MOZA01

Simulated Beam-beam Limits for Circular Lepton and Hadron Colliders

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Introduction - contents

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    - Beam-beam limit in strong-strong simulation
  - Collision with large crossing angle and crab waist
    - Beam-beam limit in weak-strong simulation
    - Beam-beam limit in strong-strong simulation

- Beam-beam limit in hadron collider
  - Collision with zero or small crossing angle
    - Beam-beam limit in weak-strong simulation
    - Beam-beam limit in strong-strong simulation

Beam-beam limit

- Luminosity

$$L = \frac{N^2 f_{rep}}{4\pi\sigma_x\sigma_y} R\left(\frac{\sigma_z}{\beta_y}, \frac{\theta_c\sigma_z}{\sigma_x}\right)$$

$N=N_+=N_-$: bunch population
f_{rep}: collision freq.
θ_c: half crossing angle

- $\frac{\sigma_z}{\beta_y}$: hourglass, $\frac{\theta_c\sigma_z}{\sigma_x}$: normalized crossing angle (Piwinski angle)
- Tune shift

$$\xi_y = \Delta\nu_y = \frac{N r_e}{2\pi\gamma}\frac{\beta_y}{\sigma_y(\sigma_x+\sigma_y)} R\left(\frac{\sigma_z}{\beta_y}, \frac{\theta_c\sigma_z}{\sigma_x}\right)$$

- Increasing N, beam size especially vertical for flat beam increases. Tune shift is saturate at a certain value. Luminosity linearly increases for N, not N^2. This situation is called Beam-beam limit.

$$L = \frac{N\gamma f_{rep}}{2 r_e \beta_y}\xi_y \qquad \sigma_x \gg \sigma_y$$

- How large tune shift is achieved in equilibrium? Do simulations predict the beam-beam limit?

Weak-strong and strong-strong simulation

- Weak-strong simulation
  - One (strong) beam is assumed to be fixed charge distribution, and the other (weak) beam is represented by macro-particles.
  - Beam-beam interaction is evaluated by tracking the macro-particles in the electro-magnetic field induced by the fixed charge distribution.
  - The strong beam is assumed to be Gaussian distribution in most cases.
- Strong-strong simulation - Both beams are represented by macro-particles.
  - Beam distribution is represented on meshed space using Particle In Cell method. Arbitrary and self-consistent distribution of two beams are treated.
  - Statistical noise of macro-particles induces an fluctuation in potential calculated by PIC. The unphysical emittance growth by the noise is cared in the strong-strong simulation.
    - As an approximation, two beams are represented by Gaussian whose sizes are determined turn-by-turn. It is called Soft Gaussian approximation.
    - Strong-strong simulation based on PIC is more popular than the soft Gaussian approximation.
- Quasi-strong-strong simulation
  - Repeat weak-strong simulation with keeping self-consistency.

Weak-strong strong simulation

- Bassetti-Erskine Formula (CERN-ISR-TH/ 80-06)- 2D Electric force induced by transverse Gaussian distribution

$$\Delta p_y + i\Delta p_x = e^{-:H_{bb}:}(p_y + ip_x)$$
$$= \frac{2N_b r_e}{\gamma}\sqrt{\frac{2\pi}{\sigma_x^2 - \sigma_y^2}}\left[w\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right.$$
$$\left. - \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) w\left(\frac{\frac{\sigma_y}{\sigma_x}x + \frac{\sigma_x}{\sigma_y}y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right]$$

- Round beam

$$\Delta p_r(s_i) = \frac{2N_{p,i} r_p}{\gamma}\frac{1}{r}\left[1 - \exp\left(-\frac{r^2}{2\sigma_r(s_i)^2}\right)\right]$$

Particle In Cell

- Beam potential

$$G(x,y) = \frac{1}{2}\ln(x^2+y^2)$$

$$\phi(\boldsymbol{x}) = -\frac{2Nr_e}{\gamma}\int d\boldsymbol{x}' G(\boldsymbol{x}-\boldsymbol{x}')\rho(\boldsymbol{x}')$$

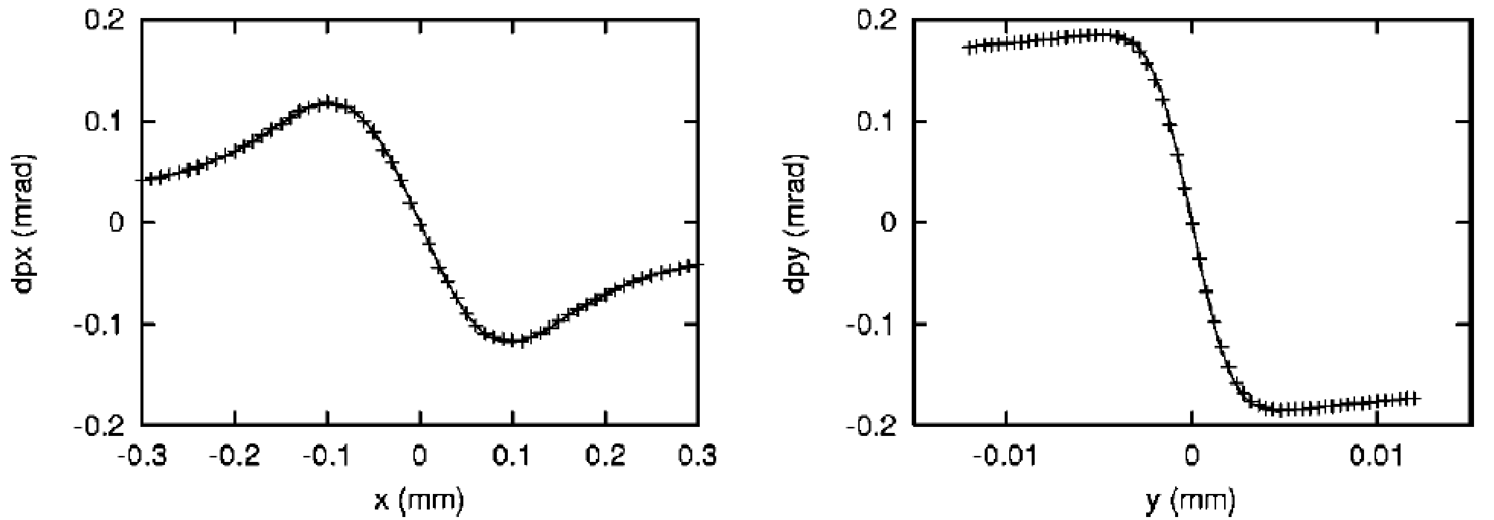
- Integrated Green function

$$G(x_i,y_j) = \int_{y_j-\Delta y/2}^{y_j+\Delta y/2}\int_{x_i-\Delta x/2}^{x_i+\Delta x/2} G(x,y)dxdy$$

$$g(x,y) \equiv \int\int \log(x^2+y^2)dxdy = -3xy+$$
$$x^2\tan^{-1}(y/x) + y^2\tan^{-1}(x/y) + xy\log(x^2+y^2)$$

Beam-beam force for flat beam

- Integrated Green function is indispensable to reproduce correct beam-beam force for flat beam, σ_x/σ_y>100.
- K. Ohmi, PRE62, 7287 (2000), PIC and Bassetti-Erskine formula



K. Yokoya had used Integrated Green function since 1980'.

Arc transformation

- Linear transfer matrix (6x6)

$$M_{2\times2\times2} = \begin{pmatrix} M_x & 0 & 0 \\ 0 & M_y & 0 \\ 0 & 0 & M_z \end{pmatrix}$$

$$M_i(s) = \begin{pmatrix} \cos\mu_i + \alpha_i \sin\mu_i & \beta_i \sin\mu_i \\ -\gamma_i \sin\mu_i & \cos\mu_i - \alpha_i \sin\mu_i \end{pmatrix}$$

