INFLUENCE OF FILLING PATTERN STRUCTURE ON SYNCHROTRON RADIATION SPECTRUM AT ANKA

J. L. Steinmann*, E. Blomley, M. Brosi, E. Bründermann, M. Caselle, N. Hiller†, B. Kehrer, A.-S. Müller, M. Schedler, P. Schönfeldt, M. Schuh, M. Schwarz, M. Siegel
Karlsruhe Institute of Technology, Karlsruhe, Germany

Abstract

We present the effects of the filling pattern structure in multi-bunch mode on the beam spectrum. This effects can be seen by all detectors whose resolution is better than the RF frequency, ranging from stripline and Schottky measurements to high resolution synchrotron radiation measurements. Our heterodyne measurements of the emitted coherent synchrotron radiation at 270 GHz reveal discrete frequency harmonics around the 100 000th revolution harmonic of ANKA, the synchrotron radiation facility in Karlsruhe, Germany. Significant effects of bunch spacing, gaps between bunch trains and variations in individual bunch currents on the emitted CSR spectrum are described by theory and supported by observations.

INTRODUCTION

With the approximation that the signal of every revolution in a storage ring is the same, the beam spectrum can be written as the convolution of an infinite pulse train, separated by the revolution time T_0, the filling pattern signal $s_\mathrm{F}(t)$ and the single pulse signal $s_\mathrm{p}(t)$ [1]:

$$s(t) = \text{Ш}_{\mathrm{T}_0}(t) * s_\mathrm{F}(t) * s_\mathrm{p}(t), \tag{1}$$

where $\text{Ш}_{\mathrm{T}_0}(t)$ denotes the Shah distribution [2]:

$$\text{Ш}_{\mathrm{T}_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\mathrm{T}_0)\,. \tag{2}$$

The filling pattern signal consists of dirac delta peaks at the position $k\mathrm{T}_\mathrm{RF}$ of the k-th bunch and height V_k corresponding to the bunch charge

$$s_\mathrm{F}(t) = \sum_{k=1}^{h} V_k\,\delta(t - k\mathrm{T}_\mathrm{RF}), \tag{3}$$

where h denotes the harmonic number ($h = 184$ at ANKA). In the frequency domain, the convolution transforms into a multiplication and the spectrum reads

$$S(f) = \frac{1}{\mathrm{T}_0}\text{Ш}_{\mathrm{f}_0}(f) \times \mathrm{DFT}\left\{s_\mathrm{F}(t), f\right\} \times s_\mathrm{p}(f), \tag{4}$$

where the finite sum of the filling pattern signal leads to a Discrete Fourier Transform (DFT):

$$\mathrm{DFT}\left\{s_\mathrm{F}(t), f\right\} \equiv \sum_{k=1}^{h} V_k\, e^{-i2\pi f k \mathrm{T}_\mathrm{RF}}. \tag{5}$$

* johannes.steinmann@kit.edu
† now at Paul Scherrer Institute, Villigen, Switzerland

Due to the properties of the DFT, the spectrum of the filling pattern signal is continuous and repeats indefinitely with period length of $f_{RF} = 1/T_{RF}$. The complete signal spectrum, however, comprises of the DFT multiplied by the Shah-distribution in frequency domain $\text{Ш}_{\mathrm{f}_0}(f)$ and the spectrum of the single pulse signal $s_\mathrm{p}(f)$.

While in time domain the convolution with the Shah distribution leads to a copy of the filling pattern signal at every revolution, in frequency domain the multiplication by the Shah distribution results in a sampling at multiples of the revolution frequency. This is known as a frequency comb.

The additional multiplication by the single pulse spectrum gives the envelope of the spectrum. For synchrotron radiation the pulse spectrum is the synchrotron radiation spectrum ranging up to the x-rays with all its features like coherent amplification for long wavelengths determined by the form factor of the bunch [3].

INFLUENCE OF DFT

The influences of the filling pattern on the beam spectrum can be seen by analyzing Eq. (5), but keeping in mind that the complete spectrum is sampled at the revolution frequency harmonics and scaled by the single pulse spectrum.

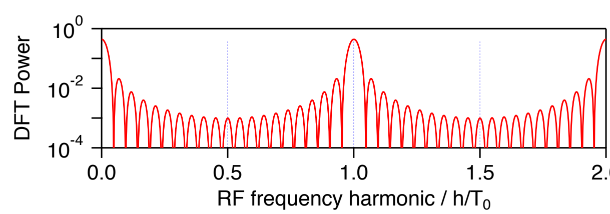

Figure 1: Discrete Fourier transformation of 20 consecutive pulses with the same intensity leads to a repeated sinc-function with zero crossings at every $h/20$ harmonic.

