SPECTRAL ANALYSIS OF TURN-BY-TURN DATA*

J. Choi†, BNL, Upton, NY 11973, USA

Abstract

With the recent technical developments, it is now popular to get the turn-by-turn data for the storage ring. Even though response matrix based analysis, like LOCO, have strong advantages in lattice analysis, the turn-by-turn data analysis is quite attractive because it takes very short time in data acquisition and many effective analyzing methods have been developed. Basically, such analysis requires accurate estimation of peaks of frequency spectra with high resolution. In this paper, we review the various accuratenesses of such estimations depending on processes using exact sinusoidal data and apply the end-matching method to simulation and measurement.

INTRODUCTION

Fourier Transformation (FT) is the main tool to obtain the tunes from the measured turn-by-turn data. Because the resolution of the tune from the raw data depends on the number of samples, we need more and more turns for the better and better resolution. However, there are certain limits in increasing the amount of data. First, the memory size is limiting the data amount. Second, if we are interested in the transition period, e.g. during the decoherence, it can happen that only several hundred or even several dozen turns should be processed.

Therefore, the interpolation and windowing methods are used to increase the resolution. The interpolation is a way of finding the more accurate peak position from several frequency spectrum with natural resolution limited by number of turns. Usual method are find the peak position from the values around the peak area [1] and for the data which is expected to have sinusoidal motion like storage ring turn-by-turn data, optimization is also used [2]. And, whatever method is used, for the given measurement, it leads the right direction and reduces the resolution.

However, the windowing is different. The windowing is introduced to improve the periodicity of the input signal. That is, the Fourier transform is performed under the assumption that the given turn-by-turn data is repeated over and over again, But, in reality, it is hardly satisfied because the both ends do not match in the usual case. These discrepancies bring about the leakage at the peak and make the resolution worse. To mitigate the situation, usually the windows smears out the both ends and improve the periodic property. Therefore, windowing surely reduces the leakage and, thereby, improves the resolution. However, the windows modifies spectrum itself, if not carefully applied, it can give the wrong tune with better resolution. Furthermore, the windowing always reducing the spectrum power even though it can increase signal to noise ratio. That is, when we repeat the measurements, windowings can give shorter error bar around the displaced peak point. Because we don't know the real value, we can misunderstood that the result is better because of the shorter error bars.

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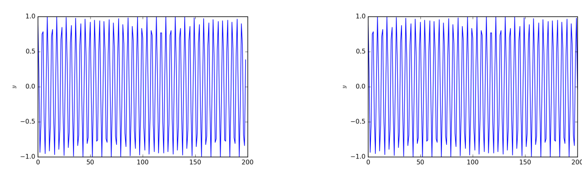
† jchoi@bnl.gov

FOURER TRANSFORM OF THE EXACT SINUSOIDAL SIGNAL

To see the effect of end points, we perform the discrete Fourier transform (DFT) for the exact sinusoidal signal, where we know the peak frequency, and compare the "measurement" with the real value.

Figures 1 and 2 are the wave signal of $y = \cos(2\pi n \cdot \nu)$ where $\nu = 0..2212769367$ which is very close to the fractional part of real NSLS-II horizontal tune. These are the same signals but Fig. 1 is shown up to $n = 199$ and Fig. 2 is shown up to $n = 201$. Even though there is just 2 points difference, end points of $n = 199$ case mach very well for periodicity while $n = 201$ case the end points are going in reverse ways.



(a) (b)

Figure 1: The sinusoidal signal up to (a) $n = 199$ and (b) $n = 201$.

Figure 2 shows the results of the frequency spectra for $n = 199$ and $n = 201$ cases and peak values with/without windowing and interpolation. For the windowing, we use Hanning window and the result does not significantly depend on the windowing method as far as the widowing is known going well with the given data type. For the interpolation we tried various methods [2] [3] and found Quinn2 method [3], which uses three points interpolation, consistently works well for any type of data and we use this method throughout the paper.

As can be seen, if the end values satisfy periodic condition, the peak leakage is very small and the windowing is not needed and if applied, the result is worse than that without the windowing. Even for the badly matching case, even though the peak leakage increased, the interpolation give very exact value and the windowing makes it worse. In both cases, the three points around peak for the interpolation are clearly identified and applying window only move the solution away from the real value.

Table 1 shows the accuracies for various number of turns with/without end-matching, interpolation, and windowing.

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Table 1: Tune Accuracies for Exact Sinusoidal Data for the Number of Turns with/without End-Matching, Interpolation, and Windowing

Turns	**20000**	**15162**	**1024**	**1026**	**300**	**303**	**30**	**27**
DFT	2.3×10^{-5}	6.0×10^{-8}	4.0×10^{-4}	2.9×10^{-5}	1.3×10^{-3}	1.5×10^{-4}	1.2×10^{-2}	9.5×10^{-4}
+Quinn2	8.3×10^{-10}	4.7×10^{-15}	4.8×10^{-7}	2.8×10^{-9}	4.3×10^{-6}	9.3×10^{-8}	7.4×10^{-4}	9.0×10^{-6}
+Hanning	8.6×10^{-6}	2.2×10^{-8}	1.5×10^{-5}	1.1×10^{-5}	4.8×10^{-4}	5.7×10^{-5}	4.9×10^{-3}	3.8×10^{-4}

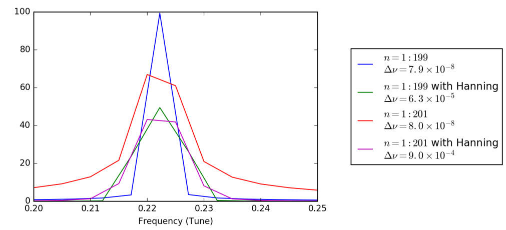


Figure 2: The spectra of DFT from the exact sinusoidal signal and solutions from with/without interpolation and windowing.