- Take into account of dispersion and x-y coupling at IP

$$M_6 = R_\eta M_{4\times2} R_\eta^{-1}$$

$$R_\eta = \begin{pmatrix} \{1 - \frac{|R_{\eta,x}|}{1+r_{\eta,0}}\} I_2 & \frac{R_{\eta,x} S_2 R_{\eta,y}^T S_2}{1+r_{\eta,0}} & R_{\eta,x} \\ \frac{R_{\eta,y} S_2 R_{\eta,x}^T S_2}{1+r_{\eta,0}} & \{1 - \frac{|R_{\eta,y}|}{1+r_{\eta,0}}\} I_2 & R_{\eta,y} \\ S_2 R_{\eta,x}^T S_2 & S_2 R_{\eta,y}^T S_2 & r_{\eta,0} I_2 \end{pmatrix}$$

- Crabbing and crossing angle, ζ

$$R_{\eta,i} = \begin{pmatrix} \zeta_i & \eta_i \\ \zeta_i' & \eta_i' \end{pmatrix} \; i = x, y$$

$$M_4 = R M_{2\times2} R^{-1}$$

$$R = \begin{pmatrix} r_0 I_2 & -S_2 R_2^T S_2 \\ -R_2 & r_0 I_2 \end{pmatrix} \qquad R_2 = \begin{pmatrix} r_1 & r_2 \\ r_3 & r_4 \end{pmatrix}$$

- Important parameters

Flat beam β_x, α_y(waist) , β_y, ν_x, ν_y, η_y, η'_y, r_1-r_4, ζ_x(crab angle),

Round beam + η_x, η'_x, α_x

Chromaticity

- Effective Hamiltonian/generating function at IP.

$$H_I(x,\bar{p},\bar{\delta}) = \sum_{n=1} \frac{a_n x^2 + 2b_n x\bar{p} + c_n \bar{p}^2}{2}\bar{\delta}^n$$

Relations between a,b,c and chromaticity, ν',α',β'.

$$\begin{aligned} a_1 = & \frac{\sin^2\mu_0}{\beta_0^2}\left[-\beta_1(\cot\mu_0+\alpha_0)(1+\alpha_0^2)\right. \\ & +\left\{-\alpha_1+\mu_1\csc^2\mu_0+2\alpha_1\alpha_0\cot\mu_0\right. \\ & \left.\left.+(\alpha_1+\mu_1\csc^2\mu_0)\alpha_0^2\right\}\beta_0\right], \\ b_1 = & \frac{\sin^2\mu_0}{\beta_0}\left[-\beta_1(1+\alpha_0^2)+\{\alpha_1\cot\mu_0\right. \\ & \left.+\ (\alpha_1+\mu_1\csc^2\mu_0)\alpha_0\right\}\beta_0\Big], \end{aligned}$$

$$c_1 = \left[\beta_1\cot\mu_0 - \beta_1\alpha_0 + (\alpha_1+\mu_1\csc^2\mu_0)\beta_0\right]\sin^2\mu_0$$

$$\mu = \mu_0 + \mu_1\delta,\ \beta = \beta_0 + \beta_1\delta \text{ and } \alpha = \alpha_0 + \alpha_1\delta$$

Transformation for chromaticity

$$\bar{x} = x + \frac{\partial H_I}{\partial \bar{p}} = x + \sum (b_n x + c_n \bar{p})\bar{\delta}^n,$$

$$p = \bar{p} + \frac{\partial H_I}{\partial x} = \bar{p} + \sum (a_n x + b_n \bar{p})\bar{\delta}^n,$$

$$\bar{z} = z + \frac{\partial H_I}{\partial \bar{\delta}} = z + \sum n(a_n x^2 + 2b_n x\bar{p} + c_n\bar{p}^2)\bar{\delta}^{n-1}$$

Crossing angle, crab cavity

- Crossing angle is equivalent to collision of two beams with xz tilt.

$$
\begin{aligned}
\bar{x} &= \tan\theta_{crs} z + \left(1 + \frac{\bar{p}_x}{\bar{p}_s}\sin\theta_{crs}\right) x \\
\bar{y} &= y + \sin\theta_{crs}\frac{\bar{p}_y}{\bar{p}_s} x \\
\bar{z} &= \frac{z}{\cos\theta_{crs}} - \frac{\bar{H}}{\bar{p}_s}\sin\theta_{crs} x \\
\bar{p}_x &= \frac{p_x - \tan\theta_{crs} H}{\cos\theta_{crs}} \\
\bar{p}_y &= \frac{p_y}{\cos\theta_{crs}} \\
\bar{p}_z &= p_z - \tan\theta_{crs} p_x + \tan^2\theta_{crs}]H,
\end{aligned}
$$

$$H = (1 + p_z) - \sqrt{(1 + p_z)^2 - p_x^2 - p_y^2}$$

$$p_s = \sqrt{(1 + p_z)^2 - p_x^2 - p_y^2}.$$

$$H = \pm\theta_{crs} p_x z \delta(s - s^*)$$

$$\bar{x} = x \pm \theta_{crs} z \qquad \bar{z} = z \pm \theta_{crs} p_x$$

- xz tilt can be controlled by crab cavity.

$$\frac{eV_{crab}\omega_{crab}}{cE_0}\sqrt{\beta_x^*\beta_{x,crab}} = \theta_{crs}$$

$$H = \frac{eV_{crab}}{E_0} x \sin\left(\frac{\omega_{crab} z}{c}\right)\delta(s - s_{crab})$$

Crab waist

$$H = \pm \frac{1}{2\theta_{crs}} x p_y^2 \delta(s - s^*) \qquad \bar{y} = y + x p_y / \theta_{crs}$$

- Transfer matrix for y

$$\begin{pmatrix} 1 & x/\theta_{crs} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\mu & \beta \sin\nu \\ -\frac{1}{\beta}\sin\mu & \cos\mu \end{pmatrix} \begin{pmatrix} 1 & -x/\theta_{crs} \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\mu - \frac{x}{\beta\theta_{crs}} \sin\mu & \left(\beta + \frac{x^2}{\beta\theta_{crs}^2}\right) \sin\mu \\ -\frac{1}{\beta}\sin\mu & \cos\mu + \frac{x}{\beta\theta_{crs}} \sin\mu \end{pmatrix}. (5$$

- α_y depending on x appears. Waist depend on x. Particle with x collide with another beam core at its waist.

$$\bar{\alpha} = -x/\beta\theta_{crs} \text{ and } \bar{\beta} = \beta + x^2/\beta\theta_{crs}^2$$

Waist shift is proportional to x. $\quad s = -x/2\theta_{crs}$

Synchrotron radiation

- Simplest D=τ_i/T_0 δ_{ij}., t=()

$$M = M_0(1 - D)$$

- Radiation matrix

$$D_r(s) = \frac{2}{3} r_e \frac{\gamma^3}{\rho^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\rho} + 2\rho K_x & 0 & \frac{1}{\rho} + 2\rho K_y & 0 & 0 & 2 \end{pmatrix}$$

$$D = \oint M_0(s^*, s)^{-1} D_r(s) M_0(s^*, s) ds$$

$$B_r(s)_{ij} = \frac{55}{24\sqrt{3}} \frac{r_e \hbar}{mc} \frac{\gamma^5}{|\rho|^3} \delta_{i6} \delta_{j6}$$

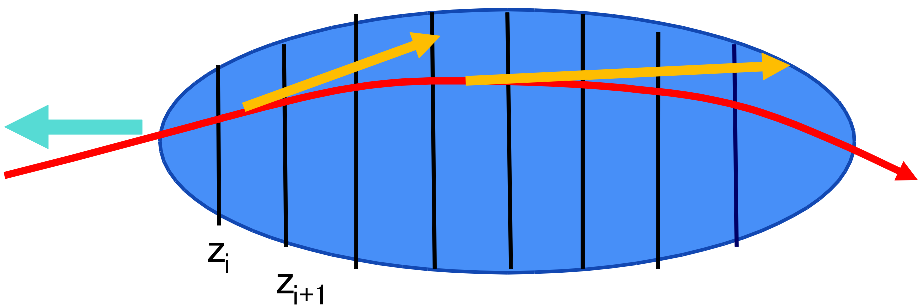
$$B = \oint M_0(s^*, s)^t B_r(s) M_0(s^*, s) ds$$

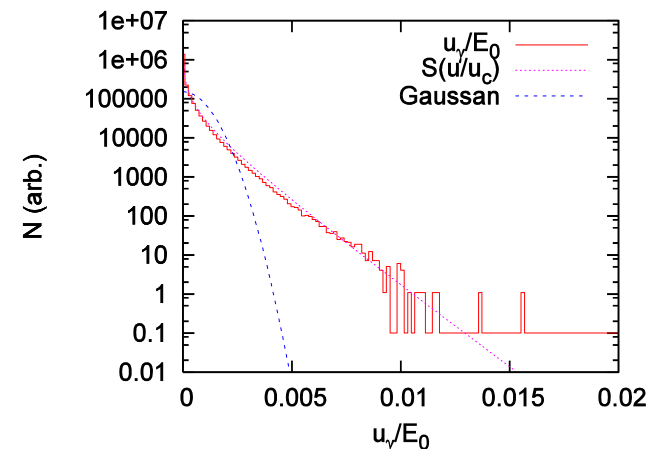
Beamstrahlung: essential for Higgs factory

- Synchrotron radiation during beam-beam collision
- Calculate trajectory interacting with colliding beam.
- Particles emit synchrotron radiation due to the momentum kick dp/ds.

