

# A TIME DOMAIN ANALYSIS METHOD FOR RF NOISE

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## Abstract

A time domain analysis method is developed for the calculation of the longitudinal oscillations caused by the RF noise in the storage ring. This method is based on the impulse response model, and it could calculate the change of transient field caused by beam oscillation and RF noise turn by turn. By means of discrete spectrum analysis, the spectrum of the beam is obtained. The synchronous oscillation of the excited by high RF source with a phase modulation is predicted in simulation program, and the corresponding experimental measurements are carried out on HLS II. The fitting results are in agreed with the experimental measurements.

## INTRODUCTION

Phase and amplitude noises in the RF system excite the beam longitudinal oscillations, it produce the longitudinal emittance growth and debase the quality of the beam [1, 2].

The effects of RF noise has always been concerned by accelerator laboratories all over the world. Hitherto, many scholars have put forward a few calculation methods [3, 4], some of these method is quite effective, such as Pederesen models and iterative equations and so on. The field caused by the beam is changed due to the longitudinal oscillations of beam. Likewise, the change of field affects the beam movement turn by turn, so the relationship between them is really complicated.

Most of the conventional RF noise analyse method are based on the longitudinal motion equation

$$\frac{dW}{dt} = eU \sin[\phi + \psi_m \cos(\omega_m t + \theta)] - \tilde{W}_r$$

$$\frac{d\phi}{dt} = -\frac{1}{2\pi} \frac{\alpha \omega_r h}{p_0 R_0} (W - W_0)$$

These method assume the effective voltage  $V_{rf}$  seen by the particle as that

$$V_{rf} = I_g R_L \cos(\omega_{rf} t + m_f \sin(\omega_m t))$$

$$= V_0 \cos(\omega_{rf} t + m_f \sin(\omega_m t))$$

However, the RF noise usually effects the current signal, not the voltage signal. So it is not very accuracy to describe the  $V_{rf}$  as a voltage modulation. In these model, two significant considerations are ignored, which are (1) beam loading (consider the detune angle as zero) and (2) the various RF cavity impedance due to the different frequency of current signal.

Presently, the most synchrotron light source is operation above 200 mA, thus the beam loading is heavy, in order to avoid the Robison unstable region, the RF cavity should be correct detuned. The effective voltage  $V_{rf}$  is the sum of beam loading voltage and generator voltage, as it

is shown in figure 1. Therefore, both of these factors cause the difference between simulation and operation, when RF system is operating under high current. On the other hand, the impedance of RF cavity would be different according to the frequency of drive signal, so impedance for the sideband is different. To calculate the effective voltage precisely, firstly, the RF drive signal

$$\sin(\omega_{rf} t + m_f \cos(\omega_m t))$$

should be expanded in each frequency component, then multiply the respective impedance of cavity.

In this paper, these two factors are taken into account to improve the model and lead to correct and informative results when actual parameters are plugged into the model equations.

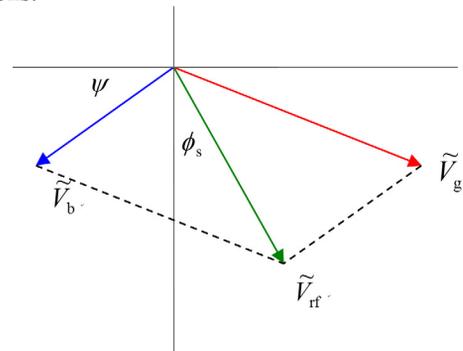


Figure 1: Voltage vector combination.

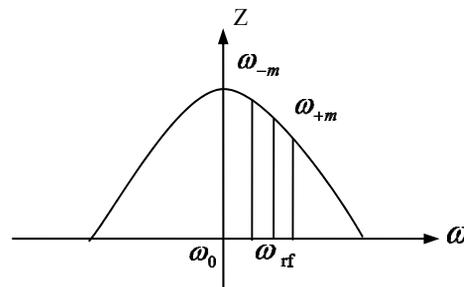


Figure 2: Impedance of RF cavity.

## PRINCIPLE OF IMPULSE RESPONSE MODEL

Inspired by the model of beam loading, if we discrete the continuous RF signal to a series of impulse, and prove the discrete signal excite the same RF field with the continuous one, then we could calculate the RF voltage by the impulse response model, which is very similar to the beam loading model.

