# PROPERTIES OF SYNCHROTRON RADIATION FROM SEGMENTED UNDULATORS BASED ON A WIGNER DISTRIBUTION FUNCTION

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*Abstract*

Three long straight sections with a double mini-$\beta_y$ lattice were designed in Taiwan Photon Source. For the purpose to understand whether the brilliance can be enhanced or not when two collinear undulators were installed in the double mini-$\beta_y$ section. The method of Wigner distribution function (WDF) is a natural way to describe a synchrotron radiation source. The brilliance is thereby calculable without a Gaussian approximation used in a conventional manner. Hence the WDF is developed to calculate the brilliance for the two collinear undulators in the double mini-$\beta_y$ section. Some important optical properties such as the degree of coherence can be directly calculated with this method. We use it as an example to investigate the properties of radiation from a segmented undulator.

## INTRODUCTION

Long undulators are installed in a storage ring to generate highly brilliant synchrotron radiation, but, for the characteristic parabolic shape of the β function in a straight section, the average size of the electron beam increases with the length of an undulator; the brilliance per unit length of an undulator thereby decreases. One way to decrease the average size of the electron beam is to add a set of quadrupole magnets in the middle of a straight section to suppress the β function in either vertical or horizontal direction so as to decrease the electron beam size. Three 12-m straight sections with a double mini-$\beta_y$ lattice [1] were built in Taiwan Photon Source (TPS) with two collinear undulators located at the minimum of the vertical β function (figure 1) was supposed to increase the brilliance. The average vertical beam size of two 3-m segmented undulators in a double mini-$\beta_y$ lattice is 55 % of a 6-m single undulator in the original lattice with a single minimum β function.

A Gaussian approximation is used in a conventional calculation of the spectrum for the diffraction-limit relation ($\sigma_r\sigma_\theta = \lambda/4\pi$) and the convolution of the photon beam from a single electron and the electron beam distribution, but it is inapplicable for a double mini-$\beta_y$ lattice because of the non-Gaussian characteristic of the radiation from the segmented undulators; the shape of the

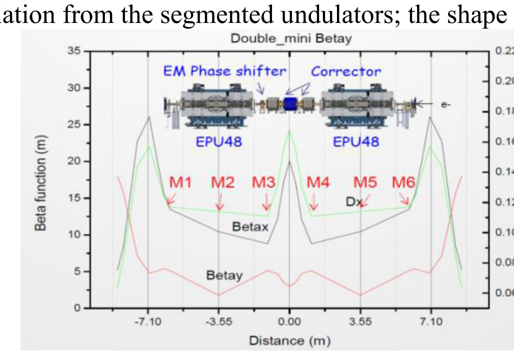


Figure 1: β function of a double mini-$\beta_y$ lattice[1].

flux density depends on the phase between the two pulses (see figure 2) of electrons in phase space at the entrance of the first undulator, which determines the length of the trajectories between the two undulators that affect the phase. The conventional way to estimate the brilliance of radiation from an undulator is thus unsuitable for a light source of this type.

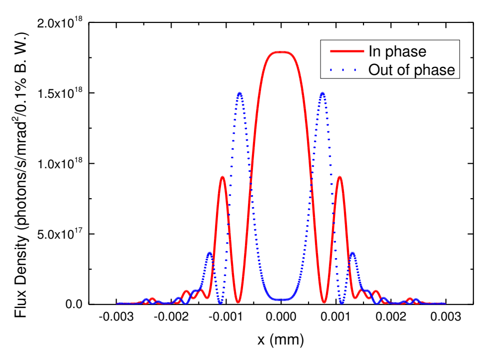


Figure 2: Flux density from segmented undulators in zero emittance electron beam.

