DEVELOPMENT OF A METHOD FOR ERRORS STUDY IN THE DESIGN OF PERMANENT MAGNET QUADRUPOLES

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Abstract

Laser-based accelerators are gaining interest in recent years as an alternative to conventional machines. Nowadays, energy and angular spread of the laser-driven beams are the main issues in application and different solutions for dedicated beam-transport lines have been proposed. In this context a system of permanent magnet quadrupoles has been realized, by INFN researchers in collaboration with SIGMAPHI company, to be used as a collection system for laser-driven protons up to 20 MeV. The definition of well specified characteristics, in terms of performances and field quality, of the magnetic lenses is crucial for the system realization and an accurate study of the beam dynamics. Hence, a method for studying the errors on the PMQ harmonic contents has been developed. It consists of different series of simulations in which magnetic and mechanical errors are introduced in the array and the harmonic content is analysed to fix the tolerances necessary to have a good beam quality downstream the system. The method developed for the analysis of the PMQs errors and its validation is here described. The technique is general and can be easily extended to any magnetic lens.

INTRODUCTION

Laser-based accelerators are a promising alternative to conventional machines as they could be less expensive in terms of construction and operation costs [1]. Anyway, the produced beams are not directly suitable for most of applications because of the huge angular and energy spread and transport beam-lines able to correct these issues have been already proposed [2, 3]. At INFN a two stage beam-line has been realized made of a permanent magnet quadrupoles (PMQs) collection system and a magnetic chicane as a selection system. Magnetic lenses are used in conventional particle accelerators to deflect, focus and/or correct the beam along the transport lines. PMQs lenses have the advantage to be relatively compact with an extremely high field gradient, of the order 100 T/m, within a reasonable big bore of few centimeters. For these reasons the interest in the application of PMQs in the beam handling of laser produced beams is growing in recent years [4–6]. A PMQs system can be used for the collection of laser-driven ion beams providing a beam with a considerable reduced divergence and also roughly selected in energy. Several PMQs designs have been proposed and developed based on pure Halbach scheme [7] or hybrid devices using saturated iron to guide the magnetic field [8]. At INFN a system based on four hybrid PMQs is foreseen. The design and realization of the quads was performed in collaboration with the SIGAMPHI company. It consist of two PMQs of 80 mm length and two PMQs of 40 mm with an active bore of 20 mm and a gradient exceeding 100 T/m. The system has been designed to improve the transmission and selection efficiencies of a magnetic energy selector based on four permanent magnet dipoles with alternating gradient [9–11]. In this context is crucial to perform a detailed error study of the PMQs in order to understand the field quality of the system and the relative effects on the beam dynamics. In this work, a model for analysing random errors in the hybrid PMQ design is breafly described considering two different error sources. The error sources considered are the mechanical assembly and the remanent magnetization of each sector of the PMQs.

QUADRUPOLE ERROR ANALYSIS

The 2D magnetic design of the hybrid quadrupoles is shown in Figure 1. The poles are set at 45 °, with respect to the horizontal axis, and are attached to four iron parts, almost saturated, used as supporting structure as well as magnetic flux guides. The poles have a rectangular main body ($13x14$ mm^2) with two smaller pieces near the bore, which allow to increase the field and, hence, the gradient inside the bore itself. The T-like magnets between two poles are modeled as three independent squared pieces ($10x10$ mm^2). The bore is 22 mm but the active aperture is 2 mm smaller as a 1 mm thick aluminum shielding pipe is set inside for the protection of the magnets. The FEM (Finite Element Method) code COMSOL has been used for the magnetic simulation of the system. The magnetic features of each part of the quadrupole are evaluated from the BH curve of the materials, NdFeB N50 for the magnets and iron XC10 for the other four parts. Each magnet has its own direction for the remanent magnetization, M_r.

