

A Lattice Correction Approach Through Betatron Phase Advance

IPAC'16 MOOCB02

May 9, 2016

Outline

1. The concept of phase correction and advantages
2. The standard algorithm and the improvement
2. Measurement precision found at NSLS-II
4. Linear lattice correction and comparison with LOCO
5. Nonlinear correction and results
6. Summary



The Concept of Betatron Phase Correction

n BPM in a ring accelerator: b1, b2, b3, bn,

Measure Phase advance ($\Phi_1, \Phi_2, \Phi_3, \dots \Phi_n$) from turn-by-turn data #

Define the phase vector:

$$\Delta\phi_m = \phi_{m+1}^{meas} - \phi_m^{meas} - (\phi_{m+1}^{mod} - \phi_m^{mod})^*$$

Quadrupole strength correction

$$\begin{pmatrix} \Delta\phi_1/\Delta k_1 & \cdots & \Delta\phi_1/\Delta k_n \\ \vdots & \ddots & \vdots \\ \Delta\phi_n/\Delta k_1 & \cdots & \Delta\phi_n/\Delta k_n \end{pmatrix} \begin{pmatrix} \Delta k_1 \\ \vdots \\ \Delta k_n \end{pmatrix} = \begin{pmatrix} \Delta\phi_1 \\ \vdots \\ \Delta\phi_n \end{pmatrix}$$

Response matrix

Measured phase error

Nonlinear correction

P. Castro, PAC'93, p2103, * R. Tomas, EPAC'06, p2023

The cons and pros of Phase Correction

Pros:

1. Fast measurement
2. Immune to BPM gain and tilt error
3. No need for stored beam

Cons:

1. Measurement of an instant lattice, susceptible to fluctuations
2. Noise to signal ratio is much higher than the orbit measurement

1. CERN, P. Castro, PAC'93, p2103
2. Cornell, D. Sagan, PRST-AB, 3, 102801(2000)
3. SSRL, X. Huang, PRST-AB, 8, 064001 (2005)
4. LHC, R. Tomas, EPAC'06, p.2023
5. RHIC, G. Wang, PAC'09, p.3859
6. Diamond, G. Rehm, BIW10, p.44
7. PSI, M. Aiba, PRST-AB 16, 012802 (2013)
8. NSLS-II, Y. Li, ICAP2015, p.6

Limit: the phase measurement error is 5-10mr

Frequency and Phase from a Sinusoidal Signal

Standard method:

$$a = \sum_i^N x_i \sin(2\pi\nu i) \quad b = \sum_i^N x_i \cos(2\pi\nu i)$$

$$\phi = \tan^{-1}(-a/b)$$

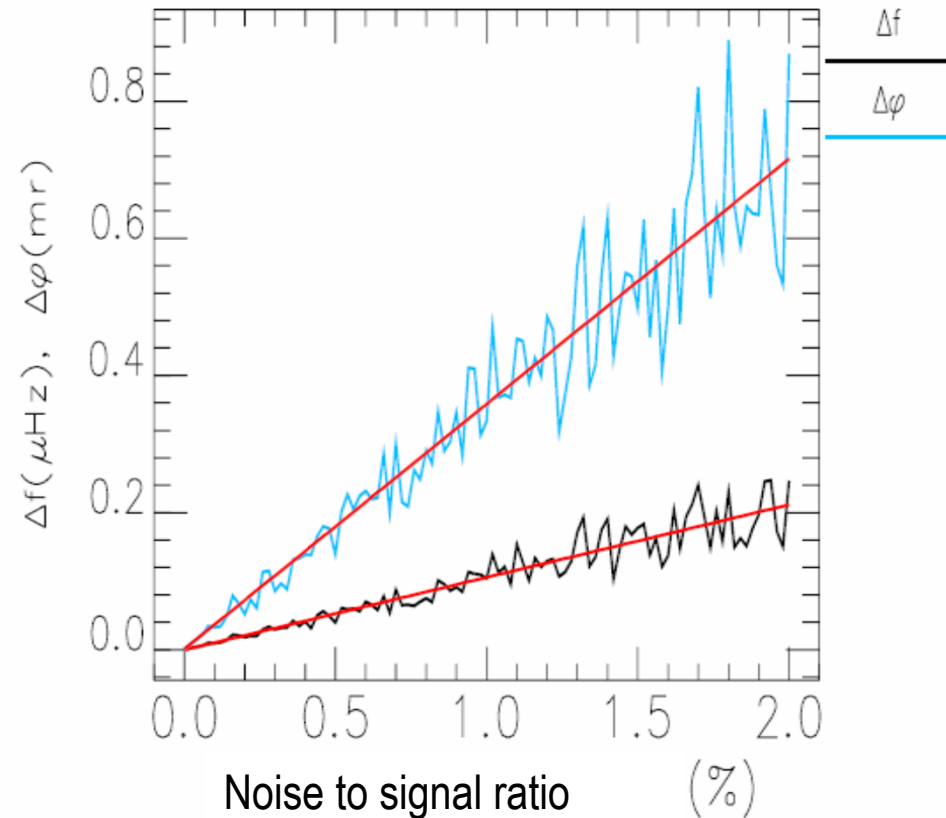
Error: $1/N + \delta\phi_e + \pi N \delta\nu$

