance driving terms
- first order cancellations

in a cell over N cells via symmetry

- sensivity matrices driving terms ↔ sextupoles (and SVD)
- extension to higher order magnetic multipoles
- extension to higher order perturative terms





Hamiltonian of betatron motion

$$H = \frac{p_x^2 + p_y^2}{2(1+\delta)} - \frac{x\delta}{\rho(s)} - \frac{1}{2} \frac{x^2}{\rho^2(s)} + \frac{b_2^2(s)}{2} (x^2 - y^2) + \frac{b_3^2(s)}{3} (x^3 - 3xy^2) + O(4)$$

$$\longrightarrow h_{jklmp}(s) = \sum_{s_i} b_N(s_i) \beta_x^{\frac{j+k}{2}} (s_i) \beta_y^{\frac{j+k}{2}} (s_i) \delta^p e^{i(j-k)\mu_x(s_i)} e^{i(l-m)\mu_y(s_i)}$$

$$A. Schoch, CERN 57-23 (1958);$$

Resonant driving terms

A. Schoch, CERN 57-23 (1958); R. Ruth, SLAC-pub 4103, (1986); J. Bengtsson, CERN 88-04 (1988);

complex coefficients s-dependent each multipole contributes to definite resonant driving terms e.g. sextupoles contribute to

2 chromaticities, 3 chromatic and 5 geometric driving terms (unavoidable 16 additional terms after correcting the chromaticity) phase relations may enhance or cancel their contribution

$$h_{11001} = h_{11001}^{*} \rightarrow \xi_{x}$$

$$h_{00111} = h_{00111}^{*} \rightarrow \xi_{y}$$

$$h_{20001} = h_{02001}^{*} \rightarrow 2Q_{x}$$

$$h_{00201} = h_{00021}^{*} \rightarrow 2Q_{y}$$

$$h_{10002} = h_{01002}^{*} \rightarrow Q_{x}$$

 $h_{30000} = h_{03000}^* \rightarrow 3Q_x$ $h_{21000} = h_{12000}^* \rightarrow Q_x$ $h_{10110} = h_{01110}^* \rightarrow Q_x$ $h_{10200} = h_{01020}^* \rightarrow Q_x + 2Q_y$ $h_{10020} = h_{01200}^* \rightarrow Q_x - 2Q_y$

all $\propto b_3$

...then higher order multipoles ∞b_4 ; ∞b_5 ; ...

...then higher perturbative orders $\infty b_3 b_3$; $\infty b_3 b_4$; ...

Compensation rules in first order

Simple rules can be establish to compensate the contribution of two sextupoles to a given resonant driving term, e.g. for (3,0)

$$h_{30000} \propto b_3 L \beta_x^{\frac{3}{2}} [e^{i3\mu_{xi}} + e^{i3(\mu_{xi}^{s} + \Delta \mu_x)}] = b_3 L \beta_x^{\frac{3}{2}} e^{i3\mu_{xi}} (1 + e^{i3\Delta \mu_x})$$
$$e^{i3\Delta \mu_x} = -1 = e^{i(2n+1)\pi}$$

We can set the phase advance to target specific resonances or, even better, to cancel the contribution to all normal sextupoles geometric resonances terms by

$$\Delta \mu_x = (2n+1)\pi$$
 $\Delta \mu_v = n\pi$

Normal and skew sextupoles resonances are cancelled by

$$S_2$$

 $3\Delta\mu_x$
 S_1
 $3\Delta\mu_x = \pi$
 S_1
 S_2

 $\Delta \mu_x = (2n+1)\pi$ $\Delta \mu_y = (2n+1)\pi$





Compensation rules in N cells

If the contribution of two sextupoles cannot be compensated within a cell some compensation can be obtained after N cells, e.g. for (3,0)

$$h_{30000} \propto b_{3}L\beta_{x}^{\frac{3}{2}} \sum_{k=0}^{N-1} e^{i3\Delta\mu_{x}k} = b_{3}L\beta_{x}^{\frac{3}{2}} \frac{1 - e^{i3\Delta\mu_{x}N}}{1 - e^{i3\Delta\mu_{x}}} = b_{3}L\beta_{x}^{\frac{3}{2}} e^{i3\Delta\mu_{x}(N-1)/2} \frac{\sin(3\Delta\mu_{x}N/2)}{\sin(3\Delta\mu_{x}/2)}$$

e.g. for N = 5 we can choose $Q_x = 2/5$ and $Q_y = 1/10$ e.g. for N = 7 we can choose $Q_x = 3/7$ and $Q_y = 1/7$ and so on... will cancel all third order resonances



