

# EXPLORING THE ULTIMATE LINEAR AND NONLINEAR PERFORMANCE OF THE HEPS HYBRID 7BA DESIGN\*

Y. Jiao<sup>†</sup>, G. Xu, Key Laboratory of Particle Acceleration Physics and Technology, Institute of High Energy Physics, Beijing 100049, China

## Abstract

The High Energy Photon Source (HEPS), a kilometre-scale diffraction-limited storage ring (DLSR) light source, with a beam energy of 5 to 6 GeV and transverse emittances of a few tens of pm.rad, is to be built in Beijing. We have obtained a hybrid 7BA lattice design, with a natural emittance of about 60 pm.rad and a circumference of about 1.3 kilometres, basically satisfying the requirement of on-axis longitudinal injection in HEPS. Nevertheless, it is interesting and necessary to explore the ultimate linear and nonlinear performance of the HEPS hybrid 7BA design. In this paper, we will introduce the multi-objective optimization with a successive and iterative implementation of the MOPSO and MOGA algorithms, and discuss certain relations between the nonlinear dynamics and linear optics of a hybrid MBA lattice. This study can provide reference for other DLSR lattice design and optimizations.

## INTRODUCTION

In the past few decades, the third generation light sources (TGLSs), based on electron storage rings with natural emittance of a few nm.rad, have been the most widely used platforms providing high quality photon beam for fundamental researches in physics, chemistry, materials science, biology, medicine, etc. Nevertheless, people never stop in pursuing better sources. Accelerator scientists proposed to construct the diffraction-limited storage rings (DLSRs) [1] with the aim to push brightness and coherence beyond the limits of existing third generation light sources (3GLSs), by reducing the emittance to approach the diffraction limit for the range of X-ray wavelengths of interest to the scientific community. Due to the high performance of a DLSR, many laboratories are now constructing storage rings with natural emittance of a few hundred pm.rad (e.g., MAX-IV [2] and Sirius [3]), or seriously considering to build new ultralow-emittance rings or to upgrade existing ring-based light sources to DLSRs.

A kilometre-scale storage ring light source with a beam energy of 5 to 6 GeV, named the High Energy Photon Source (HEPS), is to be built in Beijing. Recently a lattice design [4] based on the concept of ‘hybrid MBA’ [5] was developed for the HEPS. The ring consists of 48 identical hybrid seven-bend achromats (7BAs), with a natural emittance of 59.4 pm.rad at 6 GeV, a circumference of 1296 m and long straight sections of 6 m for insertion devices (IDs). By means of sextupole strength minimization and tune space survey, we attained large enough effective

momentum acceptance (MA, ~3%) and effective dynamic aperture (DA, ~2.5 mm in  $x$  and 3.5 mm in  $y$  plane) for on-axis longitudinal injection in HEPS [6]. Since only the ideal lattice was used in numerical tracking, we used the ‘effective’ DA and MA to measure the ring acceptance of a practical machine. Within the effective DA or MA, it is required that not only the motion remains stable after tracking of a few thousand turns, but also the tune footprint is bounded by the integer and half integer resonances closest to the working point. For this design (denoted as ‘mode I’), the layout and optical functions of a single 7BA are shown in Fig. 1 and typical parameters of the ring performance are listed in Table 1. More details of the ‘mode I’ design and related studies can be found in Ref. [7].

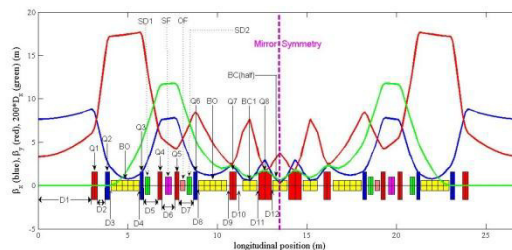


Figure 1: Optical functions and layout of a single hybrid 7BA of the HEPS preliminary design.