Improved method:

1. Naff fit the frequency
2. Naff or least square fit the phase

Results: $\Phi + \epsilon$

- At NSLS-II 10mA, 100 bunches R=1%
- Δf is found to be 10^{-7} , agrees with frequency deviation of 180 BPMs
- The ultimate single phase measurement precision is 0.4mr.

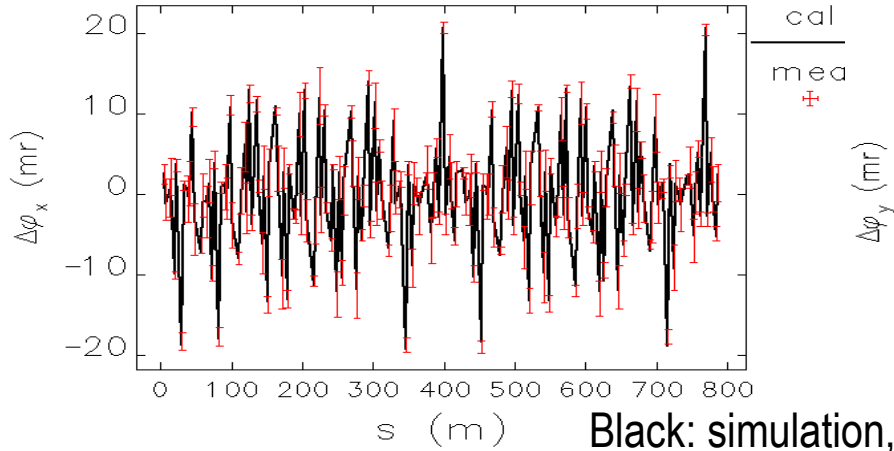


$$\Delta f = f_{\text{naff}} - f_{\text{input}}$$

$$\Delta\phi = \phi_{\text{naff}} - \phi_{\text{input}}$$

Vary a Quadrupole and Compare

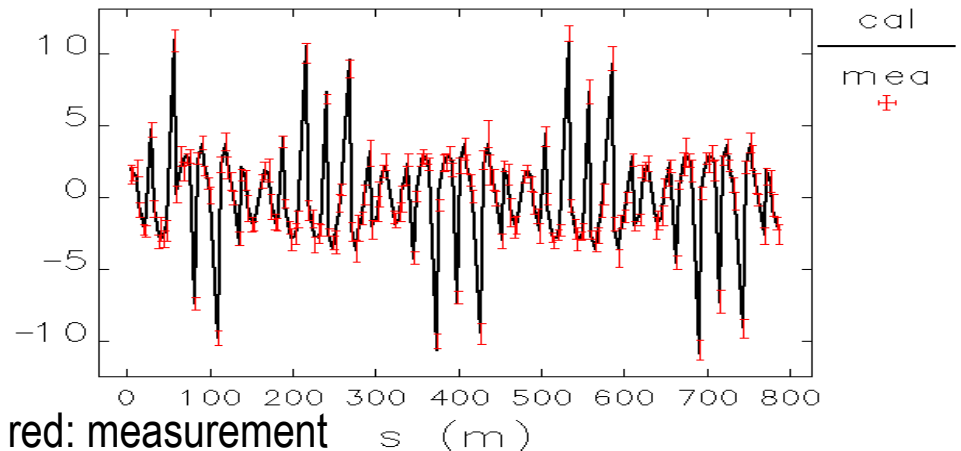
Scale factor $f_x=1.033$, error bar 1.7mr



