BEAM-BEAM SIMULATIONS WITH REALISTIC CRAB CROSSING FOR THE eRHIC RING-RING ELECTRON BEAM*

C. Montag, BNL, Upton, NY 11973, USA

Abstract

The 15 mrad beam crossing angle in the eRHIC ring-ring interaction region requires crab crossing of the 250 GeV proton beam to restore the luminosity. Since the product of the RF voltage and the RF frequency of the crab cavities is constant for a given crossing angle, higher frequencies are preferred in order to limit the required voltage. However, the 20 cm RMS proton bunch length provides an upper limit of the useable frequencies due to the significant curvature of the RF waveform over this bunch length. To study the effect of realistic crab cavities with a finite wavelength on electron beam-beam dynamics and to determine the potential need for higher harmonic crab cavities to linearize the kick a simulation code has been developed.

INTRODUCTION

The eRHIC ring-ring design is a low-risk version of a future polarized electron-ion collider based on the existing RHIC facility at Brookhaven, with an electron-proton luminosity around 10^{33} cm^{-2}sec^{-1}. The design in based on existing technologies where possible to minimize the risk in realization and reduce the required commissioning time of the new facility [1].

In order to be able to place focusing elements for both the electron and hadron beams near the interaction point (IP) the two beams have to be separated into their respective beamlines close to the IP. This separation can be accomplished by a dipole field near the IP, a finite crossing angle, or a combination of both. Due to the requirements of the electron-ion collider Physics program separation by a dipole field is not possible without a large reduction in the collider luminosity and/or a complete loss of a significant part of the Physics program [2]. Instead, the interaction region design is based on a 15 mrad total crossing angle between the two beams [3]. To largely restore the luminosity loss due to the crossing angle, crab crossing of the long hadron bunches is necessary. With an RMS bunch length of $\sigma_s = 0.2$ m a crab cavity frequency of 112 MHz has been chosen, which corresponds to a crab cavity RF wavelength of $\lambda_{\rm crab} = 2.68$ m. This particular frequency was selected because it presents a four-fold increase of the eRHIC bunch frequency of 28 MHz, thus allowing for future luminosity upgrades with twice or four times the number of bunches by bunch-splitting after injection. On the other hand, this frequency is low enough that its corresponding wavelength is large compared to the RMS proton bunch length.The proton bunch length at 250 GeV is limited by the achievable longitudinal emittance from the injectors and the requirement to keep the RMS momentum spread below $\Delta p/p = 5 \cdot 10^{-4}$ to ensure sufficient dynamic aperture. Shortening the bunches would require electron cooling at 250 GeV, which is considered a high risk. However, at proton energies of 50 and 100 GeV electron cooling is necessary to achieve these short bunches without exceeding the momentum spread limit. Since cooling at these lower energies requires a lower electron intensity and/or a shorter cooling section, it is considered moderate risk.

Crab crossing of the 20 cm long hadron bunches with 112 MHz crab cavities results in a significant deviation of the hadron bunch shape at the IP from a straight line. Using the eRHIC parameters listed in Table 1, the relative offset of the electron and hadron beam as a function of the longitudinal position s within the hadron beam is shown in Figure 1. Besides the direct geometric effect on the luminosity these large transverse offsets, which exceed the transverse RMS beam size of the proton beam, have the potential to reduce the available tune space for either beam to unsustainable levels. In this paper we present simulation results studying the available tune space for the electron beam.

Table 1: eRHIC Electron-Proton Beam Parameters

	electrons	protons
E [GeV]	20	250
β^* x/y [m]	0.38/0.27	2.16/0.27
$\epsilon_{\rm RMS}$ [nm]	53/9.5	9.5/9.5
σ_s [cm]	1.1	20

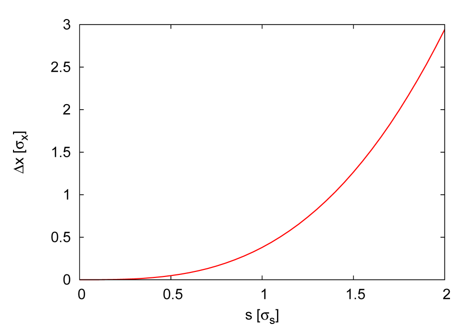

Figure 1: Horizontal offset of the short electron bunch and the 20 cm long proton bunch as function of the longitudinal position s within the proton bunch, for a total crossing angle of Φ = 15 mrad and a crab cavity RF frequency of $f_{\rm crab}$ = 112 MHz

SIMULATION METHOD

In the weak-strong treatment used in these simulation studies, the beam-beam interaction of the weak electrons with the strong proton beam is performed by slicing the long