## WIGNER DISTRIBUTION FUNCTION

To treat a synchrotron light source, Kim [2] introduced the Wigner distribution function (WDF) as a general method to describe quantitatively such a source. The characteristics of the WDF and the relations between its

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quantum and optical formalisms have been reviewed in Bazarov's work [3]. There are several definitions of the

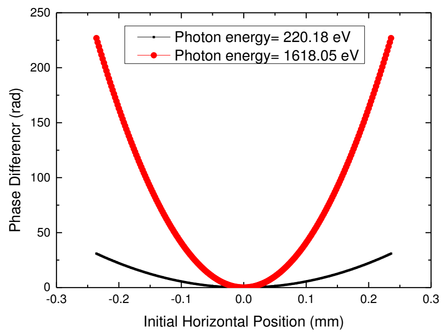


Figure 3: Phase difference of two pulses from segmented undulators for which the initial angle is set to zero; the upper and lower bounds are twice the beam size.

brilliance according to the WDF [3]; the brilliance on axis (the WDF at the origin of a 4D-transverse phase space) is chosen as the definition of brilliance in this paper. The mathematical form of the WDF and of the WDF on axis follow,

$$W(\boldsymbol{r},\boldsymbol{\theta}) = (\tfrac{1}{\lambda})^2 \int E(\boldsymbol{r}-\tfrac{\boldsymbol{r}'}{2})E^*(\boldsymbol{r}+\tfrac{\boldsymbol{r}'}{2})e^{ik\boldsymbol{r}'\cdot\boldsymbol{\theta}}\, d^2\boldsymbol{r}' \quad (1)$$

$$W_{on-axis} = W(\mathbf{0},\mathbf{0}) = (\tfrac{1}{\lambda})^2 \int E(-\tfrac{\boldsymbol{r}'}{2})E^*(\tfrac{\boldsymbol{r}'}{2})\, d^2\boldsymbol{r}' \quad (2)$$

in which $\boldsymbol{r} = (x,y)$ and $\boldsymbol{\theta} = (\theta_x,\theta_y)$ are the transverse position and angle respectively with the electric field in the frequency domain. The electric field generated from the undulators is calculated with the following equation,

$$E(\boldsymbol{r};\omega) = \frac{ie\omega}{4\pi\varepsilon_0 c}\int \frac{1}{R}[\boldsymbol{\beta} - \boldsymbol{n}(1+\frac{ic}{\omega R})]e^{i\omega(\tau+\frac{R}{c})}d\tau \quad (3)$$

in which $\boldsymbol{R}(\tau) = \boldsymbol{r} - \boldsymbol{r_e}(\tau)$ is the vector from the position of an electron to the observer, $\boldsymbol{\beta} = d\boldsymbol{r_e}/d\tau\,/c$, $\boldsymbol{n} = \boldsymbol{R}/R$ and $R = |\boldsymbol{R}|$. An ideal thin lens is put before the observer plane to remove the quadratic phase in the radiation so as to decrease the number of points required to be calculated, to save computing time [3]. The random property of an electron beam produces a result that the interference term of the electric field of the pulses from each electron cancels; only an incoherent summation term is left. The total WDF is thus a summation of the WDF of the pulse from each single electron,

$$W(\mathbf{r},\boldsymbol{\theta}) = \textstyle\sum_{N_e} W_{se}(\mathbf{r},\boldsymbol{\theta}) \quad (4)$$

in which $W_{se}$ is the WDF of the pulse from a single electron; $N_e$ is the number of electron passing through the undulator per second.

In addition, the distribution of the electric field on the observer plane is assumed to be unchanged but only shifted in position for the electrons off axis in an electron beam with finite emittance. With this assumption, the total WDF can be calculated on convoluting the WDF with the distribution of the electron beam for a single undulator. The radiation from a double mini-$\beta_y$ lattice with double undulators cannot be treated with that method because the quadrupole magnets between the undulators alter the orbits of the electrons; each electron with varied angle and initial position generates radiation with a distinct distribution on the observer plane, and the phase between two pulses depends on the trajectory. The distribution of electric field of the radiation from each undulator is still the same, but there is a phase difference between them that is then calculated with the difference of the path lengths of the trajectories between the electron on axis and each other electron.