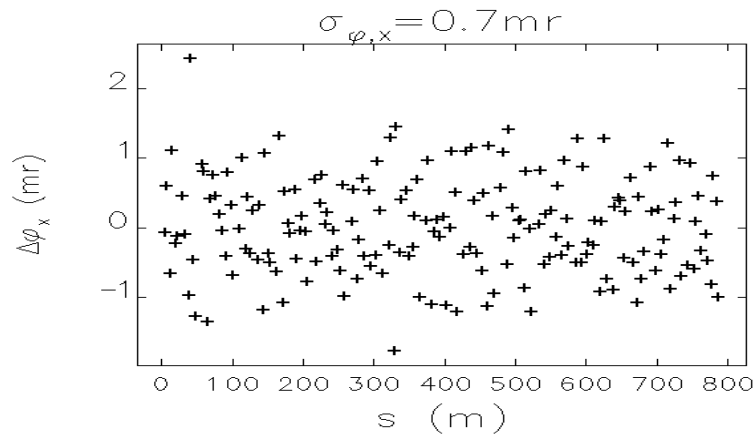
$\sigma_\phi=0.7\text{mr}$

Black: simulation, red: measurement
error bar from 10 measurements

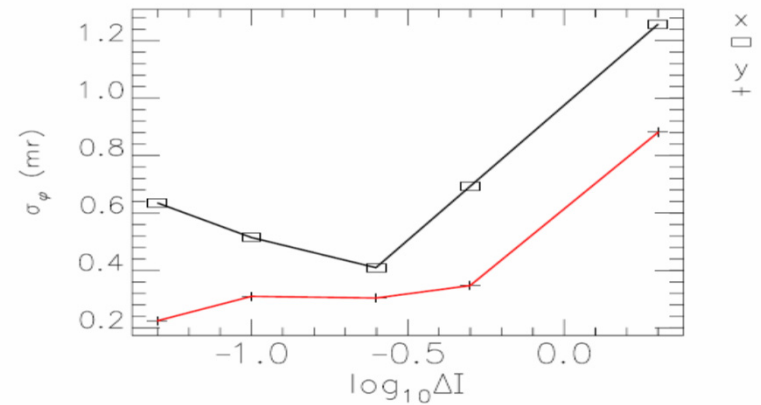
Scale factor $f_x=1.039$, error bar 0.5mr