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proton bunch into 9 slices of equal intensity. The proton distribution inside the bunch is assumed to be Gaussian in all dimensions. In the presence of a crossing angle ϕ the coordinates of the individual electrons have to be transformed into the IP by a Lorentz boost. The Lorentz boost is described as

$$\begin{aligned}
H &= (1+p_z) - \sqrt{(1+p_z)^2 - p_x^2 - p_z^2} && (1)\\
p_x^* &= (p_x - H\tan(\phi))/\cos(\phi) && (2)\\
p_y^* &= p_y/\cos(\phi) && (3)\\
\delta &= p_z - p_x * \tan(\phi) + H\tan^2(\phi) && (4)\\
H^* &= (1+\delta) - \sqrt{(1+\delta)^2 - p_x^{*2} - p_y^{*2}} && (5)\\
p_z^* &= \sqrt{(1+\delta)^2 - p_x^{*2} - p_y^{*2}} && (6)\\
x^* &= z\tan(\phi) + (1 + (p_x^*/p_z^*)\sin(\phi))x && (7)\\
y^* &= y + (p_y^*/p_z^*)x\sin(\phi) && (8)\\
z^* &= z/\cos(\phi) - (H^*/p_z^*)x\sin(\phi) && (9)
\end{aligned}$$

for the transformation into the IP, and the corresponding inverse equations after the beam-beam kick. Here, x, y, and z denote the horizontal, vertical, and longitudinal coordinates of the individual electrons, while p_x, p_y, and p_z are the corresponding momenta.

The transverse sizes of the individual slices of the proton bunch have to be modified as well to account for the crossing angle as well as their distance z_{slice} from the proton bunch center. Synchro-beam mapping is then employed to calculate the beam-beam interaction of each individual electron with each proton bunch slice, which requires a modification of the transverse electron coordinates to account for the actual interaction point not coinciding with the nominal IP:

$$\begin{aligned}
z_{\text{slice}}^* &= z_{\text{slice}}/\cos(\phi) && (10)\\
\beta_x^{\text{new}*} &= \beta_x^*\cos(\phi) && (11)\\
\beta_y^{\text{new}*} &= \beta_y^*\cos(\phi) && (12)\\
\gamma_x^{\text{new}*} &= 1/\beta_x^{\text{new}*} && (13)\\
\gamma_y^{\text{new}*} &= 1/\beta_y^{\text{new}*} && (14)\\
\epsilon_x^{\text{new}} &= \epsilon_x/\cos(\phi) && (15)\\
\epsilon_y^{\text{new}} &= \epsilon_y/\cos(\phi) && (16)\\
s &= (z - z_{\text{slice}}^*)/2 && (17)\\
\beta_x(s) &= \beta_x^{\text{new}*} + s^2\gamma_x^{\text{new}*} && (18)\\
\beta_y(s) &= \beta_y^{\text{new}*} + s^2\gamma_y^{\text{new}*} && (19)\\
\sigma_x(s) &= \sqrt{\beta_x(s)\epsilon_x^{\text{new}}} && (20)\\
\sigma_y(s) &= \sqrt{\beta_y(s)\epsilon_y^{\text{new}}} && (21)\\
x^* &= x + p_x s - z_{\text{slice}}^*\sin(\phi) && (22)\\
y^* &= y + p_y s. && (23)\\
& && (24)
\end{aligned}$$

After the beam-beam kick these transformations to the electron coordinates have to be reversed,

$$\begin{aligned}
x &= x^* - s(\delta p_x + p_x^*) + z_{\text{slice}}^*\sin(\phi) && (25)\\
p_x &= p_x^* + \delta p_x && (26)\\
y &= y^* - s(\delta p_y + p_y^*) && (27)\\
p_y &= p_y^* + \delta p_y, && (28)\\
& && (29)
\end{aligned}$$

where δp_x and δp_y are the horizontal and vertical beam-beam kicks.

In the presence of crab cavities in the proton beamline, the horizontal position of each individual proton beam slice becomes

$$x_{\text{slice}} = -\frac{\lambda_{\text{crab}}}{2\pi}\sin\left(2\pi\frac{z_{\text{slice}}}{\lambda_{\text{crab}}}\right)\tan(\phi), \quad (30)$$

and crab crossing of the electron bunch modifies the horizontal coordinate of each individual electron according to

$$x^* = x - z\tan(\phi), \quad (31)$$

assuming the electron crab cavity wavelength is large compared to the electron bunch length. This assumption is valid even for electron crab cavity frequencies of 500 MHz, which corresponds to a wavelength of 60 cm, due to the short electron bunch length of only 1.1 cm.

The electron storage ring itself is presently modeled as a linear matrix with betatron tunes Q_x and Q_y, synchrotron tune $Q_s = 0.04$, and chromaticities $\xi_x = \xi_y = 2$. Once a realistic electron ring lattice with sufficient dynamic aperture is available these studies will be repeated with that complete nonlinear lattice.

TUNE SCAN

Tune scans in the range from $(Q_x, Q_y) = (0.5, 0.5)$ to $(0.75, 0.75)$ were performed with a step size of $\Delta Q = 0.01$ in order to determine the useable tune space under different crab crossing conditions, namely with an ideal, linear crab cavity for the proton bunches, a realistic 112 MHz proton crab cavity, and with and without (ideal) crab crossing of the short electron bunches. At each working point 1000 test particles were launched and tracked for 10 transverse damping times. The resulting equilibrium RMS beam sizes σ_x and σ_y were determined by averaging over the last damping time. Using these RMS electron beam sizes and the design proton beam sizes, the resulting luminosity was then calculated.