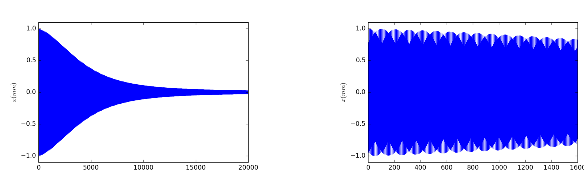
For the exact sinusoidal motion, windowing always makes the interpolated solutions worse. We can see that, if end points are matched, as for the shaded columns in the table, even using the small number of turns, we can get quite accurate results.

SIMULATION DATA

We applied the end-point matching method to the simulation data. Figure 3 shows the result of 20000 turns of 6d simulation using N2Track [4]. To add decoherence effect, we tracked 1000 particles and used the average values. However, collective effects are not included.

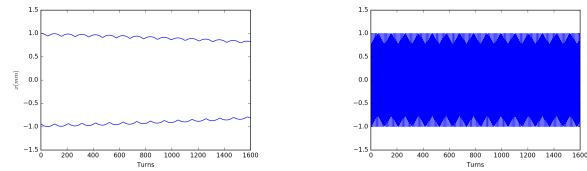
Because of the decoherence and damping envelope of the turn-by-turn data, it is not possible to find the data set with end points making the data periodic. Therefore, we find the envelope numerically and decompose the raw data into envelope and pure betatron oscillation. Figure 4 shows the decomposition for the first 1600 turns to show more detail. Now we can find the data sets with end points are matched and we perform the Fourier transform for the matched data sets.

We did not include any error in the simulation. Therefore, the data are very clean and we could perform Fourier transform even with 40 turns and 20 turns are overlapped (running window). The result is shown in Fig. 5.



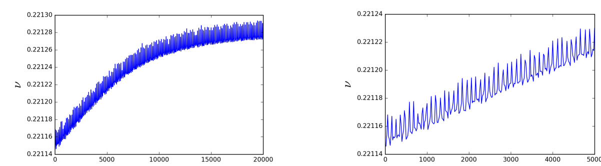
(a) (b)

Figure 3: Simulated turn-by-turn data with decoherence for (a) 20000 turns and (b) 1600 turns to show more detail.



(a) (b)

Figure 4: (a) The envelope and (b) betatron oscillation for the simulated data.



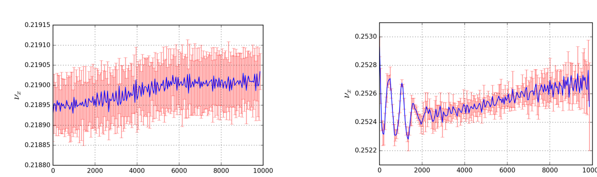
(a) (b)

Figure 5: Horizontal tune variations for the simulated turn-by-turn data when 40 turns with 20 turns overlapped are used for the Fourier transform. For (a) all 20000 turns and (b) first 1600 turns to show more detail.

MEASURED DATA

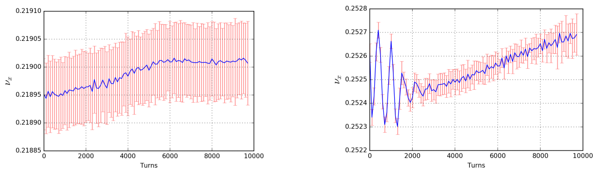
We applied the same method to the measure turn-by-turn kicked data with pingers. Because of the noise, it was not feasible to find envelopes and we used FastICA [5] to increase the signal to noise ratio. Also, the number of turns for the Fourier transform should be increased at least to 100 turns.

Figure 6 shows the horizontal and vertical tune variations with Fourier transform of 100 turns and 50 turns overlapped. Figure 7 shows the horizontal and vertical tune variations with Fourier transform of 200 turns and 100 turns overlapped. The study about the variations are now undergoing.



(a) (b)

Figure 6: (a) Horizontal and (b) vertical tune variations for the measured turn-by-turn data when 100 turns with 50 turns overlapped are used for the Fourier transform.



(a) (b)

Figure 7: (a) Horizontal and (b) vertical tune variations for the measured turn-by-turn data when 200 turns with 100 turns overlapped are used for the Fourier transform.

SUMMARY

When three peak points can be identified in the frequency spectrum, the interpolation can be effectively used and there is no guarantee that windowing helps to obtain more accurate peak values. Windowing will help when the peak and neighboring two points are not clear because of the noise and/or native high resolution. In this case, by windowing the data, the peak area of the spectrum will be narrowed and make three points more visible. And then these points can be effectively used by the interpolation.

On the other hand, by selecting the periodic data set, more accurate solution with higher resolution can be obtained without losing any information.

ACKNOWLEDGMENT

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