Nevertheless, before fixing this design as the final design of the HEPS project, it is necessary and also interesting to explore the potential of such a hybrid 7BA lattice, for instances, to look at the achievable minimum emittance and the maximum DA and MA at a specific emittance, while keeping the circumference basically unchanged, i.e., varied in +/- 1 m. It is also worthy to check whether there exist some general relations between the nonlinear performance and linear optical parameters. To this end, we optimized the ring performance using the multi-objective genetic algorithm (MOGA) and multi-objective particle swarm optimization (MOPSO) algorithms (e.g., [8-11]). In the optimization, all the possibly tuneable element parameters except the ID section length were globally scanned, including the lengths and positions of all magnets, bending angles of dipoles and gradients of quadrupoles and multipoles, within specific ranges that are determined by practical or optical constraints. Also, by comparing the performance of these two algorithms in Ref. [12], we demonstrated that for an explorative and complicated multi-objective problem with many optimizing variables and local optima (like optimization of a DLSR design), evolving the population with a successive and iterative implementation of MOPSO and MOGA would be more effective than using either of these two algorithms.

\* Work supported by NSFC (11475202, 11405187)

<sup>†</sup> jiaoyi@ihep.ac.cn

## OPTIMIZATION OF THE EMITTANCE AND SEXTUPOLE STRENGTHS

First, we looked at the optimal trade-offs between natural emittance  $\varepsilon_{x0}$  and sextupole strengths (represented with a nominal strength  $K_s$ ) with MOPSO and MOGA (see [12] for detailed description of the optimization). The quadrupole lengths were optimized such that all the quadrupole gradients are below but close to their upper limits.

The final solutions are shown in Fig. 2 (a). To look at the available maximum ring acceptance at different emittance range, the solutions were separated into six parts marked with different colours, and in each part the available maximum scaled DA area (with effective MA of not less than 3%) was obtained by comparing those with different tunes and plotted in Fig. 2 (b). Here the scaled DA area (in unit of  $\text{mm}^2$ ), i.e., the product of the horizontal and vertical effective DA sizes normalized with respect to the square root of the values of beta functions, is defined and used to measure the nonlinear performance of the ring.

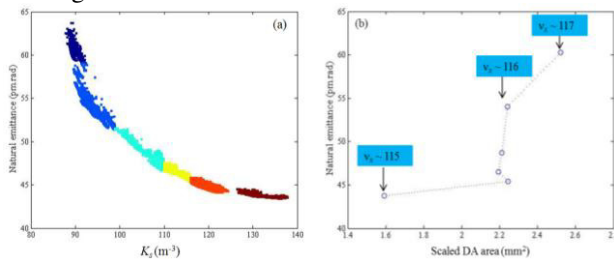


Figure 2: Solutions projected on the  $\varepsilon_{x0}$ - $K_s$  plane, and the available maximum scaled DA area at different emittance range.

It appears feasible to find solutions with scaled DA area larger than that of the ‘mode I’ design ( $1.63 \text{ mm}^2$ ), and at the same time, with emittance below  $60 \text{ pm.rad}$ . Nevertheless, the figure does not show a monotonous variation of the scaled DA area with the emittance as for  $K_s$ . The horizontal integer tunes also have significant impacts on the available scaled DA area. This suggests that the level of sextupole strengths is not the decisive and exclusive factor of the nonlinear performance of the ring, and the sextupole strength minimization followed with tune space scan may be not the best way to maximize the ring acceptance. Therefore we then implemented MOPSO and MOGA optimizations using directly the emittance and ring acceptance as optimizing objectives.

## OPTIMIZATION OF THE EMITTANCE AND RING ACCEPTANCE

In spite the solutions obtained above do not show optimal trade-offs between the emittance and ring acceptance, they provide a good start point for the new optimizations which use directly the emittance and ring acceptance as optimizing objectives.

In the new optimizations, the multipoles were split into eight families to provide six free knobs for nonlinear optimization. In this case the nonlinear performance was valued with the scaled ring acceptance (in unit of  $\text{mm}^2$ ),

i.e., the product of the scaled DA area and the effective MA (normalized by 3%). Totally 32 optimizing variables and two objectives (weighted emittance and scaled ring acceptance) were used. The initial solutions are selected from the solutions obtained above predicting larger scaled DA area than the ‘mode I’ design. To evaluate the objectives numerical tracking and frequency map analysis are needed, which takes a longer time ( $\sim 60 \text{ s}$ ) than that for the evaluation just in linear optics regime. Nevertheless, to ensure the comprehensiveness of the solutions, we chose a relatively large population size of 4000.