ρ=23.5/19.7m<< ρ_{bend}= 6,094/11,000m (CEPC/TLEP)

u_c= 164/194 MeV, N_γ=0.21/0.092





$$ds = \frac{z_i - z_{i+1}}{2}$$

$$\langle\delta\rangle = dn_\gamma\langle u\rangle = \frac{2r_e\gamma^3}{3\rho^2}ds$$

$$\frac{1}{\rho_{x,y}} = \frac{dp_{x,y}}{ds}$$

$$\langle\delta^2\rangle = dn_\gamma\langle u^2\rangle = \frac{55}{24\sqrt{3}}\frac{r_e\hbar\gamma^5}{mc\rho^3}ds$$

Beam-beam limit in Lepton colliders
Higgs factory, damping time 150 turns

- Collision with zero or small crossing angle
  - Beam-beam limit in weak-strong simulation
  - Beam-beam limit in strong-strong simulation
- Collision with large crossing angle and crab waist
  - Beam-beam limit in weak-strong simulation
  - Beam-beam limit in strong-strong simulation

Luminosity is calculate by the beam-beam simulation.
Beam-beam tune shift is estimated by the luminosity.

$$L = \frac{N\gamma f_{rep}}{2r_e\beta_y}\xi_y \quad \longrightarrow \quad \xi_L = \frac{2r_e\beta_y}{N\gamma f_{rep}}L$$

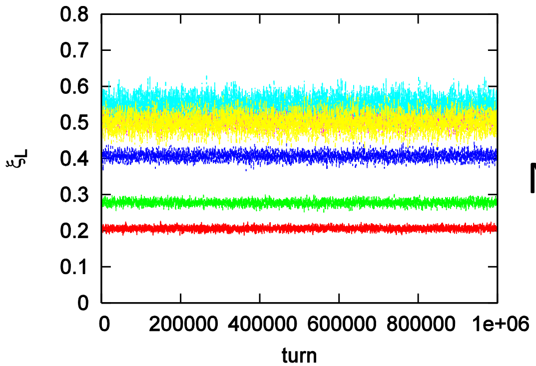
Equilibrium beam-beam tune shift
=Beam-beam parameter

Simulations are executed with scanning the bunch population;
initial beam-beam tune shift ξ_{y0}.

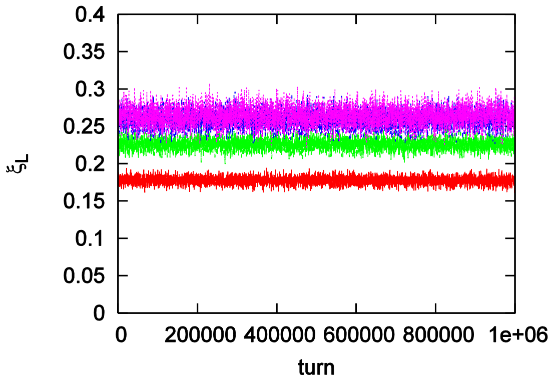
No crossing angle or small crossing angle
weak-strong simulation

Luminosity evolution for scanning bunch population

Higgs factory,
damping time is 150 turns.



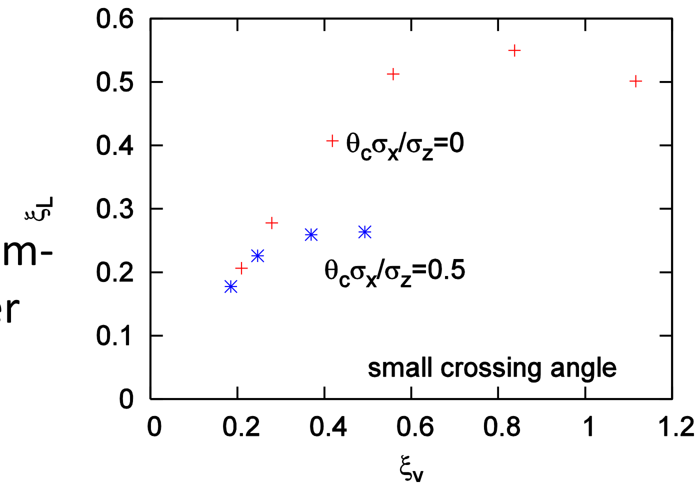
No crossing angle



Small crossing angle

$$\frac{\theta_c \sigma_z}{\sigma_x} = 0.5$$

Equilibrium Beam-beam parameter



bunch population
Initial tune shift,

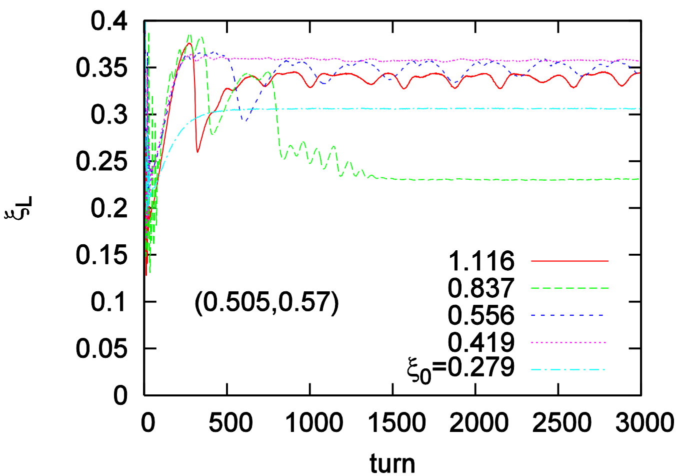
- Beam-beam parameter ξ_L=0.5 is achieved for collision with zero crossing angle in weak-strong simulation.
- Beam-beam parameter is saturated at ξ_L=0.27 for collision with small crossing angle

No crossing angle or small crossing angle

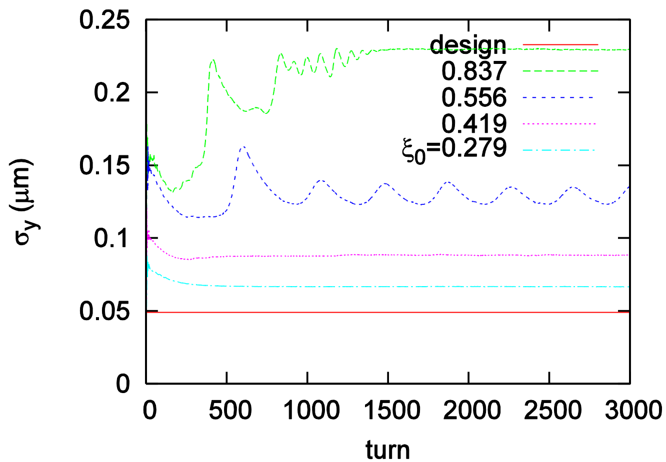
strong-strong simulation

- Collision with zero crossing angle.
- Beam-beam parameter is saturated at ξ_L=0.3-0.35.
- Vertical emittance growths of e+ and e- are simultaneous and synchronized. Non Gaussian Collective emittance growth, K. Ohmi, PRL92, 214801 (1994).
- No coherent motion, except vertical π mode seen only in ξ_0=0.837.