 $\mathbf{Q}_{\mathbf{x}} = \Delta \mu_{\mathbf{x}} / 2\pi$ $\mathbf{Q}_{\mathbf{y}} = \Delta \mu_{\mathbf{y}} / 2\pi$

Role of symmetry:

Cancellation of imaginary part of driving terms occur if the cell is symmetric



Re-adapetd from A. Streun

sensitivity matrices (and SVD inversion)

Linear relations between resonance driving terms and multipole gradients

using \mathbf{M}_{sext} sextupoles for N driving terms

J. Bengtsson, SLS 97-9, (1997)

Linear relations can be extended to octupoles, to compensate resonances or amplitude detuning terms, e.g.

$$\delta \vec{\nu}_{\text{oct}} = \mathbf{B}_{\text{oct}} \vec{b}_{4} \qquad \begin{pmatrix} \partial Q_x / \partial J_x \\ \partial Q_y / \partial J_y \\ \partial Q_y / \partial J_x \\ \partial^2 Q_x / \partial \delta^2 \\ \partial^2 Q_y / \partial \delta^2 \end{pmatrix} = \frac{3}{8\pi} \begin{pmatrix} (\beta_x)_1^2 & \dots & (\beta_x)_{N_{\text{oct}}}^2 \\ -2(\beta_x \beta_y)_1 & \dots & -2(\beta_x \beta_y)_{N_{\text{oct}}} \\ (\beta_y)_1^2 & \dots & (\beta_y)_{N_{\text{oct}}}^2 \\ 4(D_x^2 \beta_x)_1 & \dots & 4(D_x^2 \beta_x)_{N_{\text{oct}}} \\ -4(D_x^2 \beta_y)_1 & \dots & -4(D_x^2 \beta_y)_{N_{\text{oct}}} \end{pmatrix} \begin{pmatrix} b_{4,1} \\ b_{4,N_{\text{oct}}} \end{pmatrix}$$

using N_{oct} octupoles for 5 detuning terms

S. Leemann et al., PRSTAB 14, (2011)





Numerical algorithms

Based on numerical search of parameter space

- GLASS (GLobal Search of All Stable Solutions)
- gradient search, symplex, least square, ...
- genetic algorithms, MOGA, particle swarm, (or just random search ⁽ⁱ⁾)

Accurate numerical calculation of the objectives (tracking-based)

- dynamic aperture, momentum aperture (s-dependent)
- FMA, detuning terms, diffusion rates, RDT, lifetime, injection eff.
- verified experimentally: Diamond, SOLEIL, ESRF, SPEARIII, NSLS-II, ...

Realistic models can be used *directly in the optimisation stage*

- engineering apertures, IDs, full 6D motion with RF, radiation damping
- errors in magnets: fringe fields, systematic and random multipoles
- misalignments: girders, individual magnets, BPMs

Parallelised on clusters (large throughput)





MOGA

The use of MOGA in the optimisation of synchrotron light sources was pioneered by M. Borland and colleagues and implemented in elegant/pelegant

Objectives: simultaneous optimisation of **multiple objectives**, particularly apt to cases where objectives are conflicting

Parameters: suitable for large dimension parameter spaces ensuring the search of global minimum

Populations choice: a population evolves over a number of generations. Mating techniques and selection criteria mimic evolutionary processes: mutations (i.e. crossover, inversion, ...) selection of dominant solutions (dominant individuals)

Results: indicates the best trade-off between objectives in the Pareto-front

Parallel implementation of genetic algorithm and sorting (NGSA-II) are available. Explore large portion of the parameters space with high accuracy.





MOGA optimisation



John Adams Institute for Accelerator Science

Busan, Korea, 9 May 2016



Further objectives in MOGA

Many objectives have been trialled: objective choice ranges from the injection efficiency computed with 6D tracking of injected beam (slow computation!) to proxies of the DA given by driving terms (fast computation!)









MBA

MAX IV, Pep-X, ALS-U Sirius, Elettra U, ILSF, ...

Hybrid MBA ESRF, APS U, HEPS detuned TME cells with small Dx

LGB + D_x bump + paired sexts.