Firstly, the discrete generator current signal could be written in Dirac function form. The generator current signal  $I_g$  is obtained

$$I_g(t) = e \sum_n \delta(t - nT_0 + \tau_s \cos \omega_s t)$$

Secondly, this function is investigated by Fourier transformation, the FT of  $I_g$  signal is given by

$$I_g(\omega) = e\omega_0 \sum_m j^{-m} J_m(\omega\tau_s) \sum_k \delta(\omega + m\omega_s - k\omega_0)$$

where the relation

$$e^{jx \cos \theta} = \sum_m j^m J_m(x) e^{jm\theta}$$

Compared the frequency spectrum of discrete signal with the continuous signal, it's easy to find that the discretization is periodic extension of the continuous signal in frequency domain, however, the RF cavity is a narrow band device, harmonic component could not excite the RF field in cavity, that mean both of the discrete and continuous signal excite the same RF field in narrow band cavity. So we could use the discrete signal to calculate the transient RF field.

Thirdly, according to the impulse response model, the RF field excited by the single impulse could be written as

$$V(t) = \frac{2\pi R}{Q} E e^{-\frac{t}{\tau}} e^{-i\omega_0 t}$$

Finally, the effective voltage seen by the particle is given by

$$\begin{aligned} \tilde{V}_c &= \tilde{V}_g + \tilde{V}_b \\ &= \sum_{t'=0}^t \frac{2\pi}{Q} (I_g(t') + I_b(t')) R \cdot e^{-\frac{t-t'}{\tau}} e^{j(\omega(t-t')+\varphi)} \end{aligned}$$

### SIMULATION AND EXPERIMENT

To verify the impulse response method, we make a simulation program, and the actual parameters of HLS II are plugged into the simulation program.

Table 1: Parameter of HLS II

Parametr	A4 Paper
Energy	800 MeV
RF frequency	204.03 MHz
Radiation loss	16.73 keV
Current	300 mA

The effect of LLRF system have not been considered in this simulation program, so experiment should be done under open loop operation. However, without the LLRF system, it is difficult to accumulate high current in the storage ring. So we carry out the experiment and simulation at low current in the first step.

#### Simulation

The By using this program, we calculate these parameters (1) beam loading voltage (2) transient generator voltage (3) tracking of phase and energy oscillations (4) frequency spectrum of beam. Figure 3 is the transient beam loading voltage,  $\tau$  is the field filling time of RF cavity, and  $\Psi$  is the detune angle. Figure 4 is the transient voltage excited by impulse signal (generator signal). Usually, the field is deemed to stable after triple filling time, however, in Fig. 3 and Fig. 4, there are still 0.92% and 3.2% deviation at triple filling time, so we use the value at sextuple

filling time as stable RF field. The phase and energy oscillation with RF modulation tracking is shown in Fig. 5 and Fig. 6. Due to synchrotron radiation damping and RF modulation, the phase and energy deviation damp down, but could not be zero, so there must be a hole in Fig. 5, and the oscillation of energy in Fig. 6 should be serrated. Figure 7 is the frequency spectrum of beam.

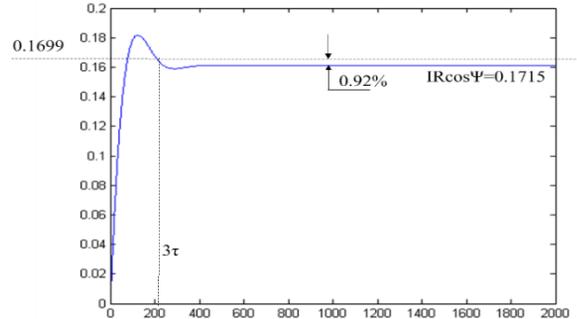


Figure 3: Transient beam loading voltage.

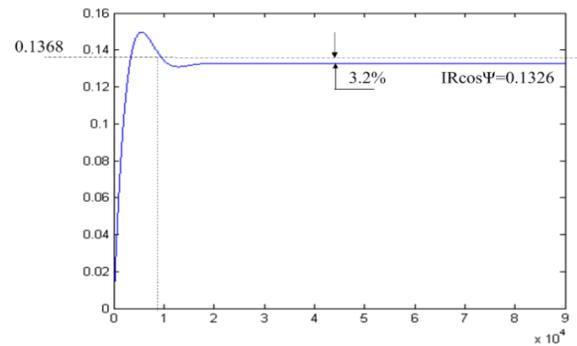


Figure 4: Transient generator voltage.

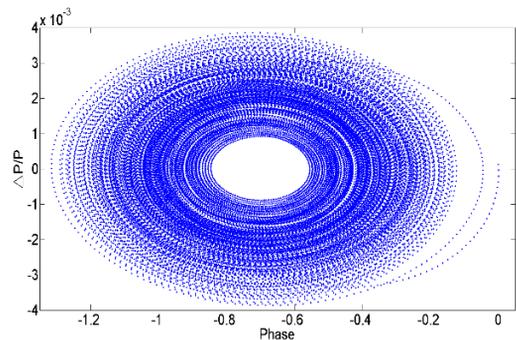


Figure 5: Phase space ellipse.