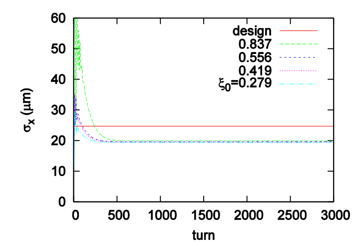
Evolution of beam-beam parameter (luminosity)



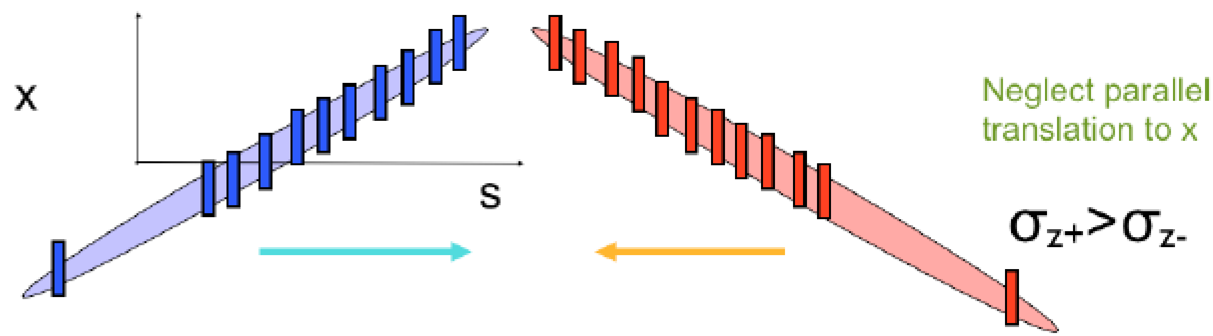
Evolution of vertical beam size at IP



Evolution of horizontal beam size at IP

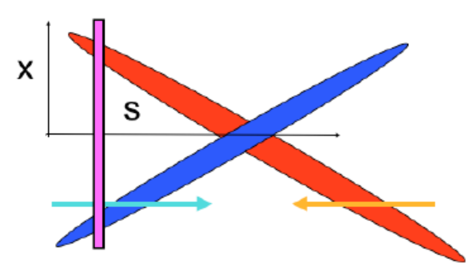


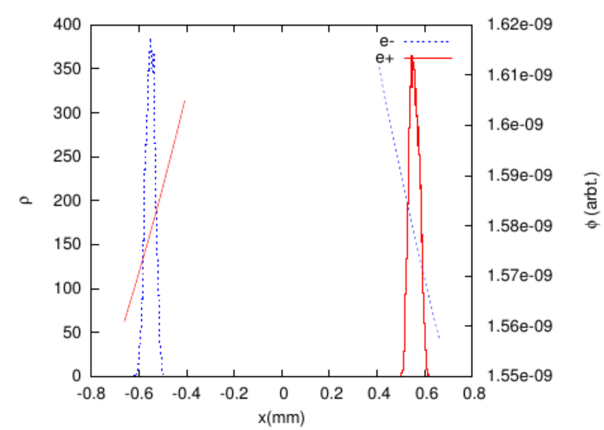
Large crossing angle



- Use shifted Green function (J. Qiang)

$$\phi(\boldsymbol{x}) = -\frac{2Nr_e}{\gamma}\int d\boldsymbol{x}' G(\boldsymbol{x} - \boldsymbol{x}' - \boldsymbol{x}_0)$$

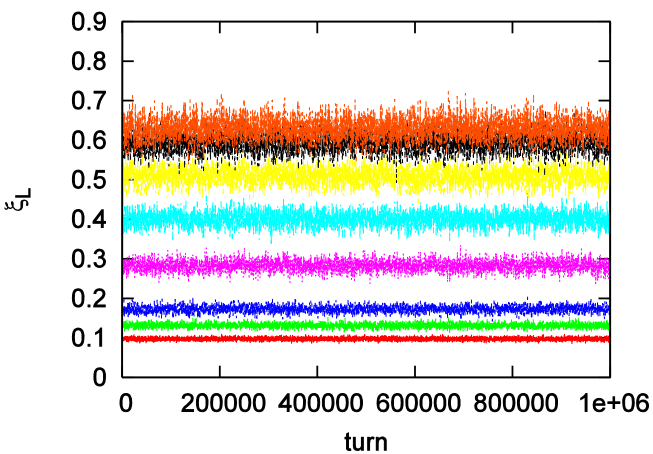




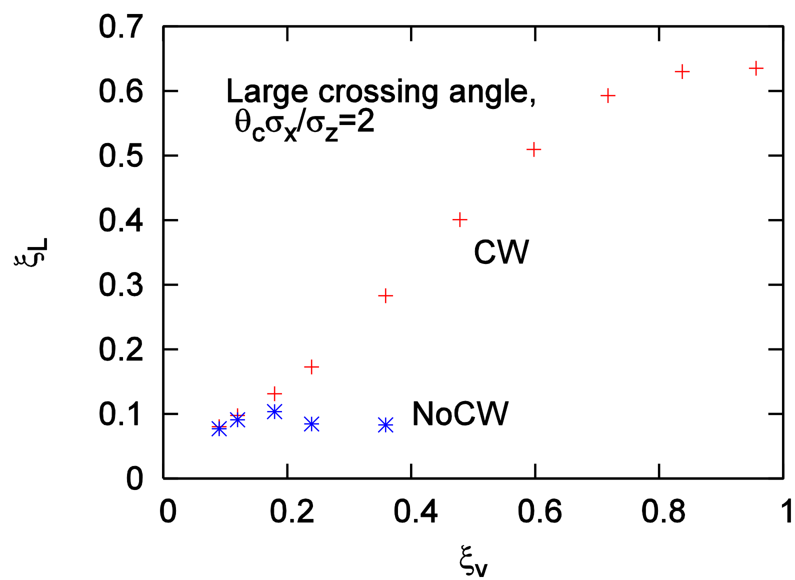
Large crossing angle and crab waist
weak-strong simulation

- Beam-beam parameter ξ_L=0.6 is achieved for collision with crab waist in weak-strong simulation.
- Beam-beam parameter is saturated at ξ_L=0.1 without crab waist.