DDBA-DTBA Diamond-II

mid-straigth section in MBA arcs (M even)

Reverse bends SLS-II

cells with reverse bends





MBA: MAX IV

MAX IV is the first low emittance light source based on 7BA lattice



Nonlinear dynamics optimisation based on

minimisation of detuning with amplitude terms \rightarrow weak octupoles minimisation of resonance driving terms via SVD of sensitivity matrix numerical evaluation of lattices based on FMA and direct tracking





MAX IV – detuning and diffusion maps

Driving terms minimisation with 5 sextupoles families distributed sextupoles 3 octupoles families to control amplitude dependent tuneshift



FMA used to study tune footprint, FM folding, resonance crossing and diffusion rates. Direct tracking with errors for robustness check

Pep-X: 7BA achromat



Nat. emittance 29 pm-rad (4.5 GeV) 5TME unit cells Cell phase advances: μ_x =(2+1/8) x 360°, μ_y =(1+1/8) x 360°

Compensation of driving terms achieved after 8 cells 10 mm DA in high beta section

All Geometrical 3rd and 4th Resonances compensated except $2v_x$ - $2v_y$ targeted explicitly using additional sextupoles (Y. Cai et al. PRSTAB 15, (2012))



MBA: ALS-U



Courtesy C. Steier, C. Sun, M. Venturini

12 cells 9BA lattice (100 pm) has quad components in all bendings and 2 families of chromatic + 2 families of harmonic sextupoles



- Lattice design work still in progress
- Options to use antibends are considered (20% reduction in ε_x)
- Small DA requires on-axis injection and small-emittance injected beam

ALS-U: extensive use of MOGA



Three objectives

- natural emittance
- weighted average diffusion rate
- momentum aperture

Integrated optimisation linear/nonlinear

Independent variables

- gradient of all focussing elements
- strength of harmonic sextupoles

DA calculations include normal and skew gradient errors (stated emittance refer to no-error

uncoupled lattice)

Constraints

 $\beta_{x,y}$ < 3 m in straight sect. g < 100 T/m

MOGA tends to push the H phase advance between the chromatic sext. to 5π

Hybrid MBA

Hybrid 7BA cell pioneered at ESRF-EBS (130pm) and adapted to APS U (67 pm)

HEPS (60 pm)

- Dispersion bump for chromatic sextupoles;
- $3\pi / \pi$ phase advance for cancellation of sextupole driving terms;
- Longitudinal gradient bend for emittance minimisation
- high beta section for injection at ESRF-EBS swap out inj. at APS-U, HEPS



HMBA optimisation strategy

Combined linear – nonlinear optics optmisation: fine readjustment of the linear optics are used to target non-linear dynamics objectives

Linear optics parameters are modified while keeping the chromaticity corrected with two sextupole families in the dispersive bump. Effect on DA and Lifetime is checked. Nonlinear elements (1 sext. and 1 oct.) are also included



Nine quadrupole per half cell set nine optics parameters selected after the assessment of the effect of each of them on the nonlinear optics. (Such choice is initially empirical, then optimised using MOGA)

HMBA: MOGA

MOGA is then further used to

control the quadrupoles and the optics functions (at fixed chromaticity) control the sextupoles

Objective functions

On-momentum dynamic aperture (special high beta injection cell) Touschek lifetime computed with Piwinski formula

e.g. MOGA at ESRF-EBS has been used with

6 sextupoles and 2 octupoles over two cells then reduced to

3 sextupole

and 1 octupole over two cells

Effect of errors is fully included (10 seeds),

misalignments, BPM, multipoles and their correction



Large chromaticities [e.g. (6,4)] show better dynamics





Diamond II: DDBA cell – 270 pm

The 4BA cell can be modified to introduce an additional straight in the middle of an arc while keeping the emittance small.



- Increase dispersion at chromatic sextupoles
- removed sextupoles in the new straight
- Longer mid-cell straight section from 3m to 3.4 m longer is unmanageable
- Problem with horizontal phase advance between chromatic sextupoles (0.8 $*2\pi$)
- DA ~ \pm 5 mm after extensive MOGA runs



IPAC16 Busan, Korea, 9 May 2016



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Diamond II: DTBA cell

A more aggressive design has been proposed that merges the ESRF HMBA concept with the Diamond DDBA mid straight section taking the best of both

Use the ESRF cell (7BA with longitudinal gradient dipoles) – removing the mid dipole to make it a 6BA with a straight at the centre



Promising design:

- Emittance 120 pm
- ~ 10mm DA
- ~ 3 h lifetime

short straight sections ~5m long straight sections ~8 m mid-straight section ~3 m

Large beta x for injection under investigation

Diamond II: DTBA cell optimisation

Following the ESRF approach we start with a linear \leftrightarrow nonlinear optimisation make a thorough scan of the linear optics parameters that provide the best DA and MA (for a fixed chromaticity)

Only quads are changed

Parameter	main target
α _y @SF	dQ _y /dJ _y
Octupole	dQ _x /dJ _x
μ_y (SF-SF)	dQ _y /dJ _x
$\beta_x @ID$	ε _x
β_x @SF	ϵ_{x} and nat. $\xi_{x,y}$
D _x @ (ID)	ϵ_x and nat. $\xi_{x,y}$

MOGA is then used to optimise 6 sextupole families in two cells.