Besides, to ensure the effectiveness of the optimization, as many constraints as possible were considered and strict criteria for the evaluation of the effective DA and MA were imposed. The constraints include a reasonable maximum value of beta function  $\beta_{x,y}$  [ $\max(\beta_{x,y}) \leq 30 \text{ m}$ ], reasonably low beta functions in ID section for high brightness ( $1.5 \text{ m} < \beta_y < 4 \text{ m}$  and  $1.5 \text{ m} < \beta_x < 15 \text{ m}$ ), stability of the optics [ $\text{Tr}(M_{x,y}) < 2$ , with  $M_{x,y}$  being the transfer matrix of ring in  $x$  or  $y$  plane], fractional tunes in  $(0, 0.5)$  that is favourable against the resistive wall instability, reasonable natural chromaticities ( $|\xi_{x,y}| \leq 5.5$  in one 7BA), etc. It was required that within the effective DA or MA, the tunes varied with betatron oscillation amplitude or momentum deviation should be away from the integer resonances by at least 0.05 and from the half integer resonances by 0.01, or namely the fractional tunes should be within  $[0.05, 0.49]$ ; the fractional tunes of the nominal working point should be separated by at least 0.015, to avoid particles trapped by coupling resonances due to the space charge effect; based on similar consideration, the tune variation curves should not be crossed within the effective MA to avoid possible trapping in coupling resonances during the longitudinal injection; and the upper limit of the sextupole strength is set to  $280 \text{ m}^{-3}$ , by assuming a larger magnet pole radius for the sextupoles ( $14 \text{ mm}$ ) than for quadrupoles ( $12.5 \text{ mm}$ ), with the aim to provide enough room for extracting the photon beam coming from the upstream IDs.

The sextupoles lengths were optimized by iterative implementations of the MOPSO and MOGA algorithms. And, during the iterations the emittance range of interest was gradually reduced such that more and more solutions had emittance of about  $60 \text{ pm.rad}$  or even lower.

Here we present only that the solutions of the last iteration of the MOPSO and MOGA in Fig. 3. It shows that in spite of limited tuning ranges of optimizing variables and various constraints in the optimization, the final solutions distribute nearly continuously in the objective space, and clearly show almost a monotonous variation of the scaled ring acceptance with the emittance. It appears that for the HEPS hybrid 7BA design,  $50 \text{ pm.rad}$  is about the turning point. As  $\varepsilon_{x0}$  is smaller than  $50 \text{ pm.rad}$  the scaled ring acceptance decreases rapidly with the emittance, while changes with a much smaller slope as  $\varepsilon_{x0}$  is above  $50 \text{ pm.rad}$ . This suggests that for HEPS it is best to choose a design with emittance of above  $50 \text{ pm.rad}$  to achieve a robust nonlinear performance, i.e., having a high tolerance to small deviations in linear optics.

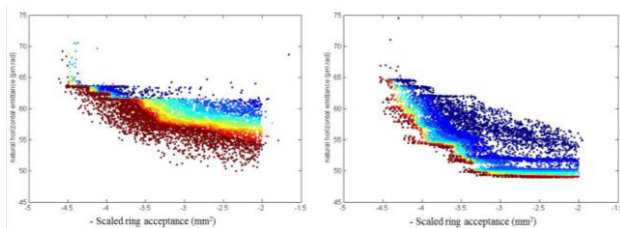


Figure 3: Solutions of the last iteration of MOPSO (left) and MOGA (right) in the objective space. The population is plotted after every 100 generations and marked with different colours.

The PSO solutions from generation 400 to 1000 and MOGA solutions from generation 100 to 1000 were selected for post analysis. We re-evaluated these solutions to obtain linear and nonlinear performance parameters, and kept those with scaled acceptance of above  $1.5 \text{ mm}^2$ , MA of above 3% and  $\varepsilon_{x0}$  of below  $65 \text{ pm.rad}$ . We could then view these solutions in different sub-parameter space. Due to limited space, those plots are not shown here.

It was found that the solutions cover quite a large integer tune range, i.e., from 111 to 112 in  $x$  and from 39 to 42 in  $y$  plane. Note that the covering range of the horizontal integer tunes is entirely different from that for optimizing the emittance and sextupole strengths (from 115 to 117). Further study showed that the initial population with large horizontal integer tunes were gradually phased out with MOPSO due to their relatively smaller scaled ring acceptance. This proves again that MOPSO is powerful in generating new solutions with different characteristic parameters from the existing solutions.