Bunches in a storage ring can be grouped in so-called trains with gaps in between. Figure 1 presents a scenario with a train of 20 bunches, each with the same current. The DFT itself is independent of h, as empty bunches do not contribute to the sum in Eq. (5). In this example the filling pattern function is a rectangular signal with a length of 20 buckets transforming to a sinc-function with zero crossings at every $h/20$ harmonic in frequency space. To get the spectrum, the DFT is sampled in frequency space at multiples of f_0/h.

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PARTICLE MOTION

In a real accelerator, the approximation that the filling pattern signal is the same for every revolution can be too optimistic. In reality, bunches perform for example transverse or longitudinal movement as well as coherent phase shifts due to beam-loading and higher order effects. However, when the storage ring is operated in user operation mode, where instabilities are avoided or damped down by a feedback system, particle motions are kept to a minimum. Especially in the case of synchrotron radiation, transverse motion has minor effects as most detectors, depending on the radiation transport line, are not sensitive to transverse position changes. The motion seen most by the detector is synchrotron oscillation, which affects the arrival time of the light pulses.

Taking the effect of coherent synchrotron motion into account, one finds for the detected spectrum [4, p. 264ff]

$$S(\omega) \propto \sum_{p,m=-\infty}^{\infty} j^{-m} J_m(p\omega_0\hat{\tau})\delta(\omega - p\omega_0 + m\omega_s), \quad (6)$$

where every revolution harmonic p has synchrotron frequency sidebands of order m. The m-th satellite has a spectral amplitude corresponding to the Bessel function of order m: $J_m(p\omega_0\hat{\tau})$ with $\hat{\tau}$ being the amplitude of the synchrotron motion seen by the detector and p the harmonic of the angular revolution frequency ω_0. Figure 2 shows the intensity

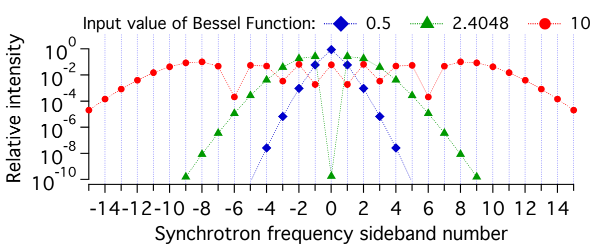

Figure 2: Synchrotron sidebands

of synchrotron frequency sidebands for three different input values of the Bessel function. Higher values lead to more visible sidebands. Due to zero crossings of the Bessel functions , also attenuated harmonics can be seen. The integrated power is constant since $\sum_{m=-\infty}^{\infty} J_m(p\omega_0\hat{\tau}) = 1$ (Eq. 8.512,1 in [5]), resulting in a leakage of the power from the main harmonic into the sidebands. As long as the detector resolution is broad enough to cover all detectable synchrotron sidebands, the measured power will be the same compared to the case without synchrotron oscillation.

Figure 3 shows how the input value of the Bessel functions in Eq. (6) leads to an increase of observable synchrotron sidebands. The number of the harmonic, where the power is below a fraction of 1000 (−30 dB) of the total power of the revolution frequency harmonic, increases almost linearly with the Bessel function's input. Observation in the sub-THz range, with undamped synchrotron oscillations in the range of some picoseconds, leads to input values of the Bessel

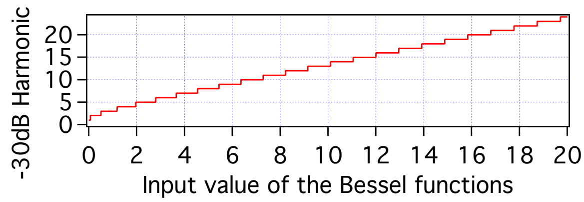

Figure 3: Input value of the Bessel function where the m-th f_s-harmonic is below −30 dB of the total power.

functions in the order of 1 to 10 (see following sections of this article).

STABILITY AND TUNING

The frequency stability of the frequency comb depends on the stability of the revolution frequency of the electrons. Due to phase-focussing in the RF cavities, the average revolution frequency has to be an integer factor (harmonic number h) of the RF frequency, determined by the length of the electron path. A change in RF frequency will result in a change of the particles energy and trajectory. The slightly different energy has minor effects on the low frequency radiation and the particle trajectory is limited by the size of the vacuum chamber. At ANKA, the frequency can be changed by ±10 kHz without major beam disturbances. As a result, above the 140th RF harmonic (≈70 GHz) the frequency comb could be shifted by more than one frequency tooth, thus covering the whole frequency range without gaps.

MEASUREMENTS

Measurements have been performed at the infrared beamline [6] at ANKA. The storage ring has a circumference of 110.4 m, which leads to a revolution frequency around 2.71 MHz. The RF frequency is 499.72 MHz and the harmonic number $h = 184$ at ANKA. To increase the power of the emitted THz radiation, the storage ring is operated in a short-pulse mode (low-alpha operation) [7]. In this mode instabilities lead to bursts of intense THz radiation at the cost of unstable synchrotron oscillation amplitudes and fluctuations in emitted power.