$\sigma_\phi=0.35\text{mr}$

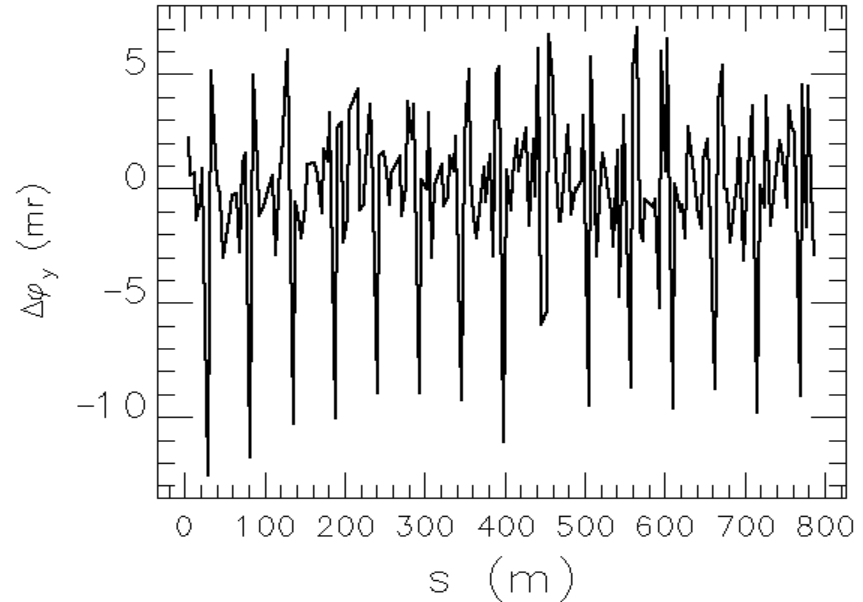
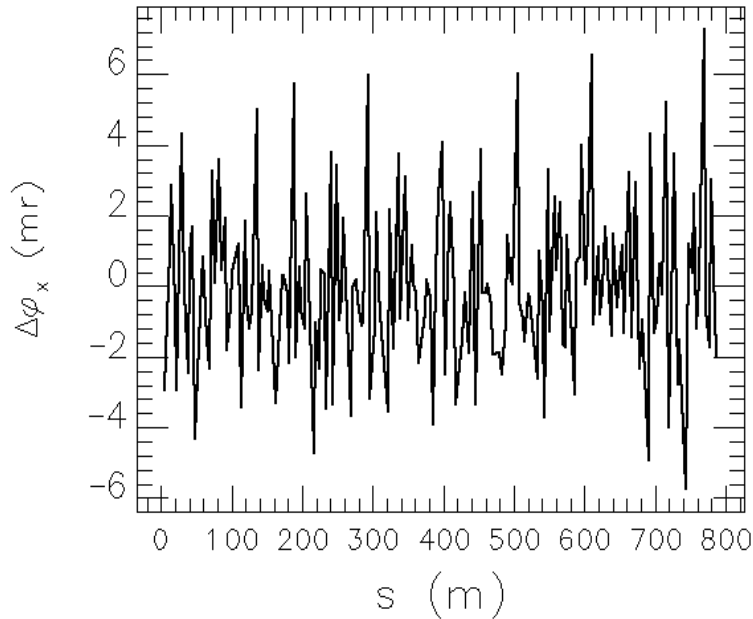


Phase Difference



Deviation at different currents

Typical Residue and Comparison with LOCO



- The phase error can be corrected to +/- 10 mr
- LOCO determines the residual beta-beat is about 1% in both planes.
- Possible reasons of the residue: energy mismatch and in-accurate modeling

Sextupole Settings Correction: the Method

1. Change a horizontal orbit corrector, measure the orbit and TBT data before and after
2. Calculate the phase vector and compare with model
3. Sextupoles produce focusing when the orbit is offset in the horizontal plane , and the difference can be detected and corrected.
4. Repeat the procedure at many correctors to break degeneracy
5. Repeat the procedure at different momentum offsets

Equivalence to Driving-term Correction

$$\Delta\phi_{x,y}(s_2) = \sum_{s_1} \Delta K_1(s_1) \beta_{x,y} [C_0(s_1) + C_1(s_1)e^{-i2\phi_{x,y}} + C_2(s_1)e^{i2\phi_{x,y}}]$$

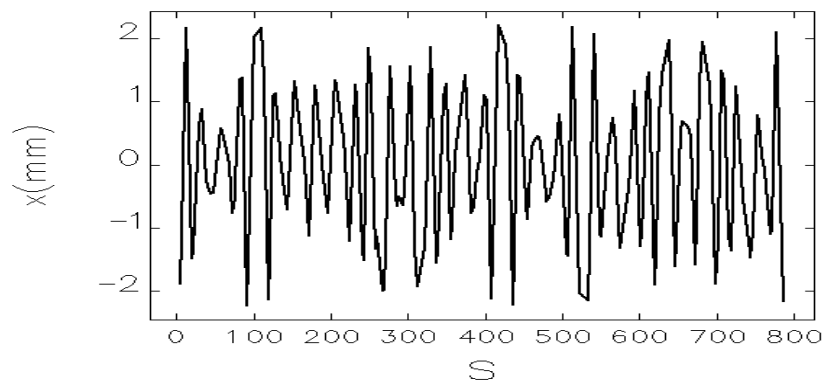
$$\Delta K_1 = \sqrt{\beta_x} K_2 (C_3 e^{-i\phi_x} + C_4 e^{i\phi_x})$$