With an ideal proton crab cavity system with infinite RF wavelength, and linear crab crossing of the electrons, the nominal luminosity is attained over a large tune space, essentially between $Q_x = 0.5$ and 0.7 in the horizontal plane, and $Q_y = 0.5$ and 0.65 in the vertical plane, with the exception of the linear coupling resonance $Q_x = Q_y$, as shown in Figure 2. When the linear proton crab cavity is replaced by a 112 MHz system the available tune space is significantly reduced, to an area bordered by $Q_x = 0.5$, $Q_y = 0.5$, and

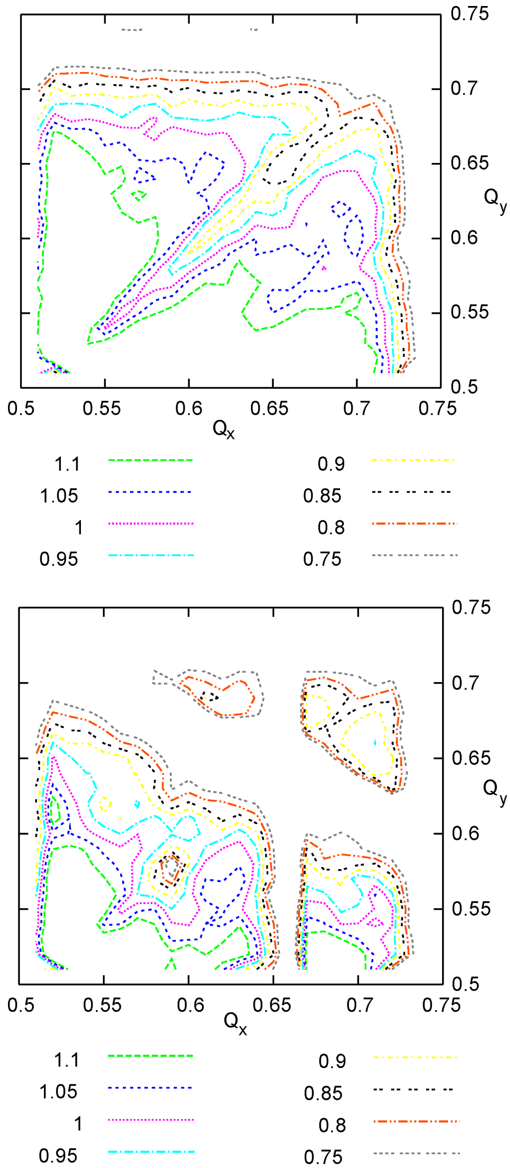

Figure 2: Luminosity contours L/L_{nominal} with an ideal proton crab cavity with infinite wavelength (top), and with a 112 MHz crab cavity system (bottom). Ideal crab crossing of the electron beam is assumed in both cases.

$Q_x + Q_y = 1.1$, Figure 2. In spite of this reduction the available tune space is likely still sufficient.

Eliminating crab crossing of the electron beam severely restricts the available tune space even more, as shown in Figure 3. While close to the half integer resonance the nominal luminosity is still reached, these areas may be too small

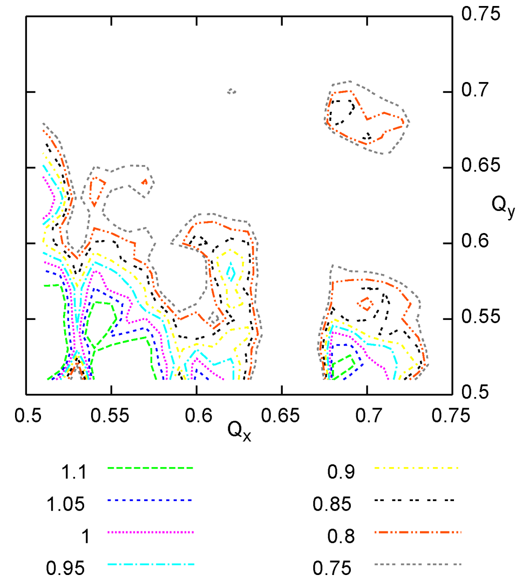

Figure 3: Luminosity contours L/L_{nominal} with a 112 MHz proton RF crab cavity, and without crab crossing of the electron bunches.

during actual operations. Further studies with the actual nonlinear electron ring lattice are needed to draw a definite conclusion.

CONCLUSION

According to simulations presented here, crab crossing with a 112 MHz crab cavity system in the eRHIC hadron ring provides a sufficiently large available tune space in which the nominal luminosity is reached, provided crab crossing is also applied to the electron beam. Without crab cavities in the electron ring the available tune space is likely too small in actual operations. Further studies using the actual nonlinear eRHIC lattice are required for a definite answer.

ACKNOWLEDGEMENTS

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