The synchrotron light is coupled out at the diagnostic port and focussed with an off-axis paraboloid mirror and a horn antenna into a waveguide of a VDI WR3.4SAX Schottky

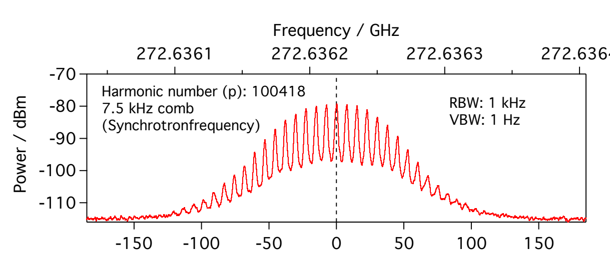

Figure 4: Averaged measurement of synchrotron frequency sidebands at the 100 418th revolution frequency harmonic.

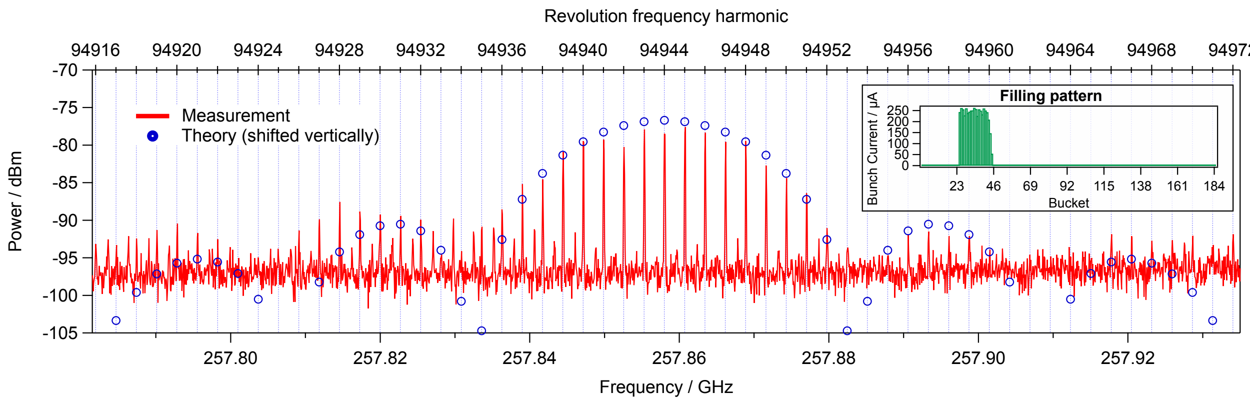

Figure 5: Measured and simulated synchrotron radiation spectrum around the 94944th revolution frequency harmonic (516th RF-harmonic) of ANKA. The filling pattern consists of 20 consecutive filled bunches (inlet). The calculated intensities have been normalized to the maximum of the measurement. The measured values match the predicted positions and amplitudes of the frequency comb. Visible peaks between revolution frequency harmonics are aliased artefacts from the double sideband mixing process.

mixer. More details about the used setup and data treatment can be found in [8]. Due to the changing synchrotron oscillation amplitude in the bursting regime, only an averaged spectrum has been taken by the use of a 1 Hz video bandwidth.

Figure 4 shows a measurement of synchrotron sidebands at the 100 418th revolution frequency harmonic. At the averaged spectrum, approximately 10 harmonics are above noise level with a 30 dB bandwidth of around 150 kHz.

A broader range has been recorded (Fig. 5) showing the discrete harmonics of the revolution frequency around the 94 944th revolution frequency harmonic, which is exactly the 516th harmonic of the RF frequency. The filling pattern consisted of a single train with 20 consecutive bunches.

The calculated intensities consider also the individual heights of the bunches, compared to Fig. 1 where equal bunch currents were assumed. However, no beamloading and particle motion effects have been considered as they were expected to be negligible at 1.3 GeV beam energy and bunch currents below 250 µA. The measured spectrum and the calculated position and amplitude show a good correlation, bearing in mind the approximations.

SUMMARY

We have presented an easy-to-use equation that allows the calculation of the frequency comb spectrum, within the made assumptions, at least up to the THz regime. The frequency comb created in a storage ring can be changed by adjusting the filling pattern. Specific harmonics can be amplified or damped and the exact frequency can be set by changing the RF frequency while the stability of the frequency comb is dependent on the RF generator that drives the accelerator cavities.

We have performed measurements with a heterodyne Schottky mixer around 270 GHz, directly observing the emitted synchrotron radiation spectrum. The observed spectrum supported our theoretical predictions.

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