$$\begin{aligned} \Delta\phi_x &= C_0 C_4 \sum_{s_1} K_2 \beta_x^{3/2} e^{i\phi_x} + C_3 C_4 \sum_{s_1} K_2 \beta_x^{3/2} e^{i3\phi_x} + \dots \\ &= C_0 C_4 h_{12000} + C_3 C_4 h_{30000} + \dots \end{aligned}$$

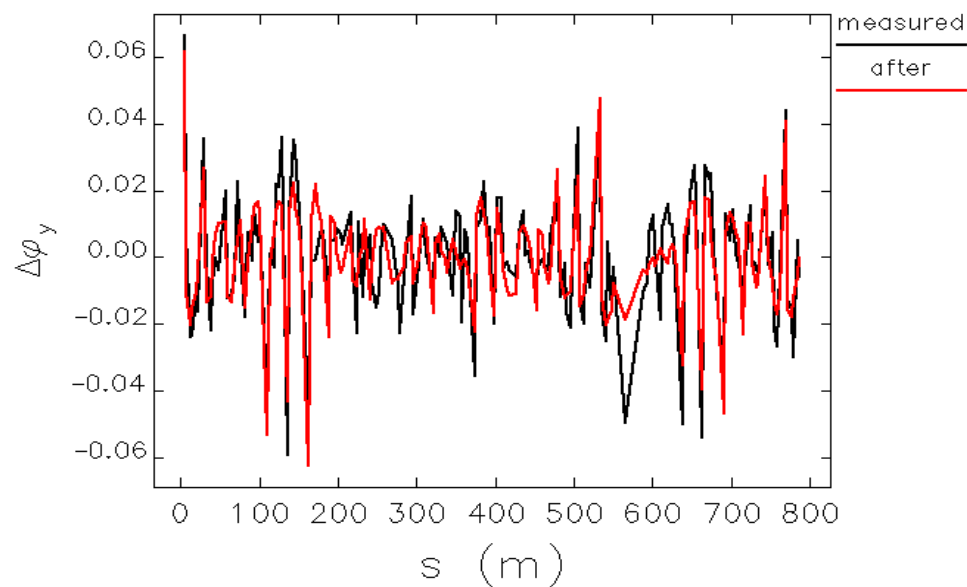
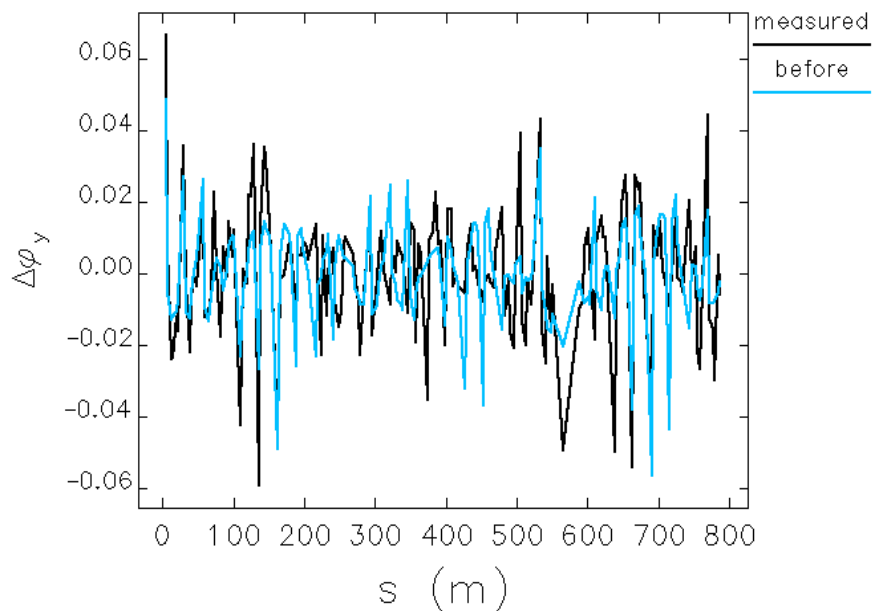
$$\begin{aligned} \Delta\phi_y &= C_0 C_4 \sum_{s_1} K_2 \beta_x^{1/2} \beta_y e^{i\phi_x} + C_1 C_4 \sum_{s_1} K_2 \beta_x^{1/2} \beta_y e^{i(\phi_x - 2\phi_y)} + C_2 C_4 \sum_{s_1} K_2 \beta_x^{1/2} \beta_y e^{i(\phi_x + 2\phi_y)} \\ &= C_0 C_4 h_{01110} + C_1 C_4 h_{01200} + C_2 C_4 h_{01020} + \dots \end{aligned}$$