Further study revealed that the integer tunes are closely related to the degree of the transverse focusing and the emittance; and they are also closely coupled with the natural chromaticities, dispersion at sextupoles, and the level of the sextupole strengths.

It was also found that the fractional tunes are dominant factors of the MA size. Choosing fractional tunes at the left bottom of the tune space can increase the effective MA up to a maximum of 3.9%.

Thus, to simultaneously attain an ultralow emittance and a largest possible ring acceptance, it needs to reach a balance among many parameters, especially the sextupole strengths and the integer and fractional tunes.

Since it is very likely to use on-axis longitudinal injection for the HEPS, the main goal of the nonlinear performance is to attain as a large MA as possible, while keeping the effective DA large enough for on-axis injection (e.g., not smaller than 50 times of the root mean square of transverse beam size at the injection point). To this end, for each specific integer tune region, we did further numerical tracking and FMA for the solutions predicting the largest MAs, rather than the largest scaled ring acceptance. One typical solution (denoted as ‘mode II’) is also listed in Table 1. Compared to the ‘mode I’ design, this design has both lower emittance and larger MA, and uses shorter multipoles. Additionally, the fractional tunes of the working point are well separated, and all the drift lengths are larger than 0.1 m.

It is worthy to note that in the integer tune of (112, 40), the coupling resonance  $2\nu_x - 2\nu_y = 48 \times 3$  is a low order structural resonance and causes a significant perturbation to dynamics. Thus, the solutions with integer tune of (112, 40) are not considered as candidate optimal designs for the HEPS.

Table 1: Typical Parameters of the Original and Optimized Hybrid 7BA Designs for the HEPS Project

Modes	Mode I	Mode II
Working point (H/V)	116.16/41.12	112.13/41.14
$\beta$ at ID section (H/V)	9/3.2 m	8.0/3.1 m
Natural chromaticities	-214/-133	-140/-137
$\varepsilon_{x0}$ (pm.rad)	59.4	55.8
Eff. MA	3%	3.7%
Eff. DA size (H/V)	2.5/3.5 mm	3.0/2.0 mm

## CONCLUSION

Taking the HEPS design as an example, in this paper we show how to effectively explore the potential of a hybrid MBA lattice from a specific design, while not necessarily requiring a deep understanding of the physics behind the lattice design and of subtle relations between the nonlinear dynamics and linear optics. The key point is to evolve the population with MOPSO and MOGA in a successive and iterative way. Since MOPSO has intrinsic ability of breeding more diversity to the population during evolution, combining MOPSO and MOGA in the optimization will be more effective and powerful in searching the global optima than using either of the two algorithms.

## REFERENCES

- [1] R. Hettel, *Journal of Synchrotron Radiation*, **21**: 843-855 (2014).
- [2] P.F. Tavares *et al.*, *Journal of Synchrotron Radiation*, **21**: 862-877 (2014).
- [3] L. Liu, *et al.*, *Journal of Synchrotron Radiation*, 2014, **21**: 904-911 (2014).
- [4] Y. Jiao, *Chin. Phys. C*, **40** (7): 077002.
- [5] L. Farvacque *et al.*, in *Proc. IPAC'13*, pp. 79-81.
- [6] G. Xu *et al.*, “On-axis beam accumulation enabled by phase adjustment of a double-frequency RF system for diffraction limited storage rings”, presented at IPAC'16, Busan, Korea, May 2016, WEOAA02.
- [7] G. Xu *et al.*, “Recent physical studies for the HEPS project”, presented at IPAC'16, Busan, Korea, May 2016, WEPOW026.
- [8] I. Bazarov and C.K. Sinclair, *Phys. Rev. ST Accel. Beams*, **8**: 034202(2005).
- [9] L. Yang *et al.*, *Nucl. Instr. Meth.*, 609 (2009) 50.
- [10] X. Pang and L.J. Rybarczyk, *Nucl. Instr. Meth.*, 741 (2014) 124.
- [11] X. Huang and J. Safranek, *Nucl. Instr. Meth.*, 757 (2014) 48.
- [12] Y. Jiao and G. Xu, “Comparing the performance of MOGA and MOPSO in optimization of the HEPS performance”, presented at IPAC'16, Busan, Korea, May 2016, HPMB019.