Luminosity evolution for scanning bunch population



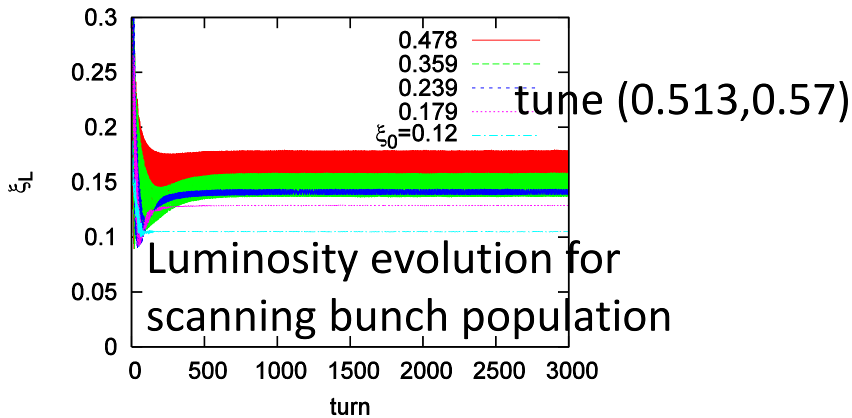
Equilibrium beam-beam parameter

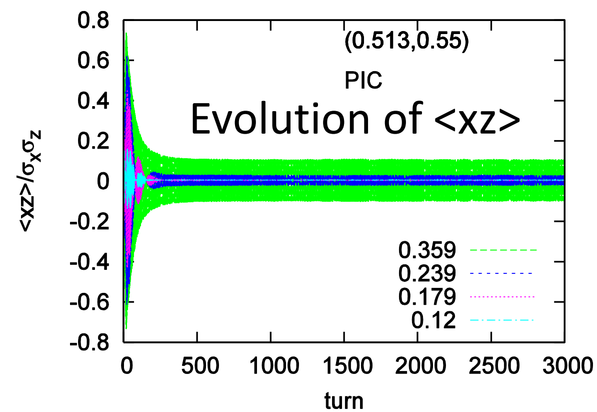


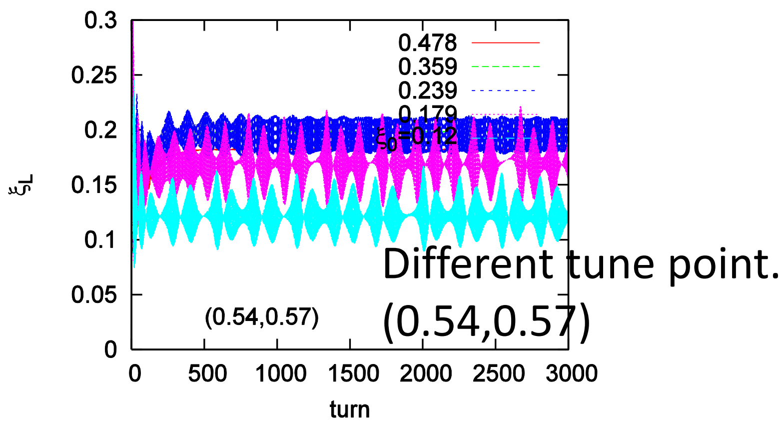
Large crossing angle and crab waist

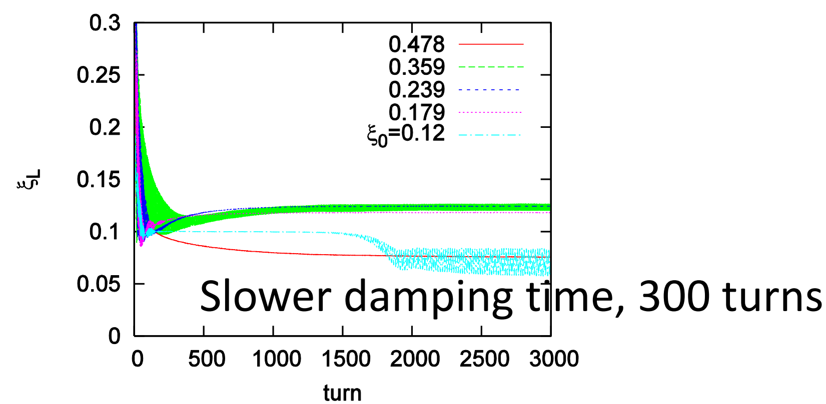
PIC based strong-strong simulation

- Large crossing angle, $\dfrac{\theta_c \sigma_z}{\sigma_x} = 2$ and crab waist.
- Beam-beam parameter is saturated at ξ_L=0.15 in strong-strong simulation.
- Luminosity fluctuates ξ_0>0.239, ξ_L>0.12. Coherent <xz> oscillation is seen in both beams (in-phase mode).



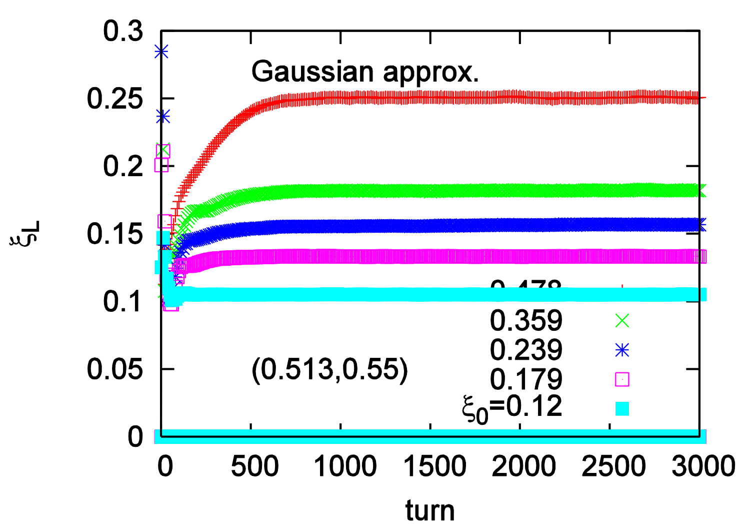


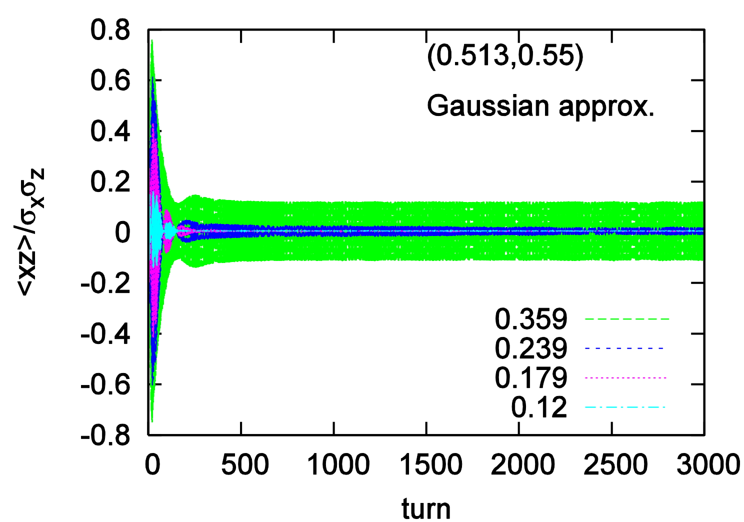




Large crossing angle and crab waist
Gaussian strong-strong simulation

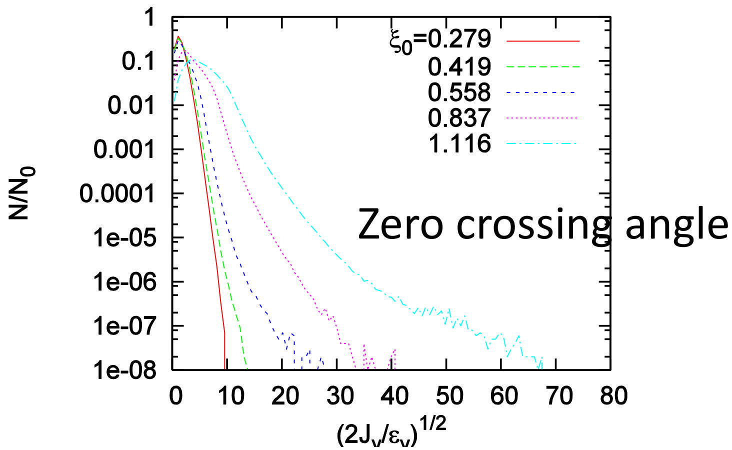
- Large crossing angle, $\dfrac{\theta_c \sigma_z}{\sigma_x} = 2$ and crab waist.
- Beam-beam parameter is saturated at ξ_L=0.15-0.25.
- Coherent <xz> motion is seen for ξ_0>0.239, ξ_L>0.15.