The x and y components of the remanence are evaluated by multiplying M_r by the sine or cosine of the wanted magnetization angle that can be expressed as $\theta = 45tk_{Hal}$ in degrees, where $k_{Hal} = 3$ is the wave-number of the magnetic Halbach array describing the magnetization pattern for a quadrupole [12], $t = 0, ..., 7$ is the ordinal number of each sector of the quadrupole and 45 is one turn divided by the number of sectors, i.e. 8.

A 2D circle of radius $r_0 = 8$ mm, centered in the origin of the model, has been defined for the post-processing of the fields and for the analysis of the harmonic content. The reason of such radius is related to the fact that the system will be working with very divergent beams (around 170 mrad half-angle) and, hence, at least within the 80 % of the bore there must a controlled harmonic content to minimize higher

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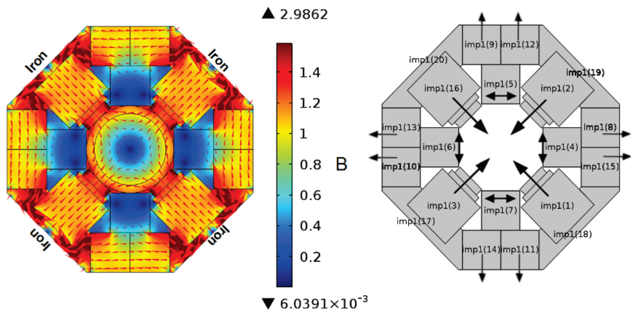

Figure 1: LHS: Magnetic flux density (colour surfaces) and directions (red arrows) evaluated in COMSOL. RHS: geometric design with id numbers and displacement (black arrows) of each block.

order effects on the beam dynamics. The harmonic content is calculated via a Fourier expansion of the radial field $B_r = (B_x * \frac{x}{r_0}) + (B_y * \frac{y}{r_0})$ [13, 14] on the reference circle and the analysis is here reported. The radial field variation is purely sinusoidal and the Fourier expansion gives its deviation from the ideal sinusoidal shape. This deviation affects the field quality and have distorting effects on the beam transport such as filamentation and growth of the emittance. Moreover, the first harmonic component, namely the dipole component, produces a steering effect on the beam, as shown in [13].

The Fourier expansion of the radial field allows to calculate the magnitude of harmonic components C_n, where n is the harmonic number, as:

$$C_n = \frac{1}{N} \sum_{k=1}^{N-1} \frac{B_{rk}}{r_0} * exp(ik(2\pi \frac{n}{N})) \qquad (1)$$

N being the number of elements in the array of the field B_r. The main quadrupole component is the $n = 2$ harmonic and, due to the symmetry of an ideal quadrupole, the allowed harmonics for an ideal quadrupole are 6, 10, 14. In the following analysis all the harmonic components, from $n = 1$ to $n = 14$, are considered as the perturbed models lose their symmetry both in terms of remanence and mechanical assembly. Anyway, the harmonics $n = 6, 10, 14$ will be referred as allowed and the higher order harmonics are negligible for this quadrupoles. The values of the harmonics for the ideal 2D case are reported in Table 1 where, in bold, are highlighted the main and the allowed harmonics.

In the first column is reported the harmonic number, in column two the modulus of complex coefficient C_n, in column three the values of C_n referred to the main harmonic in units of 10^4 and in column four the values of the normal harmonics in units of 10^4, which are the real part of the C_n components: $B_n = 2\Re(C_n)$. It is evident, from the second column of the table, that the C_2 is equal to half of the maximum field gradient and the B_2 is equal to the gradient, hence to the quadrupole field, if it is multiplied by the reference radius r_0. The values of the not allowed harmonics are negligible for the ideal model: smaller than 1 unit. The last row of Table 1 is the percentage contribution of the whole

Table 1: Harmonic Content for the 2D Ideal Model. In bold the allowed harmonics for an ideal design.