The off-momentum phase correction corrects the chromatic term:
h11001, h00111, h20001, h00201, h10002

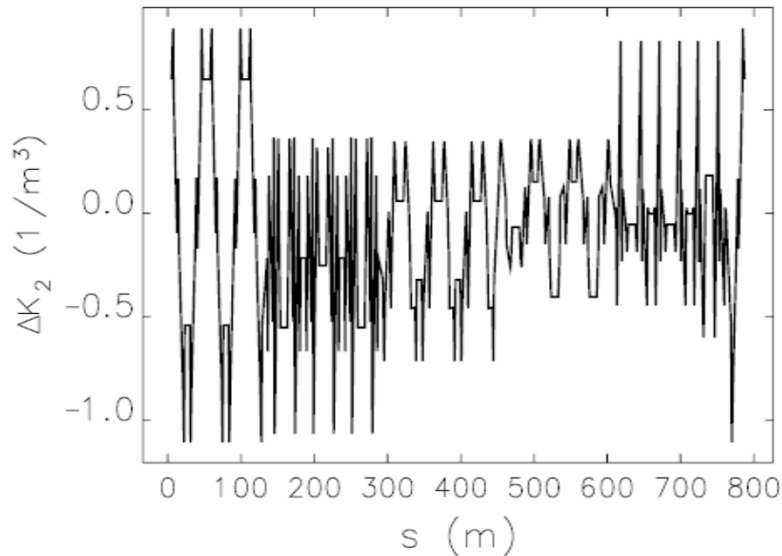
Sextupole Correction: Typical Results



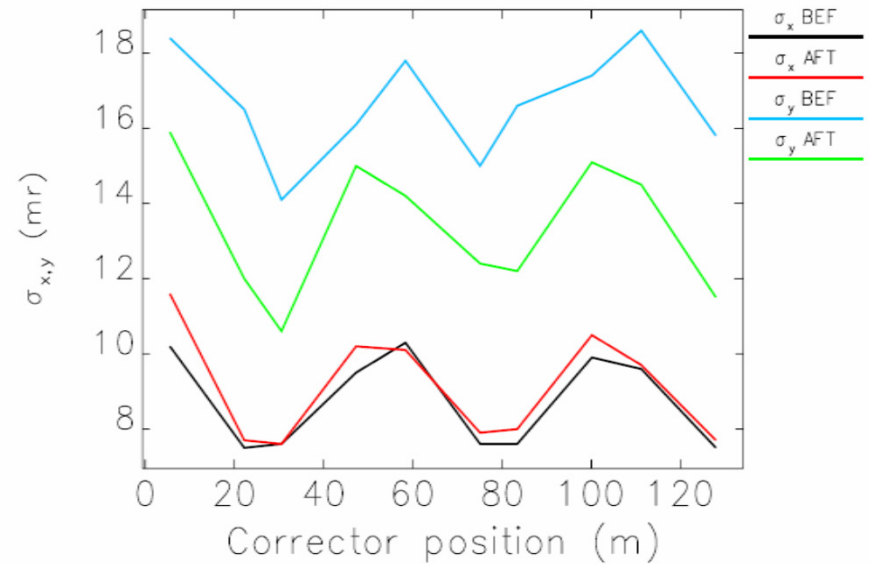
Upper Left: orbit deviation of the two lattices
Lower left: phase difference produced by sextupoles
Measurement compared with model
Lower right: phase difference after correction