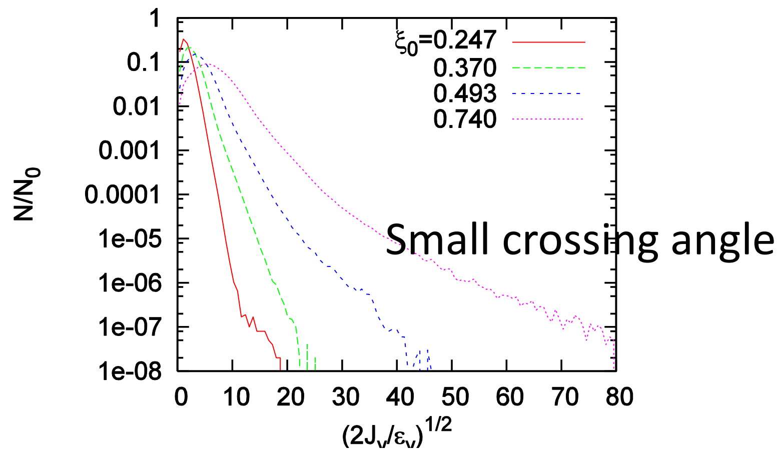


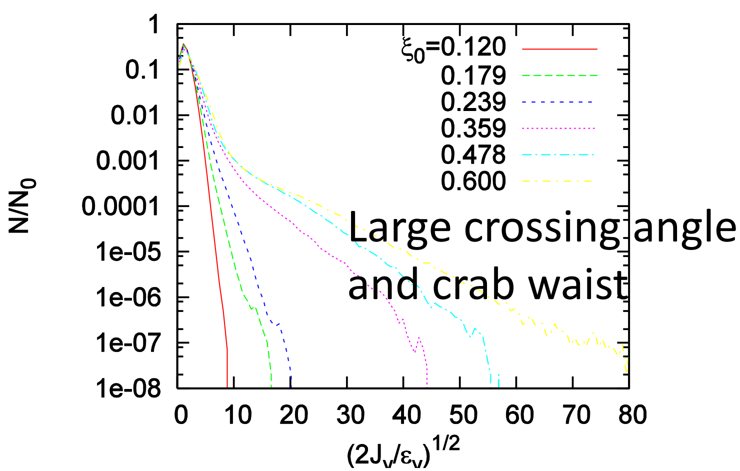


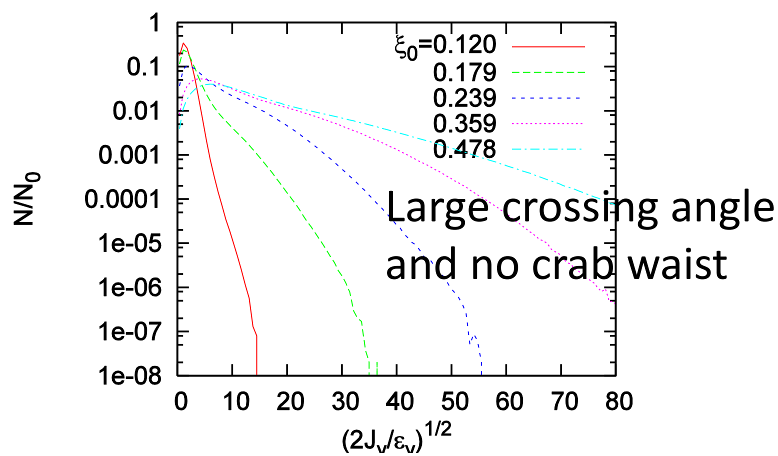
Beam hallo distribution given by w.s simulation

- Usually Vertical hallo should be taken care. No horizontal tail.
- Hallo is less serious for collision with zero crossing angle and with large crossing angle crab waist.









Lifetime given by weak-strong simulation

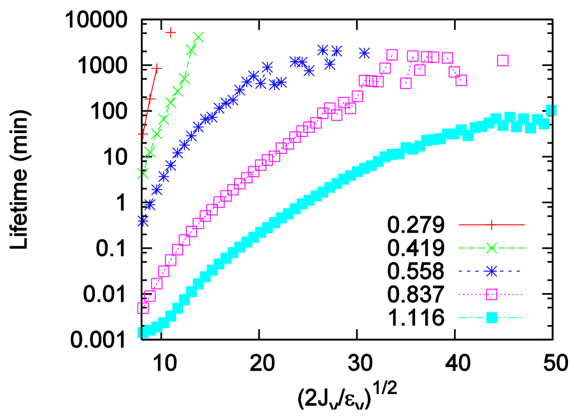
- In equilibrium, particles escape a boundary is the same number as damping from the boundary. [M. Sands, SLAC-R-121 (1970)]

$$\frac{dN}{dt} = f(J_i)\frac{dJ_i}{dt} \qquad \frac{dJ_i}{dt} = -\frac{2J_i}{\tau_i}$$

$$\tau_\ell = \frac{N}{\frac{dN}{dt}} = \frac{t_i}{2J_{i,max}f(J_{i,max})}$$

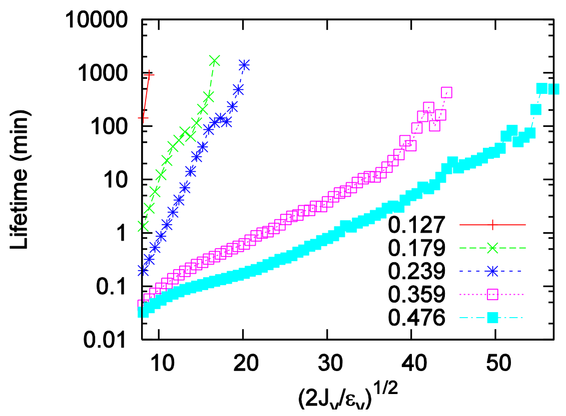
f(J): equilibrium beam distribution. For example f(J)=exp(-J/ε) for Gaussian. f(J)=N(J)/N_0 in the last slide.

Zero crossing angle



vertical aperture

Large crossing angle with crab waist



vertical aperture

How large beam-beam parameter can be achieved in Higgs factory?

Zero or small crossing angle

- For zero crossing angle ξ_L=0.5 in weak-strong ξ_L=0.37 in strong-strong. Non-Gaussian collective emittance growth is seen in s.s.
- For small crossing angle ξ_L=0.25 (w.s).
- KEKB tried crab crossing. Beam-beam parameter was ξ_L~0.09, even though expected 0.15.

Large crossing angle and crab waist

- For Large crossing angle with crab waist, ξ_L=0.6 in weak-strong ξ_L=0.15 in strong-strong. Coherent <xz> motion limits the beam-beam parameter.
- For Large crossing angle without crab waist, ξ_L=0.10 (w.s).
- The beam-beam limit will be examined in SuperKEKB.

Beam-beam limit in Hadron colliders

- No radiation damping
- Round beam
- Luminosity decrement due to emittance growth.
- One day – 10^9 revolutions in LHC, 0.25×10^9 revolutions in FCC-hh.
- Luminosity decrement $\Delta L/L_0 = 10^{-9}$/turn is a target.
- We start from studies using Weak-strong simulation.
- Since $\beta_{xy} >> \sigma_z$, small crossing angle scheme is efficient. For luminosity leveling large crossing angle is used.

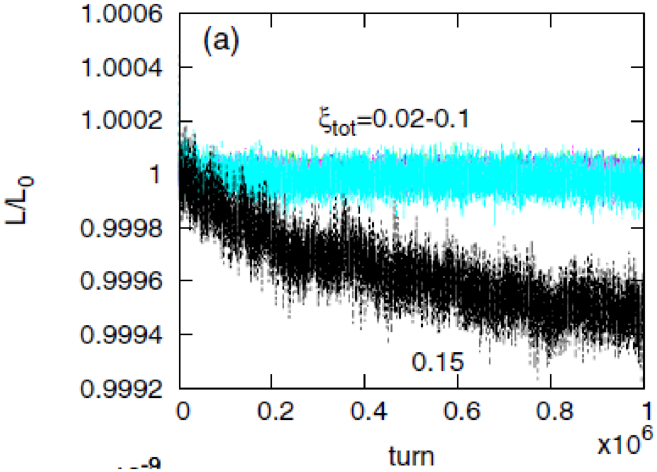
- Next, Strong-strong simulation
- Artificial emittance growth due to statistics of macro-particles

$$\frac{\Delta\varepsilon}{\varepsilon} \approx \frac{\xi^2}{N_{mp}} \times 43.4$$

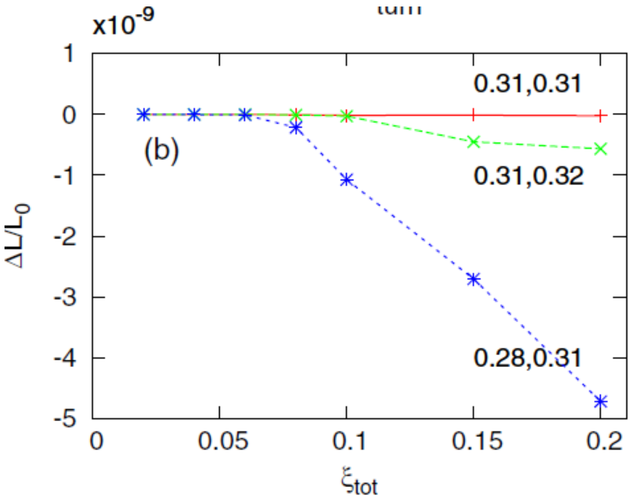
- High statistics is necessary to study high beam-beam tune shift discussed later.