n	$\lvert C_n \rvert$ [T/m]	C_n (units of 10^4)	B_n (units of 10^4)
1	6.394e-4	0.111	-0.092
2	**57.547**	**10000**	**10000**
3	8.942e-5	0.016	0.015
4	1.886e-5	0.003	-0.002
5	1.676e-5	0.003	-0.002
6	**0.38**	**66.089**	**-66.089**
7	1.281e-5	0.002	0.002
8	5.153e-5	0.009	6.005e-4
9	3.546e-5	0.006	0.002
10	**0.292**	**50.821**	**-50.821**
11	4.335e-5	0.008	0.007
12	9.066e-6	0.002	2.351e-4
13	2.819e-5	0.005	-0.002
14	**0.032**	**5.519**	**5.519**
Sum		1.22%	1.114%

harmonic content with respect to the main harmonic, given by:

$$Sum\left(\frac{C_n}{C_2}\right) = 100\left(\sum_{k=1}^{14} \frac{1000|C_n|}{C_2}\right) \qquad (2)$$

and similarly for the normal harmonics B_n.

The harmonic components for $n \neq 2$ have to be minimized in order to have a good field quality and, consequently, an acceptable control on the beam. In order to minimize the harmonic content two error sources have been considered: the mechanical positioning of each block and the remanence of the rare-earth pieces.

In order to fix the tolerances for the considered error sources, the modulus of the remanence M_r of each rare-earth piece has been multiplied for a random number, $rand1$, with a fixed seed depending on the block identification number, see RHS of Figure 1, and the ordinal number of the magnetic configuration produced (401 in total). The choice of fixing the seed of the random number generator is useful if a certain configuration has to be reproduced later, as it guarantees that the same errors would be simulated for each configuration.The number $rand1$ has been defined as uniformly distributed around the mean value 1 with a range of ± 0.03 and ± 0.06, making the remanent magnetization increasing or decreasing up to 3% in the first case, and up to 6% in the second case.

The other error source due to the mechanical assembly has been simulated introducing, in the model, a different displacement for each block, as shown in the RHS of Figure 1. The displacement of each block is due to a random number $rand2$ with fixed seed, as above. The direction has been forced to avoid overlapping of the magnets and the four iron parts are considered fixed and with no errors in their dimension as iron can be cut with high precision and this allow to simplify the model. Moreover, the T-like pieces between two poles are treated as three independent blocks, even if they are realized as a single one; this allow to take in account not only errors due to the assembly but also errors due to the machining of these parts, in fact permanent mag-

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Table 2: Harmonic Content for the 2D Model with Random Errors on Magnets Position and Remanence

Position and M_r errors	$\lvert C_n \rvert / C_2$	B_n/B_2
±100 μm - ±3%	2.5796%	1.281%

nets are very brittle materials. The number $rand2$ has been defined as uniformly distributed around the mean value 0 with a range of ±0.1 and ±0.2. In this way each block is shifted from the ideal position up to 100 μm in the first case and up to 200 μm in the second case.

In a first phase the error sources are independently modelles. This allows to fix the tolerances to the smallest ranges, as the biggest ones produce a very high contribution of the harmonic contento [13].

The combined effect of both error sources on the harmonic content has also been studied for the above tolerances fixed in the first phase, mixing each magnetic configuration with all the geometric configurations. This procedure allows to study the effects of each magnetic configuration on a certain geometry giving, as a result, the average harmonic content. Results are reported in Table 2 and Figure 2, where it is shown that the total harmonic content has a contribution smaller then 3 %.

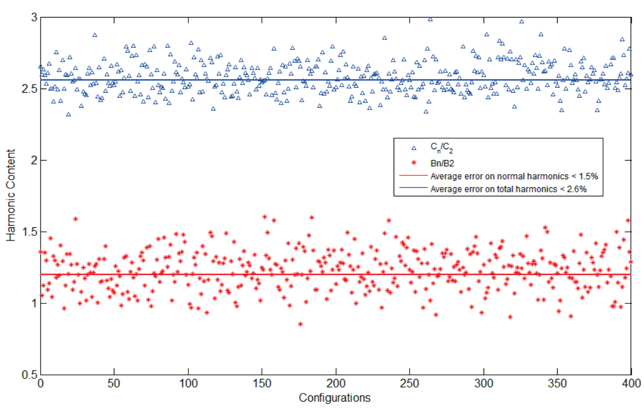

Figure 2: PMQ multipole content with random errors on magnets position and remanence.