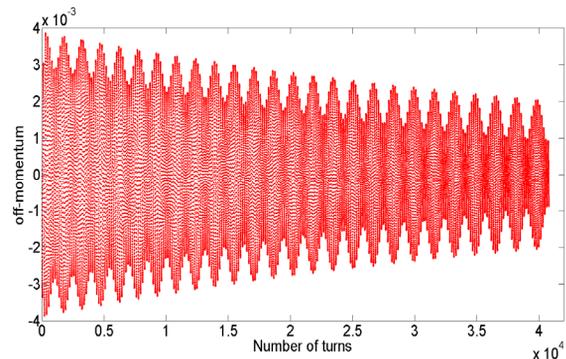


Figure 6: Tracking of energy oscillation.

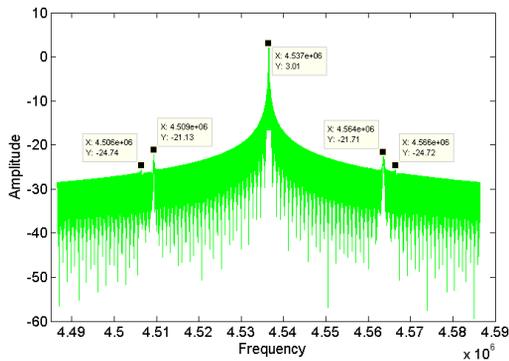


Figure 7: Frequency spectrum of beam.

### Experiment

The RF modulation experiment is done at HLS II storage ring. Because the IF signal is generated by RF signal passing through DDS (direct digital synthesizer), if we modulate the RF source directly, it would cause the signal distortions in the LLRF system, then effect the output of LLRF. So we insert a waveform generator and a modulator after the LLRF system, in order to modulate the RF signal. The layout is shown in Fig. 8.

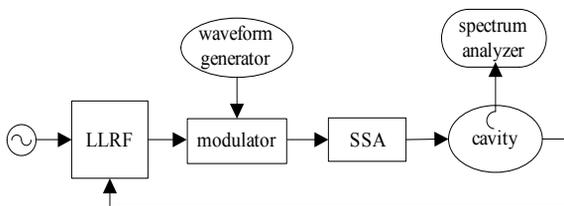


Figure 8: Layout of RF modulation experiment.

By modulating the RF voltage nearby the synchrotron frequency, the synchrotron oscillation can be excited [5, 6]. The frequency spectrum of beam is measured by TEK RSA 6100A spectrum analyzer. The measurement and the simulation are compared in the Table 2.  $m_f$  is modulation depth,  $f_m$  is modulation frequency,  $\Delta_-, \Delta_+$  is the amplitude difference between sideband and basic frequency,  $\Delta_m$  is the difference between the sidebands.

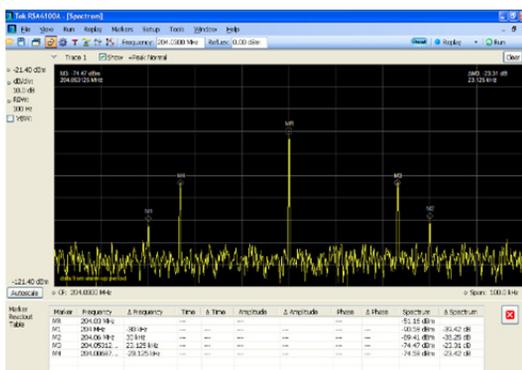


Figure 9: Frequency spectrum of beam with RF modulation.

Table 2: Results of Simulation and Measurement

Parameter	$m_f$	$f_m$	$\Delta_-$	$\Delta_+$	$\Delta_m$
Simulation	0.1	30	-27.56	-27.55	-0.01
Measurement	0.1	30	-28	-27.9	-0.1
Simulation	0.05	20	-26.7	-26.7	0
Measurement	0.05	20	-29	-32	3
Simulation	0.1	15	-25.32	-22.34	-2.98
Measurement	0.1	15	-26.4	-26.07	-0.33
Simulation	0.1	5	-21.26	-21.37	0.12
Measurement	0.1	5	-26.06	-26.06	0

### CONCLUSION

In this paper, we present a summary of impulse response model principle and simulation study. The relevant experiment is done at HLS II storage, the simulation is agreement with experiment. In next step, the effect of LLRF system would be considered in the simulation program, so we can compare simulation with experiment results at high current. With further more study, this method may help to optimise the RF noise in general operation.

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