Simplified model, Matrix transf. x BB interaction

- LHC parameter, without crossing angle.
- 2 IP horizontal/vertical crossing , $\xi_{tot}=2\xi_{0,IP}$



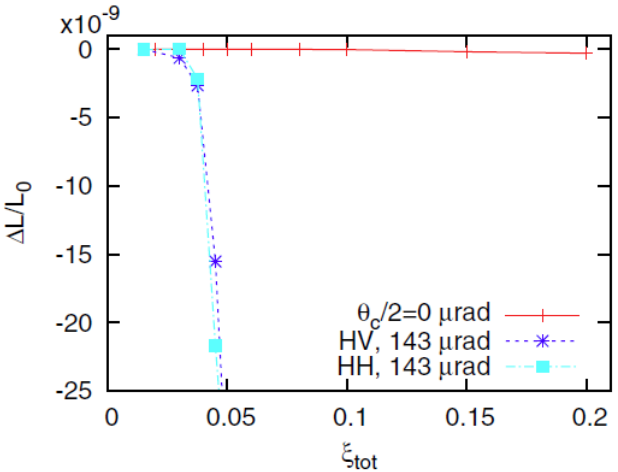
Luminosity evolution



Luminosity decrement as function of beam-beam tune shift

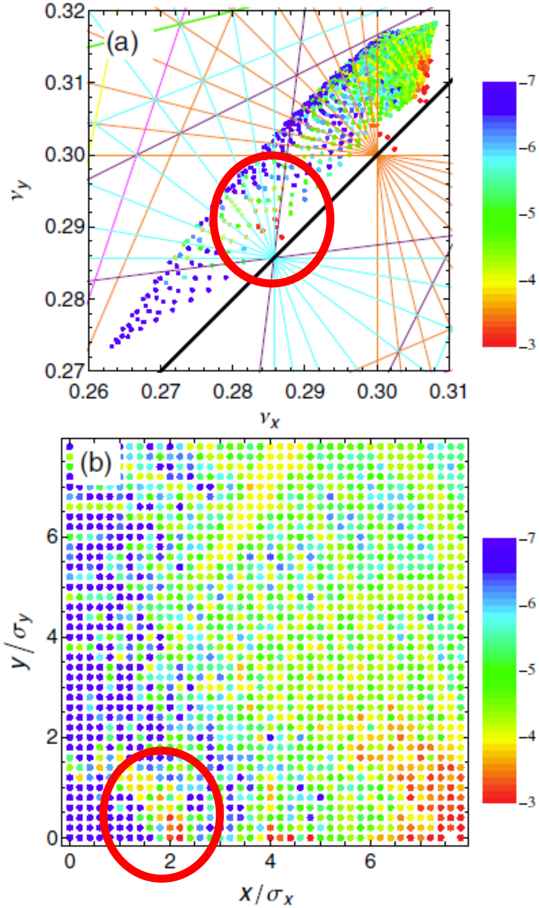
- Beam-beam tune shift for is $\Delta L/L_0=10^{-9}$ very high, $\xi_{tot}>0.2$ for LHC operating tune (0.31,0.32), and higher for (0.31,0.31).

Effect of crossing angle, $\frac{\theta_c \sigma_z}{\sigma_x} = 0.9$



Luminosity decrement as function of beam-beam tune shift

FMA analysis for crossing collision



- Beam-beam tune shift is limited at 0.035 for finite crossing angle.
- 7-th order resonances appear at $x \sim 2\sigma_x$. 10th and 14-th resonances appear only large amplitude, $x \sim (4\text{-}7)\sigma_x$.

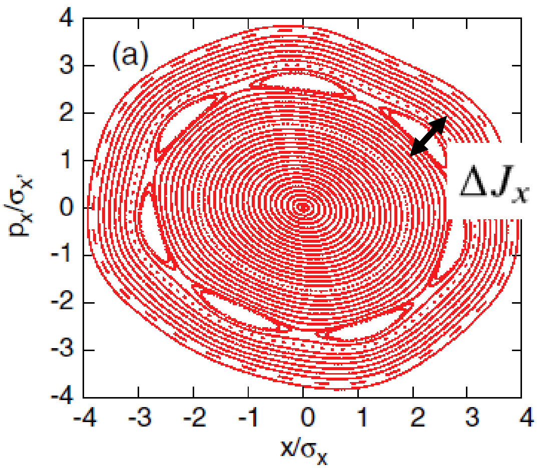
Resonance width of the 7-th order resonance induced by crossing angle

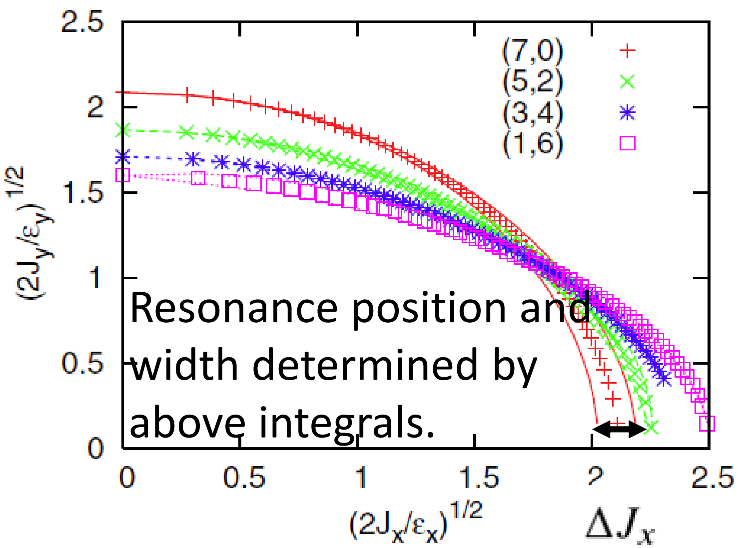
- Evaluation of the resonance width.

$$\Delta J_x = 4m_x\sqrt{\frac{U\boldsymbol{m}}{\Lambda}}.\qquad \Lambda \equiv m_x^2\frac{\partial^2 U_{00}}{\partial J_x^2} + m_x m_y\frac{\partial^2 U_{00}}{\partial J_x \partial J_y} + m_y^2\frac{\partial^2 U_{00}}{\partial J_y^2}$$

$$U_{m_x,m_y}(J_x,J_y) = \frac{\lambda_p r_p}{\gamma}\int_0^\infty \frac{du}{\sqrt{2\sigma_x^2+u}\sqrt{2\sigma_y^2+u}} \left[\delta_{m_x 0}\delta_{m_y 0} - \exp(-w_x-w_y)(-1)^{(m_x+m_y)/2} I_{m_x/2}(w_x)I_{m_y/2}(w_y)e^{-im_x\varphi_x - im_y\varphi_y}\right].\qquad ($$

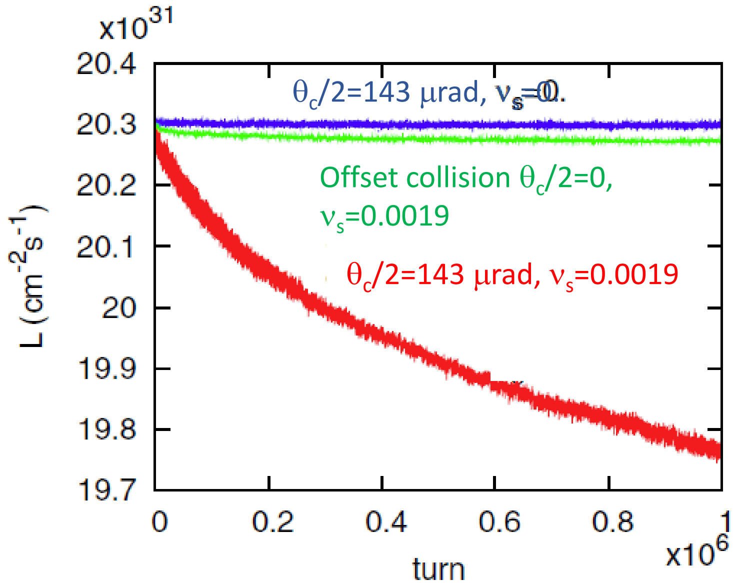
Phase space plot





Emittance growth enhanced by synchrotron motion

- Luminosity decrement for crossing collision with/without synchrotron motion.