It is important to note that the complex harmonics are more sensitive to the errors as they contain both normal and skew harmonics, respectively as real and imaginary parts. The even harmonics have a very small contribution coming from the skew components in this quadrupole, especially if compared to the values of the allowed harmonics B_6, B_{10} and B_{14}. In the odd harmonics, real and imaginary part of C_n are of the same order of magnitude.

CONCLUSION

A model for studying random errors in a permanent magnet quadrupole has been accomplished using COMSOL and MATLAB. It has been validated introducing controlled error sources and it results to be reliable. The model has been used for 2D study in order to set the maximum range of the error sources considered and then applied to the 3D model for the study of the magnetic features of the quadrupoles and the effects on the beam optics. The same model can be easily extended to any magnetic lens and more error sources, such as the angle of the remanent flux, can be introduced. More details are in [13] where also the effects on the beam dynamics are reported.

REFERENCES

[1] H. Daido *et al.*, “Review of laser-driven ion sources and their applications”, *Rep. Prog. Phys.*, vol. 75, p. 056401, 2012.

[2] S. Busold *et al.*, "Commissioning of a compact laser-based proton beam line for high intensity bunches around 10 MeV", *Phys. Rev. ST Accel. Beams*, vol. 17, p. 031302 (2014)

[3] S. Sinigardi *et al.*, " Transport and energy selection of laser generated protons for postacceleration with a compact linac", *Phys. Rev. ST Accel. Beams* vol. 14, p. 121301 (2011)

[4] S. Becker*et al.*, "Characterization and tuning of ultrahigh gradient permanent magnet quadrupole", *Phys. Rev. ST Accel. Beams*, vol 12, p. 102801 (2009)

[5] H. Sakaki *et al.*, "Simulation of Laser-Accelerated Proton Focusing and Diagnosis with a Permanent Magnet Quadrupole Triplet", *Plasma Fusion Res.* vol. 5, 009 (2010)

[6] M. Schollmeier*et al.*, "Controlled Transport and Focusing of Laser-Accelerated Protons with Miniature Magnetic Devices", *Phys. Rev. Lett.* vol. 101, p. 055004, (2008)

[7] K. Halbach, "Physical and optical properties of rare earth cobalt magnets", *Nucl. Instrum. Methods*, vol. 187, p. 109 (1981)

[8] T. Mihara *et al.*, "Superstrong Adjustable Permanent Magnet for a Linear Collider Final Focus", *Proc. of the 12th Linear Accelerator Conference*, SLAC Report No. SLAC-PUB-10878.

[9] F. Schillaci *et al.*, "ELIMED, MEDical and multidisciplinary applications at ELI-Beamlines", *J. Phys.: Conf. Ser.* vol. 508 p.012010 (2013)

[10] V. Scuderi *et al.*, "Development of an energy selector system for laser-driven proton beam applications", *NIMA, Proceeding of EAAC Workshop 2013*, vol. 740, p. 87-93, 2014.

[11] M. Maggiore *et al.*, "Transport and energy selection of laser produced ion beams for medical and multidisciplinary applications", presented at IPAC’14, Dresden, Germany, Jun. 2014, paper TUPME034.

[12] K. Halbach, "Design of Permanent Multipole Magnets with Oriented Rare Earth Cobalt Material", *Nucl. Instrum. Meth.*, vol. 169, p.1, (1980).

[13] F. Schillaci *et al.*, "Errors and optics study of a permanent magnet quadrupole system", *JINST*, vol. 10, p. T05001, 2015.

[14] J. Tanabe, "Iron Dominated Electormagnets Design, Fabrication, Assembly and Measurements", *World Scientific Publishing Co. Pte. Ltd.*, 2005.