For circularly polarized light generated with an elliptical undulator, the WDF is defined as follows [3]:

$$W(\mathbf{r},\boldsymbol{\theta},\Omega) = \boldsymbol{\Omega}\cdot\mathbf{S}(\mathbf{r},\boldsymbol{\theta}) \quad (5)$$

$$\boldsymbol{\Omega} = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3}sin\chi cos\varphi \\ \sqrt{3}sin\chi sin\varphi \\ \sqrt{3}cos\chi \end{pmatrix} \quad (6)$$

$$\begin{aligned} S_0(\mathbf{r},\boldsymbol{\theta}) &= W_{xx}(\mathbf{r},\boldsymbol{\theta}) + W_{yy}(\mathbf{r},\boldsymbol{\theta}) \\ S_1(\mathbf{r},\boldsymbol{\theta}) &= W_{xy}(\mathbf{r},\boldsymbol{\theta}) + W_{yx}(\mathbf{r},\boldsymbol{\theta}) \\ S_2(\mathbf{r},\boldsymbol{\theta}) &= i\left[W_{xy}(\mathbf{r},\boldsymbol{\theta}) - W_{yx}(\mathbf{r},\boldsymbol{\theta})\right] \\ S_3(\mathbf{r},\boldsymbol{\theta}) &= W_{xx}(\mathbf{r},\boldsymbol{\theta}) - W_{yy}(\mathbf{r},\boldsymbol{\theta}) \end{aligned} \quad (7)$$

$$W_{kl}(\mathbf{r},\boldsymbol{\theta}) = (\tfrac{1}{\lambda})^2 \int E_l^*(\boldsymbol{r}-\tfrac{\boldsymbol{r}'}{2})E_k(\boldsymbol{r}+\tfrac{\boldsymbol{r}'}{2})e^{ik\mathbf{r}'\cdot\boldsymbol{\theta}}\, d^2\mathbf{r}' \quad (8)$$

The WDF is a vector mapping ($\Omega$) on the Poincaré sphere of generalized Stokes's parameters ($S$), with components $S_0$, $S_1$, $S_2$ and $S_3$ corresponding to the intensities of total flux, linearly polarized in horizontal and vertical, linearly polarized at $\pm 45^{\circ}$, circularly polarized light, respectively.

## NUMERICAL CALCULATION

In the calculation of a WDF for an electron beam with finite emittance, distribution $E(\boldsymbol{r};\omega)$ of the electric field on the observation plane 30 m from the middle of the straight section of an electron on axis from each segmented undulator was initially calculated and set to be in phase. 1000 electrons were generated randomly by the distribution function of an electron beam in phase space at the entrance of the first undulator; the trajectories between the undulators were then calculated with a matrix method for each electron, and the path lengths were compared with that on axis to obtain the phase difference. The electric field was derived by the shift in the position according to the initial position and angle of an electron in phase space at each entrance of the undulators and a shift in phase due to the phase difference of the electric field of the one on axis. The WDF on axis was calculated for the pulses from each electron and summed incoherently.

## NUMERICAL RESULTS

The following results are based on elliptical polarized undulator with period length 48 mm. The angular flux densities on axis and the brilliance of a single undulator from the authors' numerical code were compared with the results of a conventional method with SPECTRA [4] in figures 4 and 5. The brilliance of the zero-emittance beam for a circularly polarized mode with the authors' code is about 1.3 times larger than the conventional method. For

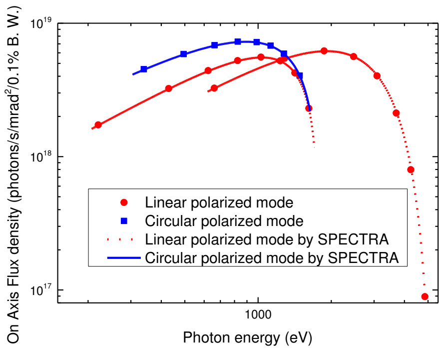


Figure 4: Comparison of on-axis flux density of a single undulator.