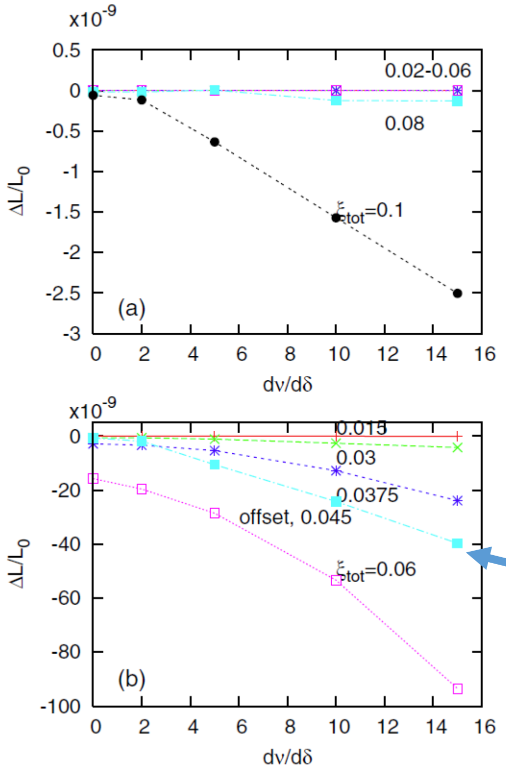


Offset collision induces similar level of resonances as that of crossing angle. In offset collision, beam-beam force does not depends on z.

- Mechanism of Emittance growth
  1. Resonances with considerable width.
  2. The beam-beam force depends on z.
  3. Synchrotron motion changes z. Resonance amplitude and width are modulated by synchrotron motion.

Another example of emittance growth

- Offset collision + chromaticity/chromatic beta
  - Offset collision induces 7-th order resonances.
  - Chromaticity changes tune depending on synchrotron phase, δ.
  - Resonance amplitudes is modulated by synchrotron motion.
- Crossing angle collision + chromaticity/chromatic beta
  - Resonance amplitudes is modulated by synchrotron motion.



Luminosity decrement for collision without crossing angle. Very weak decrement.

Luminosity decrement for collision with crossing angle. Enhanced decrement.

Luminosity decrement for offset collision without crossing angle. Enhanced decrement.

Emittance growth due to Turn-by-turn collision offset noise

- In Proton colliders without radiation damping, even small noise could affect the luminosity performance.
- Emittance growth rate/luminosity decrement due to turn-by-turn collision offset noise, Δx.

$$\frac{\Delta\varepsilon}{\varepsilon} = -\frac{\Delta L}{L} \approx \left(\xi\frac{\Delta x}{\sigma_r}\right)^2 \times 21.7$$

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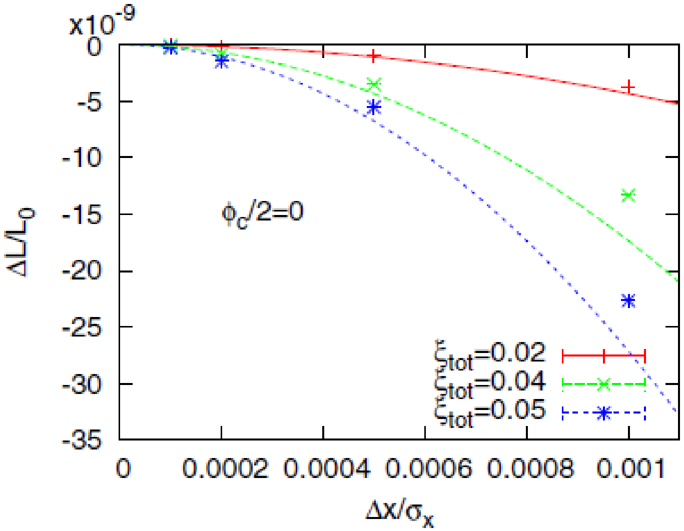


W.s. simulation containing turn-by-turn noise.
Fairly agreement with above formula
Noise should be controlled up to $10^{-4}\ \sigma_x$. level for ξ=0.05.

Strong-strong simulation and statistical noise

- Strong-strong simulation contains statistical noise for macro-particle number.
- When we use the turn-by-turn noise formula, minimum macro-particle number for a target tune shift

$$\frac{\Delta\varepsilon}{\varepsilon} = -\frac{\Delta L}{L} \approx \left(\xi \frac{\Delta x}{\sigma_r}\right)^2 \times 21.7$$

$$\Delta x/\sigma_x \sim \Delta y/\sigma_y \sim 1/\sqrt{N_{mp}}$$

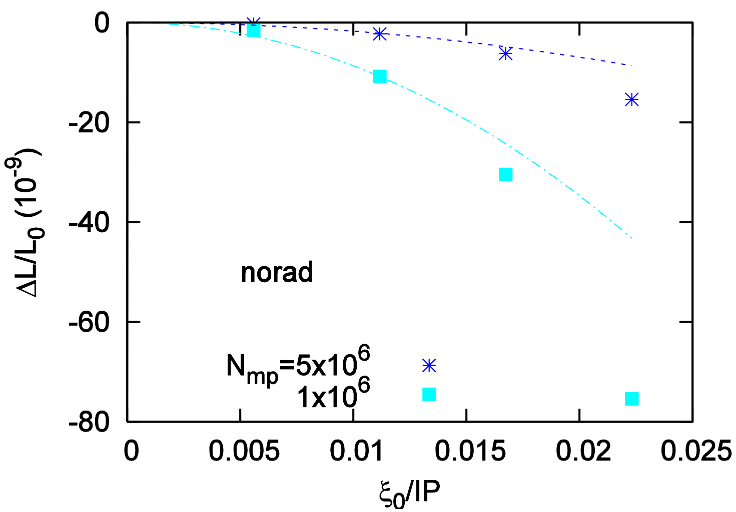
$$\frac{\Delta\varepsilon}{\varepsilon} \approx \frac{\xi^2}{N_{mp}} \times 43.4$$

- Simulation for ξ=0.03, $\Delta L/L_0=10^{-9}$ requires 4×10^7 macro-particles in minimum.

- In lepton colliders, radiation excitation exists $\Delta x^2=2\sigma_x^2/\tau_x$ implicitly. Statistical noise lower than radiation excitation does not matter.

Strong-strong simulation for a proton collider

- Luminosity decrement is calculated as function of tune shift for two macro-particle numbers.
- Luminosity decrement depends on macro-particle statistics 10^6 and $5x10^6$; this result is unphysical. Lines are given by the formula for turn-by-turn noise.



- The decrement are fairly agree.
- Simulation gave stronger decrement at higher tune shift.
- Some kind of strong-strong effect? No decrement in weak-strong.

Summary

- Beam-beam limit in lepton colliders are studied using Higgs factory parameters.
- Weak-strong simulation gave very high beam-beam parameter $\xi_L>0.5$ for collision without crossing angle and with large crossing angle and crab waist.
- Strong-strong simulation showed lower beam-beam parameter, $\xi_L>0.375$ for collision without crossing angle, and $\xi_L>0.15$ for collision with large crossing angle and crab waist. Strong-strong effects, collective emittance growth and coherent instability in <xz>, are the sources, respectively.
- To make better precision for bam-beam limit prediction in FCC-ee, elaboration of strong-strong simulation is indispensable.
- Lattice nonlinearity is another issue (D. Zhou, IPAC'16).

- Beam-beam limit in hadron colliders are studied based on HiLum LHC parameters.
- Emittance growth is caused by resonances with considerable width and their modulation due to synchrotron motion.
- Collision offset noise is one source of emittance growth.
- Strong-strong simulation contains emittance growth due to the numerical noise of macro-particle statistics and may contain some other (physical?) effects.
- There are many other effects to degrade luminosity. Lattice nonlinearity…

Thank you for your attention