Sextupole correction and DA improvement



Sextupole Strength change

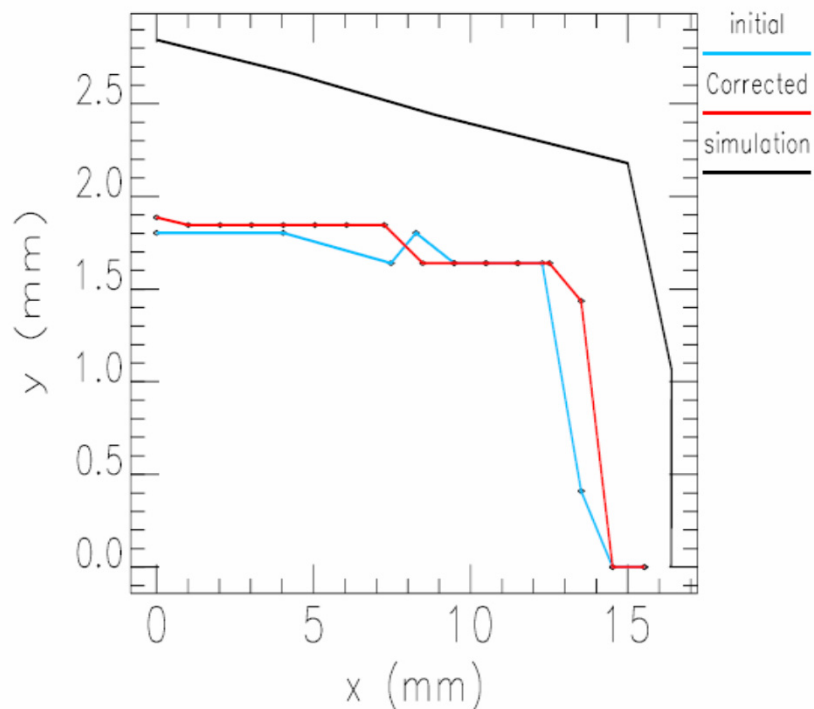


The phase error improvement of 10 measurements

- The horizontal phase error remains at about 10mr. The vertical phase error can be improved by 10%.
- 6 Sextupoles are powered in series

Dynamic Aperture Measurement Results

DA at Injection point



- Lifetime and injection efficiency did not change
- Reasons for small improvement:
DA is already large, sextupoles powered in series orbit leaking into the vertical plane