A. Alekou et al, in WEPOW044



SLS II – reverse bends

Combining longitudinal gradient bends and reverse bends to reduce the emittance



Courtesy A. Streun

SLS II - optimisation

SLS-II is the based on a 7BA lattice with longitudinal bends, superbend in the mid-dipole and reverse bends

288 m, 140 pm bare lattice (12 cells) distributed sextupoles in the arc for chromatic correction
4 chromatic; 9 geometric; 10 octupoles

Nonlinear dynamics optimisation based on

- minimisation of driving terms via phase cancellation in 1st order
- minimisation of amplitude dependent tuneshift using octupoles
- and some 2nd order driving terms
- tracking with errors
- MOGA used for DA on momentum and DA at ±dp/p=3% (three obj.)

DA -4mm/5mm; MA almost 5% (4.5 h)



Conclusions

Since the approval of MAX IV in 2009

ESRF was funded in 2012 (commissioning 2020) SIRIUS is under construction (commissioning 2018) many upgrade programmes and new rings proposal developed based on MBA design with many variants

Many key drivers: but a crucial one is

growing confidence on nonlinear dynamics optimisation strategies

deterministic algorithms and numerical algorithms (MOGA) provide good solutions

+

accurate calculation of key physical quantities (and expt. agreement) +

robust optimisation includes the effect of errors

The development of ultra low emittance rings is now seriously tackled by a large community, in EU, US and Asia.

Low emittance ring community

The last IPACs counted about 20 new studies around the world

The LOWεRING network sponsored by EuCARD2 is playing an important role in fostering these developments

- ICFA Low Emittance Rings Workshops (LowERing 2010, 2011)
- XDL 2011 Workshops for ERLs and DLSRs, Cornell, June 2011
- Beijing USR Workshop, Huairou, October 2012
- DLSR Workshop, SPring-8, December 2012
- Low Emittance Ring Workshop, Oxford, July 2013
- DLSR Workshop, SLAC, December 2013
- Workshop on collective effects (TWIICE), Paris, 2014
- Workshop on Low Emittance Rings Technology (ALERT), Valencia, 2014
- Low Emittance Rings Workshop (LER2014), Frascati, September 2014
- DLSR Workshop, Argonne, November 2014
- Workshop on Low emittance ring design, Barcelona (2015)
- Low Emittance Rings Workshop (LER2015), Grenoble, September 2015
- Workshop on collective effects (TWIICE-2), Abingdon (UK), 2016
- DLSR workshop, Hamburg, (2016)
- Workshop on Low Emittance Rings Technology (ALERT-2), Trieste, 2016



- WP6: low emittance rings
- Y. Papahilippou (CERN)
- S. Guiducci (INFN)
- R. Bartolini

Acknowledgments

Acknowledgments:

```
R.P. Walker, P. Raimondi (ESRF)
```

for Diamond II

A. Alekou, M. Apollonio, I. Martin, T. Pulampong, S. Liuzzo (ESRF), N. Carmignani (ESRF)

for material and many discussions

C. Steier (ALS), M. Venturini (ALS), C. Sun (ALS), A. Streun (SLS), D. Einfeld (ESRF), S. Leemann (MAX IV), E. Karantzoulis (ELETTRA), L. Lin (SIRIUS), Y. Li (NSLS-II), Y. Papaphilippou (CERN)





Acknowledgments

Acknowledgments:

R.P. Walker, P. Raimondi (ESRF)

for Diamond II

Thank you for your attention !

for material and many discussions

C. Steier (ALS), M. Venturini (ALS), C. Sun (ALS), A. Streun (SLS), D. Einfeld (ESRF), S. Leemann (MAX IV), E. Karantzoulis (ELETTRA), L. Lin (SIRIUS), Y. Li (NSLS-II), Y. Papaphilippou (CERN)