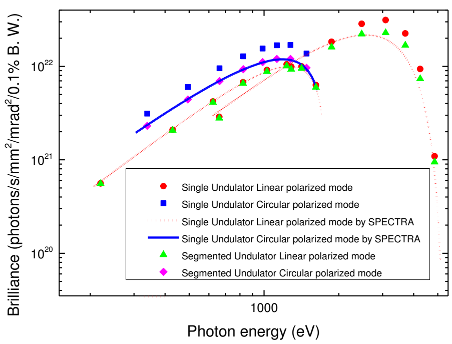


Figure 5: Comparison of brilliance of an electron beam of zero emittance.

the linearly polarized mode the results from both single undulator and segmented undulator are consistent except in the region, at high energy, of the third harmonic. The brilliance of a segmented undulator was smaller than that of a single undulator (figure 6). Because the brilliance of a segmented undulator decreased more rapidly than that of a single undulator as the initial position and angle are away from zero (figures 7 and 8).

## CONCLUSION

The optical properties of the radiation from segmented undulator have been studied base on the WDF. Due to triple quadrupole magnets are located between the segmented undulator in the double mini-$\beta_y$ straight section. The distribution of the electric field on the observer plane become complicated and hence could not be analysed by conventional method. This issue can naturally be treated by the WDF. For an electron beam of zero emittance, the brilliances of the linear polarization in a single and a segmented undulator are almost the same. However, the brilliances of the circular polarization in a segmented undulator is smaller than that of a single undulator obviously. If we consider the finite emittance, the brilliance both in linear and circular polarization of a segmented undulator will be much smaller than that of a single undulator.

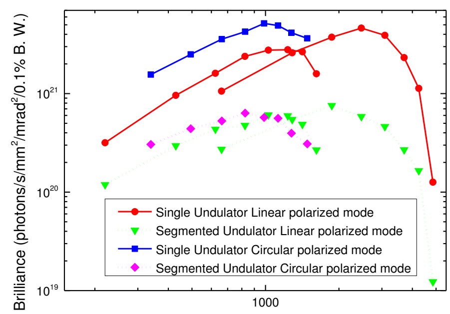


Figure 6: Comparison of brilliance of an electron beam of finite emittance(solid curve: single undulator, dot curve: segmented undulator).

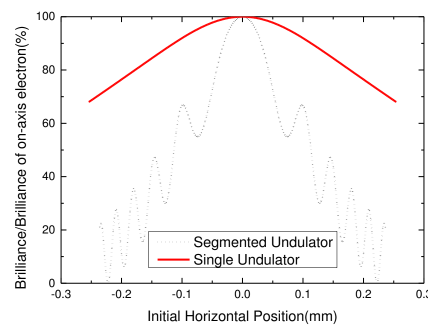


Figure 7: Comparison of brilliance of a single and of a segmented undulator for varied initial horizontal position in phase space (with initial horizontal and vertical angle, and vertical position are all zero).

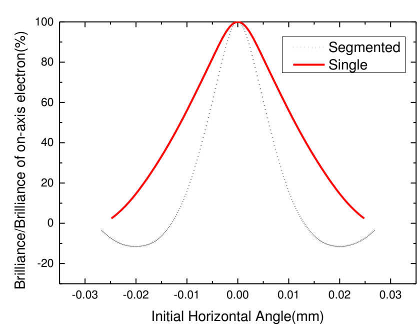


Figure 8: Comparison of brilliance of a single and a segmented undulator for varied initial horizontal angle in phase space (in this case, the initial horizontal position is varied but keep zero of vertical angle and position).

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