Dynamic aperture before and after correction
And comparison to simulation

Summary

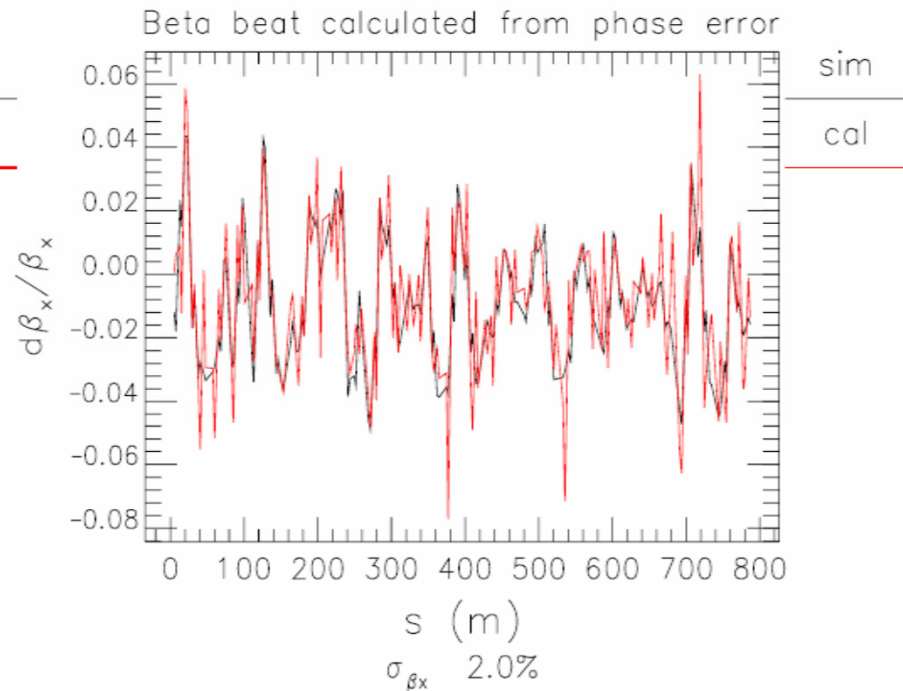
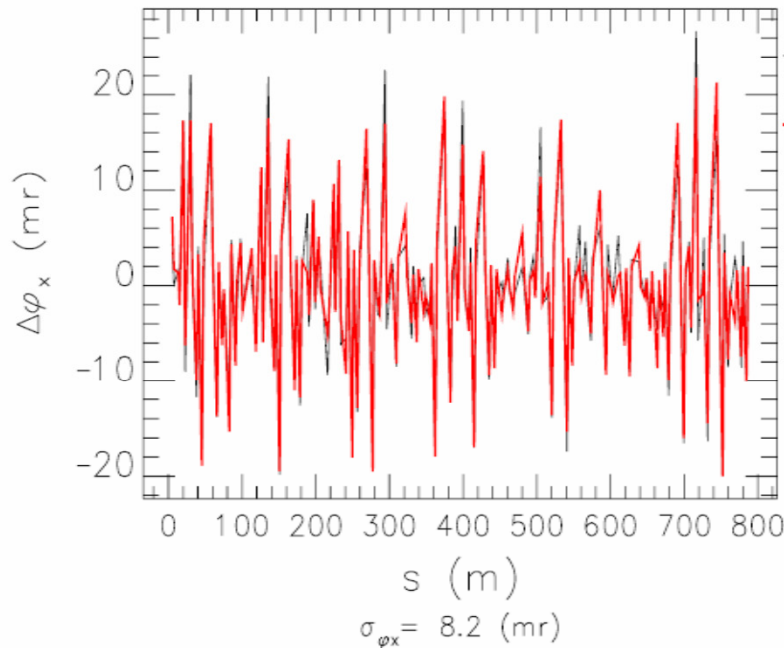
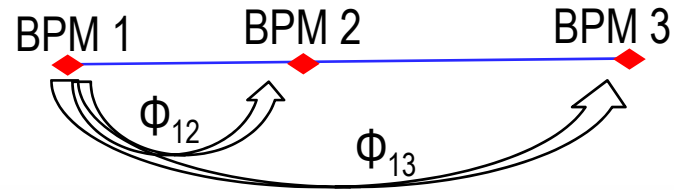
- Phase correction is equivalent to beta function correction; however, phase measurement is fast and not affected by BPM calibration
- It was determined at NSLS-II that the measurement precision of phase is 1mr.
- The linear lattice can be corrected to $\Delta\beta/\beta \sim 1\%$ (rms) within half an hour
- Limited improvement has been seen in the nonlinear lattice correction, future improvement is underway.

Acknowledgement

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F. Willeke, X. Yang, L. Yu

Comparison with Formula

$$\beta_1^e = \beta_1 \frac{\cot \phi_{12}^e - \cot \phi_{13}^e}{\cot \phi_{12} - \cot \phi_{13}}$$



- The formula is an approximation
- At NSLS-II $\Delta\beta/\beta = 2.5 \Delta\phi$